tentials of the individual particles. Upon substituting ϵ_1 for ϕ into (5.2), multiplying the resulting equation by $n\, d\tau$, and integrating over $d\tau$ at a constant time, we obtain, by the aid of (2.3), (2.6), (2.7), (2.11) to (2.15) and (3.1) to (3.3), and by considerations of symmetry,*

$$\begin{aligned} \frac{d}{dt}\int n\langle\epsilon_1\rangle\, d\tau + \int \sum_{i=1}^{3} \frac{\partial}{\partial q_i}\left(H_i + \sum_{j=1}^{3} S_{ij}u_j\right) d\tau + \int \sum_{i=1}^{3} \frac{\partial}{\partial q_i}\left\{\frac{n}{m}\langle p_i'\varphi_1^{(i)}\rangle\right\} d\tau \\ = \int \sum_{i=1}^{3} \frac{n}{m}\langle p_iF_i\rangle\, d\tau, \end{aligned} \tag{5.6}$$

where

$$\langle\epsilon_1\rangle = \frac{1}{2}\, m(u_1^2 + u_2^2 + u_3^2) + \varphi_1^{(e)} + \langle\varphi_1^{(i)}\rangle + \frac{3}{2}\, kT. \tag{5.7}$$

Upon transforming the second and third volume integrals on the left hand side of (5.6) into surface integrals, we see that this equation may be interpreted as stating that the time rate of increase of total energy of the system is equal to the rate at which the non-conservative body forces F_i and surface forces S_{ij} do work on the system, augmented by the rates of transport of thermal energy of disordered motion and of internal potential energy through its boundaries. The importance of this latter portion of the energy flux, arising from the strong intermolecular forces in the case of a liquid, has been pointed out by Born and Green.[11] This flux is not contained in the expression (2.15) for the components of the heat current.

*In the equation obtained by letting $\phi = \epsilon_1$, in (5.2), we interchange the phase coordinates of the representative particle with those of each of the other $N - 1$ particles in turn, and add the resulting equations. Then, by considerations of symmetry and by the aid of (2.3), (2.6), (2.7) and (2.11), $\int n\langle\epsilon_a\rangle d\tau = (N/\mathbf{N}) \int f\epsilon_a\, d\Omega_0 = (N/\mathbf{N}) \int f\epsilon_1\, d\Omega_0 = \int n\langle\epsilon_1\rangle\, d\tau$. A similar method of treatment applies to the other terms on the left hand side of the combined equation. The right hand side of this equation becomes $(N/\mathbf{N}) \int f(D\epsilon)\, d\Omega_0 = (N/\mathbf{N}) \int f\ \Sigma_{i=1}^{3N} (p_iF_i/m)\, d\Omega_0 = N \int\ \Sigma_{i=1}^{3} (m/n)\langle p_iF_i\rangle\, d\tau$, by the aid also of (3.2) and (3.3).

[11]M. Born and H. S. Green, Proc. Roy. Soc. London (A) **190**, 455 (1947).

NOTE ON THE HAMEL-SYNGE THEOREM*

By F. H. VAN DEN DUNGEN (*Université Libre de Bruxelles*)

The theorem given by Synge[1] for a plane motion of a compressible viscous fluid is easily extended to a three dimensional motion.

Consider a compressible viscous fluid which moves inside a fixed closed surface B, on which the velocity vanishes. Our theorem is: A velocity $\mathbf{v}(x, y, z)$ is consistent with the foregoing boundary condition if and only if

$$\int_V (\mathbf{A}\cdot\operatorname{curl}\mathbf{v} - f\operatorname{div}\mathbf{v})\, dx\, dy\, dz = 0, \tag{1}$$

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[1]Q. Appl. Math. **8**, 107-108 (1950)

where f is an arbitrary harmonic function and $\mathbf{A}$ a conjugate harmonic vector such that

$$\operatorname{grad} f + \operatorname{curl} \mathbf{A} = 0. \tag{2}$$

Proof. We have

$$\int_V (\mathbf{A} \cdot \operatorname{curl} \mathbf{v} - \mathbf{v} \cdot \operatorname{curl} \mathbf{A})\, dx\, dy\, dz = -\iint_S \mathbf{A} \cdot (\mathbf{v} \times \mathbf{n})\, dS$$

(here $\mathbf{n}$ is the normal unit vector) and

$$\int_V (f \operatorname{div} \mathbf{v} + \mathbf{v} \cdot \operatorname{grad} f)\, dx\, dy\, dz = \iint_R f\mathbf{v} \cdot \mathbf{n}\, dS,$$

and therefore it follows that

$$\iint_B (\mathbf{A} \cdot (\mathbf{v} \times \mathbf{n}) + f\mathbf{v} \cdot \mathbf{n})\, dS \tag{3}$$

vanishes if and only if Eq. (1) is satisfied. However, (3) will vanish for arbitrary harmonic f only if a continuous $\mathbf{v}$ vanishes everywhere on the surface B, and thus (1) is necessary and sufficient for the satisfaction of the viscous boundary condition.

NOTE ON ITERATIONS WITH CONVERGENCE OF HIGHER DEGREE*

By HASKELL B. CURRY (*Pennsylvania State College*)

In a recent paper[1] E. Bodewig has derived a general expression for a function $F(x)$ such that the sequence

$$x_{n+1} = F(x_n) \tag{A}$$

converges in a degree at least m in the neighborhood of every root of a polynomial $f(x)$, provided the latter has only simple roots. A part of this argument can be simplified, while another part can be made somewhat more natural.

In regard to the first point, the operator P used by Bodewig is the same as d/dy where $y = f(x)$. Further, since his r is $1/f'$, we have $r = dx/dy = Px$. Hence the identity between Bodewig's formula (14a) and the Euler inverse Taylor expansion (15) follows immediately; it is not necessary to use the recurrence formula for the higher derivatives of an inverse function.

In regard to the second point, if we have a sequence of functions $F_1(x)$, $F_2(x)$, $\cdots$ such that the sequence (A) converges in the degree at least m for $F = F_m$, then any $F(x)$ giving rise to a sequence converging at least in the degree m must be of the form

$$F = F_m + g_m f^m.$$

*Received August 29, 1950.

[1]E. Bodewig, *On types of convergence and on the behavior of approximations in the neighborhood of a multiple root of an equation*, Q. Appl. Math. **7**, 325-333 (1949).