# —NOTES—

## THE FUNDAMENTAL THEOREM OF ELECTRICAL NETWORKS*

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I wish to thank Messrs. L. C. Robbins, Jr. and W. K. Saunders for drawing my attention to what may appear to be a flaw in the reasoning of the paper with the above title.[1] The proof there given is not convincing, and I take this opportunity to supply a fuller proof, in which the reasoning is a little delicate, but which seems to establish the result.

Consider the two propositions:

(A) $$\mathbf{e} = 0 \quad \text{implies} \quad \mathbf{i} = 0.$$

(B) $$\mathbf{Z}' \neq 0.$$

It is stated on p. 127 of the paper cited that (A) implies (B); that is the result we have to establish here.

We have the following equations:

(1) $$\mathbf{i} = \mathbf{C}\mathbf{i}', \qquad \mathbf{e}' = \mathbf{C}_t\mathbf{e}, \qquad \mathbf{e}' = \mathbf{Z}'\mathbf{i}'.$$

These equations embody all our knowledge about the behavior of the network, and any set of vectors $\mathbf{i}$, $\mathbf{i}'$, $\mathbf{e}$, $\mathbf{e}'$ consistent with them are to be regarded as possible. But we must bear in mind the definition of the mesh currents $\mathbf{i}'$ as branch currents (components of $\mathbf{i}$ in branches-out-of-tree); this means that the matrix $\mathbf{C}$ is such that

(2) $$\mathbf{i}' \neq 0 \text{ implies } \mathbf{i} \neq 0.$$

Choose $\mathbf{e} = 0$ and $\mathbf{e}' = 0$. Then (1) read

(3) $$\mathbf{i} = \mathbf{C}\mathbf{i}', \qquad 0 = 0, \qquad 0 = \mathbf{Z}'\mathbf{i}'.$$

Any vectors $\mathbf{i}$, $\mathbf{i}'$ satisfying these equations are possible.

Suppose that (B) is false, i.e. suppose $\mathbf{Z}' = 0$. Then there exists a non-zero $\mathbf{i}'$ satisfying the last of (3), and, by (2), the $\mathbf{i}$ given by the first of (3) is non-zero. Thus, on the assumption that (B) is false, we have a solution of (1) with $\mathbf{e} = 0$ and $\mathbf{i} \neq 0$. Hence, if (B) is false, (A) is false. Therefore if (A) is true, then (B) is true. In other words, (A) implies (B), which is the required result.

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[1]Q. Appl. Math. 9, 113-127 (1951).