



2014 Qualifying Test Round Two Part Two [Answers in brackets in red]

1. How many real solutions are there to the equation  $\cos x = \ln x$  ? [1]
2. What is the distance between the foci of the hyperbola  $y = \frac{1}{x}$  ? [4]
3. The general solution to the cubic equation was published in 1545 in *Ars Magna*, which was authored by which mathematician?
  - a. Bombelli
  - b. Cardano
  - c. Descartes
  - d. Pascal
4. How many real solutions are there in the interval  $[0, 2\pi]$  to the equation  $2 \sin^2 \theta + 2 \sin \theta = 1$ ? [2]
5. Which of the following is the negation of the statement "P implies (Q and R)"?
  - a. P and (Q implies not R)
  - b. P and not (Q or R)
  - c. (Not P) and (Q implies not R)
  - d. (Not P) and (Q implies R)
6. In a group of 20 randomly chosen people, what is the probability that exactly one person has the same birthday as you? (Assume 365 days in a year, and that birthdays are distributed uniformly.)  
 [20\*364^19/(365^20)] (also assume you're not in the group)
7. An ice cream cone shape is formed by gluing the base of a cone with radius 2 cm and height 3cm to the base of a hemisphere with the same radius. What is the total volume of the ice cream cone (in cubic cm)? [28pi/3]
8. A set of  $n + 1$  numbers is chosen from the set  $\{1, 2, 3, \dots, 2n\}$  where  $n > 1$  and the greatest common factor (greatest common divisor) of each pair of numbers from the chosen set is determined. What is the minimum value of these greatest common factors (greatest common divisors)?
  - a. 1
  - b. 2
  - c.  $\left\lfloor \frac{n}{2} \right\rfloor$  (the greatest integer part of  $\frac{n}{2}$ )
  - d. It depends on  $n$  and the chosen set.
9. How many integers larger than 500 (in base ten) consist of only distinct even digits? [216]
10. The rhombus (a quadrilateral with four congruent sides) pictured has sides of length 1 and a diagonal of length 1. What is the radius of the inscribed circle? [sqrt(3)/4]

