The second theorem is stated and proved on page 91 of Seifert and Threlfall's *Variationsrechnung im Grossen.*

The third theorem is stated and proved on pages 42 and 43 of Lusternik and Schnirelmann's monograph *Méthodes Topologiques dans les problèmes variationnels,* Première Partie, Paris, Hermann et C^{ie}, 1934. (Actualités Scientifiques et Industrielles, No. 188.)

NOTE ON THE KERNEL $\exp(-|x-y|)$*

By HARRY POLLARD (*Cornell University*)

In a recent issue of this Quarterly a theorem was stated which, after some trivial changes and correction of a typographical error, reads as follows.[1] In order that the function $f(x)$ have the form

$$f(x) = \frac{1}{2}\int_0^\infty e^{-|x-y|} g(y)\, dy, \qquad x \geq 0, \tag{1}$$

it is necessary and sufficient that

$$f(0) = f'(0); \tag{a}$$

$$g(x) = f(x) - f''(x); \tag{b}$$

$$g(x) = 0(e^{dx}),\ x \to \infty, \text{ for some } d < 1. \tag{c}$$

Now the example $f(x) = e^x$ shows that the conditions are certainly not sufficient even if (c) is replaced by the condition

(c′) the integral in (1) exists,

which the author regards as equivalent to (c).

On the other hand the example $g(x) = e^x(x^2+1)^{-1}$ shows that the condition (c) is not necessary either. It also shows that (c) and (c′) are far from equivalent.

The following is a correct version of the theorem. The only difficulty is the discovery of a condition to replace (c). Once this is done the proof is straightforward, and is therefore omitted.

THEOREM. *In order that $f(x)$ have the form* (1) *where $g(y)$ is a prescribed function integrable on each finite interval it is necessary and sufficient that $f'(x)$ exists and is absolutely continuous on finite intervals, and moreover that*

$$f(0) = f'(0); \tag{a}$$

$$g(x) = f(x) - f''(x) \text{ for almost all } x > 0; \tag{b}$$

$$f'(x) = o(e^x),\ x \to \infty. \tag{c}$$

Remarks. (i) The condition (c) cannot be replaced by $O(e^x)$ as the example $f(x) = e^x$ shows. (ii) If the word "prescribed" and condition (b) are omitted simultaneously the theorem remains true.

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[1] H. P. Thielman, Q. Appl. Math. **6**, 443-448 (1949), Theorem I.