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the rectangle is longer than the other. It may be of interest to list the following special cases. Denoting the factor in brackets by $1 + \delta$, we find that when

$$a \ll b, \qquad 1 + \delta \sim 1 + 2(a/R)^2 - 3(a/R)^4, \tag{25a}$$

and when

$$b \ll a, \qquad 1 + \delta \sim 1 - 2(a/R)^2 + (a/R)^4. \tag{25b}$$

It may be noted when $a \ll b$ then δ assumes a maximum value of $5/27$ for $(a/R)^2 = 1/3$, whereas when $b \ll a$ then δ is always negative.

CORRECTIONS* TO THE PAPER

ON A CLASS OF SINGULAR INTEGRAL EQUATIONS OCCURRING IN PHYSICS

QUARTERLY OF APPLIED MATHEMATICS *6*, 443-448 (1949)

BY H. P. THIELMAN (*Iowa State College*)

The limits on the integral in Eq. (B), p. 445 have been omitted. They should have been indicated as 0 and ∞.

Equation (a) of Theorem I, p. 445 should read $kf(0) - f'(0) = 0$ and not $kf(0) - f''(0) = 0$ as stated. It should have been stated that $f''(x)$ in Theorem I, and $f^{iv}(x)$ in Theorem II are assumed to be of order $o(e^{kx})$ as x goes to infinity.

*Received June 6, 1949.

BOOK REVIEWS

Proceedings of a symposium on large-scale digital calculating machinery. Jointly sponsored by the Navy Department Bureau of Ordnance and Harvard University at the Computation Laboratory. Harvard University Press, Cambridge, Massachusetts, 1948. xxix + 302 pp. $10.00.

This is a collection of papers and discussions of papers presented at a symposium on large-scale digital computing machinery held at Harvard University on January 7-10, 1947. The meeting was sponsored jointly by the Navy Department Bureau of Ordnance and Harvard University. The book contains numerous photographs and drawings. The technical addresses covered eight sessions dealing with the general topics of "Existing Calculating Machines", "The Logic of Large Scale Calculating Machinery", "Storage Devices", "Numerical Methods and Suggested Problems for Solution", "Sequencing, Coding and Problem Preparation", "Input and Output Devices", "Conclusions and Open Discussion". The state of the art seems to have been well surveyed.

ROHN TRUELL

Physics in industry: The measurement of stress and strain in solids. Institute of Physics, London, 1948. x + 114 pp. $4.00.

This small book is based on the Proceedings of a Conference held in July 1946 by the Manchester and District Branch of the Institute of Physics. The topics covered include wire-resistance and other electrical strain gages and instrumentation, acoustic gages, two and three dimensional photoelasticity, and X-ray diffraction. Most of the papers are mainly summaries of the highlights of existing information and so are of more interest to British than to American workers who have the Proceedings of the Society for Experimental Stress Analysis and several books on some phases of the field available. However, there are a number of fundamental points concerning the physical behavior of wire-resistance gages which are raised and answered by Mr. Eric Jones in his very interesting and important contribution which should be read by all. A few of the other features which have not received much attention here are high frequency strain gages, the acoustic gage, and the precision setting of distance from film to specimen instead of using a reference metal in X-ray back reflection.

D. C. DRUCKER

Heat conduction with engineering and geological applications. By L. R. Ingersoll, O. J Zobel, and A. C. Ingersoll. McGraw-Hill Book Company, Inc., New York and London, 1948. xii + 278 pp. $4.00.

The subject of heat conduction is treated in two ways in various texts. As part of the general subject of heat transfer, heat conduction is treated from a practical point of view in various books on engineering applications of heat transfer. The mathematical theory, on the other hand, has been treated extensively and rigorously in several good texts. The intermediate region which gives sufficient mathematical background for a real understanding of the problems involved and yet a sufficient discussion of practical applications so as to make obvious the mode of application of the theory has been occupied by the book by Ingersoll and Zobel for many years.

The present work is a thorough revision of the older text with further engineering and geological applications. No attempt is made to give a rigorous mathematical foundation for the subject. On the other hand, several of the more advanced methods of solution of steady and non-steady heat flow problems are presented in a form which makes their ready use possible. Problems of increasing difficulty are treated through the book in the following order: Steady-state one-dimension, steady-state more than one dimension, periodic heat flow in one dimension, general non-steady heat flow in one dimension, and general non-steady heat flow in more than one dimension. A chapter is devoted to the classical treatment of the more complex problem of the two conducting media with phase change at the interface as in the formation of ice. Two final chapters deal with numerical and graphical methods for solving various types of heat conduction problems and a brief discussion of the methods of measuring the various thermal properties. Although the major applications are taken from geological problems, the book is a good one regardless of the field of application in which the reader is interested. The authors state as their aims, ". . . to develop the subject with special reference to the needs of the student who has neither time nor mathematical preparation to pursue the studies at greater length . . ." and ". . . to point out the many applications of which the results are susceptible. . ." The book succeeds admirably in living up to these stated purposes and is well worth consideration by anyone interested in applications of heat conduction.

H. W. EMMONS

Applied mathematics for engineers and scientists. By S. A. Schelkunoff. D. Van Nostrand Co., Inc., Toronto, New York and London, 1948. xi + 472 pp. $6.50.

This latest addition to the rapidly increasing number of books on "Applied Mathematics for the Engineer, etc.", seems particularly appropriate for those scientists whose training has not been that of a mathematician. The more elementary material includes a presentation of the notions of differentiation,

integration, and the algebra of complex numbers and vectors. The former topics are not treated at great length but no knowledge of the calculus is taken for granted. Chapters are devoted to interpolation, solution of algebraic and transcendental equations, power series, and the introduction of exponential and related functions. For reference use a chapter on coordinate systems (listing the conventional operators in these systems) is included as well as chapters on Bessel functions, Fresnel integrals, and other transcendental functions. An extensive chapter on second order ordinary differential equations contains methods of solution ranging from the most elementary to those for obtaining asymptotic developments. The material on partial differential equations stresses the product-series solution representation. Conformal mapping (largely a discussion of certain explicit mappings), the use of the Laplace transform, and contour integration are treated, and the book ends with the derivation, from many physical problems, of the appropriate differential equations. Nearly all procedures (from integration to approximation techniques) are related to physical processes. Those analogs are usually electrical in character and the non-electrically trained may well prefer the mathematical method *per se*.

Each new topic is introduced in an extremely simple and clear manner at the minor expense, for example, of scattering bits of complex variable theory over several chapters. Much of the important material in the topics given above has been presented, and the book should be valuable to those interested in both introductory material and the conventional techniques applicable to many physical problems.

G. F. CARRIER

Computation curves for compressible fluid problems. By. C. L. Dailey and F. C. Wood. John Wiley & Sons, Inc., New York and Chapman & Hall, Ltd., London. x + 33 pp. $2.00.

This set of computation curves is intended as a supplement to the text of Liepmann and Puckett "Introduction to Aerodynamics of a Compressible Fluid". Following a brief exposition concerning the equations used, there are three sets of charts which are frequently useful in speeding up computational work. The first set of graphs are plots of the conventional point functions of the Mach Number (e.g. Pressure/stagnation pressure v.s. Mach No., etc.). for example, (1) the shock angle, and (2) the stagnation pressure ratio, are plotted as functions of incoming Mach number and deflection angle. In section 3 the corresponding information on conical shocks is given. In the latter section, γ is taken as 1.405. In the others $\gamma = 1.4$.

G. F. CARRIER

Tables of the Bessel functions of the first kind of orders forty through fifty-one. By The Staff Of The Computation Laboratory. Harvard University Press, Cambridge, 1948. 620 pp. $10.00.

The Bessel Functions $J_{40}(x)$, $J_{41}(x)$, $\cdots$, $J_{51}(x)$, are tabulated to ten decimal places. The argument varies in steps of .01 for $0 \leq x < 100$.

G. F. CARRIER

Contributions to applied mechanics (Reissner Anniversary Volume). Edited by The Staff Of The Department Of Aeronautical Engineering and Applied Mechanics Of The Polytechnic Institute of Brooklyn. J. W. Edwards, Ann Arbor, Michigan, 1949. viii + 493 pp. $6.50.

This testimonial to the great contributions of Professor H. Reissner to aeronautical and structural engineering, applied mathematics, mechanics, and to physics contains excellent original and expository papers by many of his eminent colleagues and friends. Lack of space prevents the inclusion of even the

titles of all the papers and permits only the listing of the authors under the general classification of their subject. AERODYNAMICS: S. Bergman, R. Paul Harrington and Paul A. Libby, Henry G. Lew, D. P. Riabouchinsky, Walter Tollmien. DYNAMICS: Martin Goland, Paul Lieber and M. E. Hamilton, Rufus Oldenberger, D. Williams, S. W. Yuan and M. Morduchow. ELASTICITY AND STRUCTURES: L. H. Donnell, K. O. Friedrichs, R. Gran Olsson, Eric Reissner, A. Schleusner, George Schnadel, J. J. Stoker, N. J. Hoff, V. L. Salerno, Harold Liebowitz, Bruno A. Boley, Sebastian V. Nardo. ELECTRICITY: Ronald M. Foster, Reinhold Rüdenberg. MATHEMATICAL METHODS: Hilda Geiringer, R. Grammel, Alexander Weinstein. PLASTICITY: R. v. Mises, A. Nadai, Folke K. G. Odqvist. PROPULSION: Theodore von Kármán, Paul Torda.

D. C. DRUCKER

Tables of generalized sine- and cosine-integral functions: Part I. By The Staff Of The Computation Laboratory. Harvard University Press, Cambridge, 1949. vii + 462 pp. $10.00.

The twenty-eight introductory pages of this volume are concerned with the definitions of the functions to be tabulated, a discussion of the computation methods, the interpolation technique, a brief application list with bibliography, and tables of certain coefficients used in computing the tables which comprise the remainder of the book. Six functions are computed: They are $\int_0^x F^n(a, s)ds$ where the six integrands in question are $u^{-1} \sin u$, $u^{-1}(1 - \cos u)$, $u^{-1} \sin u \sin x$, $u^{-1} \sin u \cos x$, $u^{-1} \cos u \sin x$, $u^{-1} \cos u(1 - \cos x)$, with $u(x, a) = (x^2 + a^2)^{1/2}$.

These functions are tabulated in the range $0 \leq a \leq 1$ in steps of .01 for $0 \leq x \leq 1$ with increments $\Delta x = .01$. When a is a multiple of .05, they are also given for $1 \leq x \leq 2$ with $\Delta x = .02$ and $2 < x \leq 5$ with $\Delta x = .05$. For certain other values of a the tables for $1 < x < 2$ are also included. For $1 < a < 2$, the increments are in general larger but not uniform and values of the functions are available for $0 \leq x \leq 25$ when a is a multiple of .1.

G. F. CARRIER

Tables of generalized sine- and cosine-integral functions: Part II. By The Staff Of The Computation Laboratory. Harvard University Press, Cambridge, 1949. 560 pp. $10.00.

This volume is a continuation of the tables described in the foregoing review. The range of a is $2 \leq a \leq 25$, $0 \leq x \leq 25$. The increments in a are: .05 for $a < 5$, .1 for $5 < a < 10$, .2 for $a > 10$. The increments in x are also: .05 for $x < 5$, .1 for $x < 10$, .2 for $x < 25$. However, the tables run to $x = 25$ only for intermittent values of a. For some values of a they terminate at $x = 5$, for others at $x = 10$.

G. F. CARRIER

Vectorial mechanics. By E. A. Milne. Interscience Publishers Inc., New York, 1948. xiii + 382 pp. $7.50.

This text furnishes a thorough and unified treatment of mechanics by Gibbs' vector and dyad analysis. The author's decision to use only this tool, limits the scope of the text to some extent. Thus, the following topics are omitted: (1) the Lagrange generalized coordinates; (2) variational method; (3) the integration of the rotational equations of motion of a rigid body in terms of the Eulerian angles, which are defined in the text. Further, the author's very complete introduction to Gibbs' vector analysis leads to the use of such uncommon terms as, "tensor of a vector", "vector of a tensor". However, these disadvantages are more than compensated for by the author's clear and unified treatment of topics.

In particular, the following discussions should be of interest to the reader: (1) a vector technique for solving linear vector differential equations with constant coefficients; (2) an elegant vector treatment of Euler's theorem on rotations about a point in a rigid body; (3) a simple and interesting vector decomposition proof of the triple vector expansion; (4) a very general tensor formulation of the Gauss and Stokes integral theorems. The author's treatment of the last mentioned topic is facilitated by introducing the index notation and the basic concepts of tensor algebra.

The following topics are discussed in the order listed; (1) vector and tensor analysis; (2) systems of line vectors with applications to statics and small displacements of a rigid body; (3) kinematics and dynamics of a particle and a rigid body. The material of the text is fully illustrated by numerous problems. Many of these problems are of intrinsic interest and are completely solved by the author. In the general theory and the problems, the author's methods illustrate the power of the vector approach to mechanics.

In short, the author has written an excellent text, which even the expert may profitably peruse.

N. COBURN

Advances in applied mechanics. Edited by Richard von Mises and Theodore von Kármán. Academic Press Inc., New York, 1948. viii + 293 pp. $6.80.

This book is the first of a series intended to present the results of current research in various fields of Applied Mechanics in the form of collections of expository monographs and summaries. The parts of this volume are described separately.

Hugh L. Dryden, "Recent advances in the mechanics of boundary layer flow". The subject is discussed with particular reference to the problem of the stability of a laminar boundary layer and to the nature of turbulent flow in boundary layers. The nature of the laminar boundary layer and the effects of boundary layer suction are also discussed. An extensive bibliography is presented.

N. Minorsky, "Modern trends in non-linear mechanics". After a brief history of the development of non-linear mechanics the modern approach is presented. The first section deals with topological methods and gives the results obtained and the inherent limitations of this attack. The second section treats the analytical methods of Poincaré, van der Pol, Kryloff and Bogoliuboff, and others. The third section deals with non-linear resonance and associated phenomena.

C. B. Biezeno, "Survey of papers on elasticity published in Holland 1940-1946". As indicated by the title this contribution is in the nature of a review. The works discussed are diverse and lie in all of the subject fields of elasticity.

J. M. Burgers, "A mathematical model illustrating the theory of turbulence". A simplified mathematical model is devised with the intention of simulating the problem of turbulence in an incompressible fluid. The model does not have the same complicated geometrical character as the turbulence problem but retains the essential non-linearity and, in analogue, the characteristics of energy transfer and dissipation. From the mathematical treatment results analogous to those of the isotropic theory of turbulence are obtained and suggestions are made as to other characteristics of turbulent fluid motion.

H. Geiringer, "On numerical methods in wave interaction problems". The problem of calculating the unsteady one-dimensional flow fields in a perfect gas is investigated. The principal treatment is based on the Hugoniot shock relations; the approach of J. von Neumann is also presented.

R. von Mises and M. Schiffer, "On Bergman's integration method in two-dimensional compressible fluid flow". Part I presents the general theory of S. Bergman based upon the equation for the stream function in the hodograph plane. Part II presents the particular treatment where a simplified pressure-density relation is assumed.

This book and future volumes of the same series make possible the publication of papers of expository, review, or developmental nature in monograph length. This is particularly valuable because of the present dearth of monograph journals and the shortage of space in the regular journals in applied mechanics and mathematics. The future of the venture will, of course, depend upon demand and upon the quality of profferred contributions.

To quote from the preface—"The present volume might be considered as indicative of the topics

to be dealt with in future issues and as an example for the diversified kinds of approach we wish to cultivate. In both respects, however, the Editors reserve a certain freedom of choice. Suggestions are invited and the offering of contributions will be appreciated."

WALLACE D. HAYES

Calcolo tensoriale e applicazioni. By Bruno Finzi and Maria Pastori. Nicola Zanichelli Editore, Bologna, 1949. vii + 427 pp. Lire 2000.

The authors' purpose, as stated in the introduction, is to present the concepts and principal methods of the tensor calculus, in order to facilitate its application by mathematicians, physicists, and engineers. The reader's minimum mathematical background is presupposed to be a (first course) knowledge of the differential and integral calculus. The exposition is careful and orderly, proceeding by easy stages; and the authors have indeed achieved their aim.

The book consists of an introduction, ten chapters, and a bibliography. The introduction is in the nature of an orientation; the reader is "briefed" on the ground to be covered and acquainted with the concepts that will occur again and again, e.g., vectors, tensors, contravariant superscripts, covariant subscripts, etc. A pleasant feature of the book is the table of principal formulas ("formule notevoli") which follows every chapter. The first three chapters deal with the tensor calculus proper: vector fields, tensors and tensor algebra, and "omografie vettoriali" (linear vector functions). The next four chapters are concerned with applications to differential geometry: tensor fields in Euclidean and non-Euclidean spaces, surface geometry and Riemannian geometry. The last three chapters contain the applications to mathematical physics referred to in the first paragraph of the introduction. The authors have succeeded in compressing an unusually large amount of material into comparatively few pages. The chapter headings are: VIII Mechanics of Deformable Continua, IX Electromagnetic Theory, and X Relativity Theory. The book concludes with a bibliography of representative works on various phases of the subject.

J. B. DIAZ

Supersonic flow and shock waves. By R. Courant and K. O. Friedrichs. Interscience Publishers, Inc., New York and London, 1948. xvi + 464 pp. $7.00.

This book is primarily concerned with the hyperbolic problems associated with the behavior of continua. Except for brief remarks about shallow water waves and discontinuous waves in plastic media, attention is confined to gas flows. The opening chapter provides the fundamental notions (thermodynamic concepts, and basic mechanical laws) which form the basis of the analysis of these phenomena. This is followed by a chapter on the mathematics of hyperbolic systems of second order. Unsteady one-dimensional problems are then considered; shock interaction, detonation, and plastic waves are discussed in this section. Chapter IV deals with steady plane flow; it utilizes the hodograph method, to present certain isentropic flows, and contains the analysis of shock interactions, and treats the flow past obstacles, etc. by the perturbation method. The book ends with briefer discussions of nozzle flows, jets, axially symmetric flows, spherical waves and certain conical flow problems.

The book seems well adapted for the presentation to students of that portion of compressible fluid theory which is essentially hyperbolic. It is obviously not intended as a self contained text on the general field of compressible flow theory.

G. F. CARRIER