

QUARTERLY

OF

APPLIED MATHEMATICS

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BOOK REVIEWS

Applied mathematics for engineers and physicists. By Louis A. Pipes. McGraw-Hill Book Co., Inc. New York and London, 1946. xiii+618 pp. \$5.50.

The mathematical techniques which a beginning graduate student in Engineering should master are outlined and exemplified in this book. The material on such subjects as Laplace transforms, matrices, finite differences, and conformal mapping, is quite complete and occupies a large percentage of the portion of the book which deals with specific problems. An adequate section on special functions is given and the classical equations of mathematical physics are discussed. There appear to be, in fact, only two unfortunate omissions. The calculus of variations is almost entirely neglected and the absence of Sturm-Liouville theory (and hence a discussion of the special functions arising from the wave equation) makes the book definitely less interesting to physicists than to engineers. It should, however, provide an excellent text for a first graduate course in the techniques of applied mathematics.

G. F. CARRIER

Fundamental theory of servomechanisms. By LeRoy A. McColl. D. Van Nostrand Co., Inc., New York, 1945. XVII+130 pp. \$2.25.

This book beautifully fulfills its announced purpose of discussing the fundamental theory of servomechanisms. The author succeeds simultaneously in presenting a clear discussion of servomechanisms and in giving the reader a vivid picture of the philosophy of the control problem.

The body of the book consists of eleven chapters, an appendix, and a bibliography. A foreword written by Dr. Warren Weaver sets the stage for the discussion. For the most part, the author discusses linear systems and accordingly makes efficient use of the concepts developed in studying feed-back amplifiers. This point of view combines operational methods with certain straightforward ideas from the theory of complex variables to describe the performance of control system.

Starting with a very simple type of control mechanism, the author builds up the phenomenological and theoretical aspects of the problem, giving in the seventh chapter a detailed analysis of a particular system. The remaining four chapters discuss more specialized topics in linear systems. The appendix discusses some aspects of non-linearity and presents a study of a particular on-off servomechanism.

J. A. KRUMHANSL

Tables of Fractional Powers. Prepared by the Mathematical Tables Project conducted under the sponsorship of the National Bureau of Standards. Official Sponsor: Lyman J. Briggs. Project Director: Arnold N. Lowan. Columbia University Press, New York, 1946. xxx+486 pp. \$7.50.

The first part of this useful volume contains tables of the values of A^x for $A=2(1)9$, $x=[0.001(.001)0.01(.01)0.99; 15D]$; $A=10$, $x=[0.001(.001)1.000; 15D]$; $A=\pi$, $x=(0.001(.001)1.000; 15D, 15S]$ and $\pm x=1/4, 1/3, 1/2, 2/3, 3/4$ and $1(1)12$; $A=[0.01(.01)0.99]$, $x=[0.001(.001)0.01(.01)0.99, 15D]$ and $A=10^{-3}P$ where P is a prime number between 100 and 1000, $x=[0.001(.001)0.01(.01)0.99; 15D]$. The second part contains tables of the values of x^a for $\pm a=1/4, 1/2, 3/4$, $x=[0(.01)9.99; 15D]$; $\pm a=1/3, 2/3$, $x=[0(.01)10; 15D]$ and $a=[0.01(.01)0.99]$, $x=[0(.01)0.99; 7D]$. In the Foreword, F. Bernstein discusses problems the solution of which is facilitated by the use of these tables.

W. PRAGER