PROPOSED SYMBOLS FOR THE MODIFIED COSINE AND EXPONENTIAL INTEGRALS

By S. A. SCHELKUNOFF (*Bell Telephone Laboratories*)

The standard sine and cosine integrals are defined as follows

$$\mathrm{Si}\, x = \int_0^x \frac{\sin t}{t}\, dt, \qquad \mathrm{Ci}\, x = \int_\infty^x \frac{\cos t}{t}\, dt.$$

The cosine integral has a logarithmic singularity at $x=0$. Now in problems of electromagnetic radiation x is proportional to the frequency but the impedance functions involving Ci x are free from logarithmic singularities at $f=0$. Thus one expects and actually encounters logarithmic functions which cancel the singular parts of the cosine integrals.

For this reason the more suitable function is the following modified cosine integral

$$\mathrm{Cin}\, x = \int_0^x \frac{1-\cos t}{t}\, dt$$

which is an entire function. This function has already been used quite frequently, and we wish only to suggest that a standard notation be adopted for it.

Inasmuch as one is frequently interested in the analytic properties of impedance functions over the entire oscillation constant plane, the following modified exponential integral is suggested

$$\mathrm{Ein}\, z = \int_0^z \frac{1-e^{-w}}{w}\, dw.$$

The independent variable z will be proportional to $p=\xi+i\omega$ where $\omega=2\pi$ times the frequency. Then, on the imaginary axis we have

$$\mathrm{Ein}\,(iy) = \mathrm{Cin}\, y + i\, \mathrm{Si}\, y,$$

where y is proportional to the frequency.

The even part of Ein z may be designated as Cinh z and the odd part Sih z.