

2020 Who Wants to Be a Mathematician Round Two Qualifying Test

KEY

- For how many prime numbers $p < 300$ is $p^3 - 2p^2$ a positive perfect square (square of a positive integer)? 4
- Compute $\cos\left(\tan^{-1}\left(\frac{-\sqrt{7}}{3}\right)\right)$ (where \tan^{-1} denotes the inverse tangent function). 3/4
- What proportion of the nine-digit numbers that can be formed by permuting the digits of 123456789 are divisible by 36? (Write your answer as a fraction.) 2/9
- Let $f(x) = (2x + 3)^3$ and $g(x) = x^3 + x^2 - x - 1$. What is the sum of the coefficients of the polynomial $f(g(x))$? 27
- The *incircle* (or *inscribed circle*) of a triangle is the circle that is tangent to all three sides of the triangle. What is the radius of the incircle of a triangle with side lengths 13, 14, and 15? 4
- The integers 3, 4, 5, 6, 12, and 13 are arranged, without repetition, in a horizontal row so that the sum of any two numbers in adjacent positions is a perfect square (square of a positive integer). What is the sum of the first and last numbers in the arrangement? 11
- Which of the following numbers is the product of three consecutive prime numbers? (circle one)
a. 1223 b. 1309 c. 1989 d. 2431 e. 2717 **d.**
- What is the sum of all real solutions of $\sqrt{x + 15 - 8\sqrt{x - 1}} = 2$? 42
- A jar contains ten balls, numbered 1 to 10. Three balls are randomly drawn from the jar without replacement. What is the probability that no two of the three balls are labeled with consecutive integers? 7/15
- (Tie-breaking question) What positive integer is closest to $(e + 2\pi)^{9/2}$? 19,697