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CORRECTIONS TO MY PAPER

## ON AN EXTENSION OF THE VON KÁRMÁN-TSIEN METHOD TO TWO-DIMENSIONAL SUBSONIC FLOW WITH CIRCULATION AROUND CLOSED PROFILES*

QUARTERLY OF APPLIED MATHEMATICS, 4, 291–297 (1946)

BY C. C. LIN (*Brown University*)

Page 292, line 3: for "*is now an a parallel footing*" read "*is now on a parallel footing.*"

Page 293, line 8: for "*in the region exterior to*" read "*in the region $R_0$ exterior to.*"

Page 293, line 9: for "*such that $R_0$*" read "*such that.*"

* Received Jan. 20, 1947.

CORRECTIONS TO MY PAPER

## THE ANALOGY BETWEEN MULTIPLY-CONNECTED SLICES AND SLABS*

QUARTERLY OF APPLIED MATHEMATICS, 3, 279–290 (1946)

BY RAYMOND D. MINDLIN (*Columbia University*)

Equations (2.4), (2.6), (6.2), (6.3): replace $(1+\nu_1)$ by $E_1$ in the coefficient of $\nabla^2 T$.

Equation (2.6): replace $\alpha$ by $\alpha_1$.

Equation (6.2) is obtained from (6.1) and (2.4) and not from (6.1) and (2.6).

The reference to Biot's analogies at the end of Sec. 4 should read "analogies . . . between gravity loading and boundary pressures and dislocations and between thermal loading and dislocations."

The integration by parts following Eq. (4.6) is incomplete with the result that the following corrections should be made:

Equations (4.9), (4.10), (7.5), (7.15): multiply the last integral by 2.

Equations (7.13), (7.20): replace $[(1-\nu_1)(1-\nu_2)-1]$ by $[(1-\nu_1)(1-\nu_2)-2]$.

* Received April 22, 1947.

## BOOK REVIEWS

*Mathematical Theory of Elasticity*. By I. S. Sokolnikoff with the collaboration of R. D. Specht. McGraw-Hill Book Co., New York and London, 1946. xi+373 pp. $4.50.

There are five chapters in this book, three of them dealing with the fundamentals of the theory of elasticity, one dealing with the torsion and flexure of homogeneous beams, and the last chapter dealing with variational methods in elasticity.

In the chapter on analysis of strain we find a careful discussion of the properties of infinitesimal strain including Cesaro's proof for the *sufficiency* of the six equations of compatibility. A brief section is added on finite strain, showing the difference between the Eulerian and Lagrangian approach and giving selected references to contempory work. To this reader it seems regrettable that no mention is made

of the early work by Kirchhoff, Boussinesq and the Cosserats on this subject and no indication given with regard to the nature of the progress which has been made since then.

There follows a chapter on analysis of stress, containing all the essential facts on this subject. The third chapter is devoted to the subject of stress-strain relations. Among the topics discussed are the generalized Hooke's law together with a description of types of elastic symmetry; various forms of the complete system of differential equations of the linear theory for the isotropic body; the strain energy function including thermodynamic considerations often not included in elasticity texts; Kirchhoff's uniqueness proof for the solutions of the linear theory; and finally Saint Venant's principle.

In the foregoing three chapters the author uses his own distinctive symbols for stresses and strains, and tensor notation to the extent of writing for instance the three differential equations for the stresses in the form $\tau_{ij,j}+F_i=0$. No use is made of results of the theory of tensors and in order to study specific problems the author restates the fundamental equations of the theory in the notation which is customary in the American literature.

Nearly one-half of the book is devoted to the theory of torsion and flexure, mainly on the basis of Saint Venant's theory. The classical solutions of the torsion problem for the ellipse, rectangle, equilateral and isosceles right triangle are derived in the usual way. About twenty pages are devoted to an introduction of complex-variable theory, as a preparation to Mushelisvili's method for the boundary problem for the two-dimensional Laplace equation. As applications of this method one finds solutions of the torsion problem for the cardioid, for a loop of the lemniscate, for the section bounded by two circular arcs and for the inverse of an ellipse. There follows a discussion of the membrane analogy, a section giving references to experimental work, and a section on the torsion problem for multiply connected cross sections, with the example of Bredt's formula for the thin walled section without partitions.

In order to treat the Michell theory of torsion of shafts with variable circular cross section there is included a section on orthogonal curvilinear coordinates (without the use of tensor notation). As an example the solution for the conical shaft is given.

After this there is presented the Bessel-function solution for the torsion of the circular cylinder twisted by load distributions other than that of the Saint Venant theory, and also the relation between the torsion problem for the nonisotropic beam and for the isotropic beam.

There follows a treatment of the flexure problem with emphasis on the importance of leaving arbitrary the coordinate system in the cross section instead of choosing the system of principal axes through the centroid of the section. The *center of flexure* is defined as the load point giving zero mean local twist. Here one misses mention of the possibility of alternate definitions of this point and of some of the recent literature on the subject. A discussion of the center of *twist* would have been welcome also.

Explicit solutions of the flexure problem are presented for beams which have as cross sections an ellipse, a rectangle, an equilateral triangle, the section bounded by two concentric circles, and the cardioid.

It is somewhat of an anticlimax to have the work of this chapter conclude with a treatment of the elementary ("technical") theory of bending of beams with straight axis, including the solution of some very simple examples.

In the last section we find references to special extensions of the theory of torsion and flexure. Some topics are omitted such as the work on torsion with variable twist of thin walled sections, or of the problem of the effective width of I, T and box beam flanges, or of the effect of transverse shear deformation on the bending of beams.

The last chapter of the book is devoted to a study of variational methods. Following the example of Trefftz in the "Handbuch der Physik" there are given careful statements of the theorem of minimum potential energy (assuming infinitesimal strains and Hooke's law), and of the corresponding minimum theorem for the stresses. In somewhat unusual order the principle of virtual work is derived from the principle of minimum potential energy and stated (without qualification!) to be equivalent to the latter. There follows a proof of the reciprocity theorem; an introduction to the direct methods of the calculus of variations with the example of the twisted bar of rectangular cross-section; and a study of error estimates for the direct methods.

The final section of the chapter on variational methods describes the method of finite differences, including some recent developments, as applied to the two-dimensional Laplace equation.

Within the limits which the authors have set themselves—limits which to the reviewer appear to be somewhat narrower than the title of the book indicates—their work can be said to represent an interesting addition to the literature on the theory of elasticity.

E. REISSNER

*An index of mathematical tables.* By A. Fletcher, J. C. P. Miller and L. Rosenhead, McGraw-Hill Book Company Inc., New York and Scientific Computing Service Ltd., London, 1946. viii+450 pp. $16.00.

The need of an extensive index of available mathematical tables has long been felt by workers in applied mathematics. The usefulness of such an index is felt, not only in making possible the numerical solution of a mathematical problem by the use of existing tables but also in influencing the choice between various mathematical formulations of a problem to take advantage of such existing tables. Moreover, as pointed out in the preface, the index will be useful also to makers of mathematical tables, in showing what has been done, thus avoiding duplication and stimulating work in the direction of filling the main gaps.

The work is divided into two parts. Part I contains an index according to functions. Part II is a bibliography listed alphabetically by authors. An alphabetic index to Part I is also included. While Part I is the cardinal feature of the book, Part II gives a concise reference list of books and papers containing tables or dealing with such things as errors, in particular tables or the bibliography of tables.

Part I indicates references, cross-references, scope of these references, and eventual errors on such material as—arithmetical tables, mathematical constants, tables of algebraic and elementary functions, tables of the higher functions, tables of integrals, tables of solution of transcendental equations, tables for numerical computations by differences, etc.

In appraising the task undertaken by the authors, one should consider the widely scattered character of the information collected, the enormous quantity of material accumulated, its duplication and its eventual reliability and finally, the fact that the authors did not confine themselves to mere indexing but have indicated in a great many cases the errors involved and accomplished a selection by discarding material of mere historical value.

The preface states—"I hope that this index will be a useful servant to the many scientists the world over who are working to build up a better civilization. To them this work is respectfully dedicated." The international character of the accomplishment will not escape those scientific workers who have tried to obtain copies of material which is scarce or available only in a geographically remote location.

It is essential to continue work along the lines of this index in keeping it up-to-date and also perhaps to facilitate speedy access to the material which might be urgently needed by the reader and might otherwise be difficult to obtain. Encouragement and help in accomplishing these further aims seem to fall particularly well within the responsibility of the newly created United Nations Educational Scientific and Cultural Organization.

M. A. Biot

*Contributions to the study of oscillatory times series.* By M. G. Kendall. Cambridge, at the University Press; New York, The Macmillan Company, 1946. 67 pp. $1.75.

This work is an outline of the application of certain common schemes of statistical analysis to a few sequences of the type encountered in studies of economic time series. The viewpoint is that of the statistician.

The book is divided into seven chapters. The first is concerned with the qualitative discussion of methods to be employed, and with the construction of three time series from random sampling numbers according to linear autoregression equations of the second order. Chapters two through six are concerned with the analysis of these series by the correlogram, the periodogram and the variate difference schemes. Chapter seven contains the author's summary, in which he concludes, as one might suspect, that schemes of analysis not based on considerations of the generating processes give rise to meaningless results. An appendix contains tables for harmonic analysis.

As an account of inadequacies of the methods discussed, the paper will interest statisticians, but mathematicians and physicists who have been interested in the general time series problem in the broader sense will notice the lack of reference to a large body of the work done in this field.

J. Krumhansl