where λ is Taylor's **micro**-scale of turbulence. However, this is actually incorrect. If we write (20) in the form

$$K = \int u n_x \, dS, \tag{22}$$

then it becomes evident that

$$\langle K^2 \rangle \sim \langle u^2 \rangle (lL)^2. \tag{23}$$

The ratio between the two estimates (21) and (22) is lL/λ^2. It is clear that the direct application of the volume formula over-estimates the integral because of the small scale introduced by the differentiation process. Such occurrences are frequent in the statistical theory of turbulence. For example, the divergence of the Reynolds stress $\tau_{ij} = -\rho \langle u_i u_j \rangle$ is

$$\frac{\partial \tau_{ij}}{\partial x_j} = \frac{\partial}{\partial x_j} \{ -\rho \langle u_i u_j \rangle \} = -\rho \left\langle u_j \frac{\partial u_i}{\partial x_j} \right\rangle.$$

Based on the left-side, the estimate is

$$\frac{\partial \tau_{ij}}{\partial x_j} \sim \frac{\tau}{l}$$

while the right-side would give the (incorrect) estimate

$$\frac{\partial \tau_{ij}}{\partial x_j} \sim \frac{\tau}{\lambda}.$$

In all such cases, the lower estimates are to be taken.

BOOK REVIEWS

Gasdynamik. By K. Oswatitsch. Springer-Verlag, Wien, 1952. viii + 456 pp. $18.60.

To write a comprehensive treatise on a rapidly expanding field of knowledge is not an attractive task: there is always the possibility that new developments may soon make the treatment appear dated or even incomplete. On the other hand, the lack of such a treatise may seriously impede the recruiting of new scientific talent, because newcomers will find it increasingly difficult to work their way through numerous important papers written in various languages and scattered over a great number of technical periodicals. In producing this comprehensive treatise on the dynamics of compressible fluids, the author has therefore rendered a significant service to all interested in the development of this branch of mechanics of continua.

Chapter I contains the necessary thermodynamic background. Chapters II and III are concerned with steady and unsteady flows in one dimension. The fundamental integral theorems are established in Chapter IV. These integral theorems remain valid in the presence of shocks; the differential equations of motion are readily derived from the integral theorems. Mechanical similarity is discussed, and various vortex theorems are presented. Chapter V illustrates the application of the integral theorems to technical problems. Chapter VI is devoted to the general equations for steady inviscid flow and to exact particular solutions of these equations (Prandtl-Meyer flow, axially symmetric conical flow, transformations of Molenbrock and Chaplygin, linearization of Prandtl and Glauert). Chapter VII is concerned with steady subsonic flows (plane or axially symmetric). In particular, the methods of Krahn, Janzen-Rayleigh, Kármán-Tsien, and Ringleb are discussed. Chapter VIII is devoted to steady supersonic flows in two dimensions (slightly disturbed parallel flow, shocks and their interaction, method of character-

istics). Chapter IX surveys the available results regarding transonic flow. Chapter X is concerned with particular steady and unsteady three-dimensional flows (lifting surfaces, conical flows, delta-wing, decelerated wedge). The influence of viscosity is briefly discussed in Chapter XI, and a survey of experimental techniques is given in Chapter XII.

W. PRAGER

An introduction to linear programming. By A. Charnes, W. W. Cooper, and A. Henderson. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London. ix + 74 pp. $2.50.

The first part of the volume contains an economic introduction to the theory of linear programming. The fundamental concepts are presented in connection with a specific problem ("nut mix problem"), and the mathematical argument is kept as simple as possible. The second part is devoted to the mathematical theory of linear programming.

While all concepts with which the non-mathematical reader is not likely to be familiar are fully explained, the second part, nevertheless, requires greater mathematical experience from the reader than the first part. Obviously, there is some duplication of the mathematical work in the two parts, but this is entirely justified in a work which is aimed at such a diversified group of readers.

W. PRAGER

A selection of tables for use in calculations of compressible airflow. Prepared on behalf of the Aeronautical Research Council by the Compressible Flow Tables Panel. The Clarendon Press, Oxford, 1952. viii + 143 pp. $8.00.

The panel consisted of L. Rosenhead (Chairman), W. G. Bickley, C. W. Jones, L. F. Nicholson, C. K. Thornhill and R. C. Tomlinson.

This book of tables is the first of two volumes. A complementary volume consisting of graphs will be published in the near future. Among other tables, there is a series of tables labeled isentropic flow tables, applying to the steady isentropic flow of air; a series of tables, labeled characteristic tables, applying to steady isentropic supersonic flow of air in two dimensions or in three dimensions with axial symmetry; and a series of tables, labeled shock tables, applying to normal and oblique shocks in air.

P. CHIARULLI

Advanced mechanics of materials. By F. B. Seely and J. O. Smith. John Wiley and Sons, Inc., New York, and Chapman and Hall, Ltd., London, 1952. Second edition. xvii + 680 pp. $8.50.

This second edition of the book first published in 1932, and then by Seely only, is essentially a new book. It is written for advanced undergraduate and first year graduate students in engineering. Special emphasis is laid on the engineering evaluation of analyses of problems. Results are sometimes quoted from cited references, e.g., in the discussion of thin plates, and thereby the mathematics is kept at an elementary level. The discussion of energy methods and of plastic instability of struts is commendably clear. Problems and references are provided.

The book is in six parts [Preliminary considerations. Special topics on the strength and stiffness of members subjected to static loads. Localized stress—stress concentration. Energy methods. Influence of small inelastic strains on the load-carrying capacity of members. Introduction to instability—buckling loads.] with three Appendixes [A brief introduction to the mathematical theory of elasticity. The elastic membrane (soap film) analogy for torsion. Properties of an area.]

H. G. HOPKINS

Theory of elasticity and plasticity. By H. M. Westergaard. Harvard University Press, Cambridge Mass., and John Wiley & Sons, Inc., New York, 1952. xii + 176 pp. $5.00.

Professor Westergaard's original plan was to write a definitive work on two and three-dimensional elasticity and to include plasticity. A chapter on history covers analytical and experimental developments in elasticity, photoelasticity, plasticity, analogies, structures, and mechanics of materials. Realizing he would not have time to finish, Dr. Westergaard made a great effort to complete the part which has been published posthumously. An interesting feature is the emphasis placed upon the displacements, strains, and the strain functions from which they may be derived rather than upon stress. The approach throughout employs vector notations and descriptions wherever possible. Galerkin vector solutions of the problems of Kelvin, Boussinesq, Cerruti, and Mindlin are discussed in detail as is the author's own method of the twinned gradient.

D. C. DRUCKER

Mechanics of vibration. By H. M. Hansen and P. F. Chenea. John Wiley & Sons, Inc., New York and Chapman & Hall, Ltd., London, 1952, xiii + 417 pp. $8.00.

This book is intended as a textbook for both undergraduate and graduate engineering students. In the authors' words, it is to be a "... textbook that covers the all-important basic principles in a thorough fashion and yet is suitable for a student who has had nothing more than an elementary course in dynamics and the standard instruction in mathematics offered to engineering students today". This reviewer is of the opinion that the authors have accomplished their purpose in large measure insofar as undergraduate students are concerned, but that the book is not well suited as a text for graduate students.

Part I treats at great length a) free vibrations without damping, b) harmonic forced vibrations without damping, c) the steady state solution of harmonic forced vibrations with damping, of linear systems having one degree of freedom.

Part II treats systems of several degrees of freedom. In chapter five there is a brief discussion of Lagrange's equations and of the solution of the equations of small oscillations. In the reviewer's opinion this is the weakest chapter in the book because the all-important subject of the fundamental theory of small oscillations is discussed only very sketchily. Evidently the authors were prevented by space limitations from giving a fuller account of this theory. This reviewer believes that a condensation or omission of other topics, for the sake of a more thorough presentation of this theory, would have been desirable, particularly since the book is intended to be suitable for graduate as well as undergraduate students. This chapter contains a few inaccuracies.

Chapter six is the most valuable portion of the book. It contains an exhaustive discussion of the mobility method for the analysis of certain types of systems of several degrees of freedom. The treatment of various lever systems, branched systems and multimass systems will be very useful to anyone faced with the investigation of problems to which this method applies. Since the method is based on the concept of impedance which is most conveniently developed in connection with electrical systems, the reviewer believes that it is unfortunate that no electrical systems are mentioned.

Part II closes with a discussion of two methods of approximate calculation of the frequencies, viz., Holzer's and Graeffe's method.

Part III, entitled "Special Topics", consists of three chapters, one on continuous systems, one on vibrations of transient character, and one on non-linear vibrations. These are well written, but, being relatively short, barely touch on the subjects involved.

There is a large and remarkable collection of useful problems with answers.

The book as a whole is clearly written. The mathematical tools used are all elementary. This reviewer was surprised at the almost complete lack of literature references.

G. W. MORGAN

Elasticity in engineering. By Ernest E. Sechler. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1952. ix + 419 pp. $8.50.

This book is intended to bridge the gap between texts on strength of materials and engineering treatises on the theory of elasticity. It consists of three parts: Fundamental Equations and Analysis Methods, Engineering Problems in Stable Structures, Engineering Problems in Instability. The subject matter was evidently selected with a view toward the interest of structural engineers, in general, and aeronautical engineers, in particular.

The emphasis on theory and understanding is rather light. A few examples may suffice to illustrate the point. The local transformation theory for a state of stress is treated on the assumption of a homogeneous stress field and under "neglect" of the body forces. In a chapter on stress functions Maxwell's stress functions are unearthed and applied exclusively to the representation of a uniform stress field. Under the heading "Uniqueness of Energy Solution" the author claims to prove that for an elastic body in equilibrium under a single concentrated force the displacement at the point of application of the force is unique. This reviewer is unable to follow the argument.

E. STERNBERG

Mechanics. By A. Sommerfeld. Translated from the German by M. O. Stern. Academic Press, Inc., New York, 1952. xiv + 289 pp. $6.50.

This book constitutes volume 1 of the author's *Lectures on theoretical physics.* It is concerned with the mechanics of systems of a finite number of degrees of freedom. (Systems with an infinite number of degrees of freedom are treated in vol. 2 entitled *Mechanics of deformable bodies* an English translation of which was published in 1950.) The following chapter headings will give an idea of the scope of the book: Mechanics of a particle—Mechanics of systems, principle of virtual work, and d'Alembert's principle—Oscillation problems—The rigid body—Relative motion—Integral variational principles of mechanics and Lagrange's equations for generalized coordinates—Differential variational principles of mechanics—The theory of Hamilton—Problems. There is no need to comment here on the merits of Sommerfeld's exposition; the translation is well done.

W. PRAGER

Grenzschicht-Theorie. By H. Schlichting. G. Braun, Karlsruhe, 1951. xv + 481 pp. $10.00.

The key papers on boundary layer theory are scattered over many technical journals some of which are not readily accessible. Moreover, development has been very intense in this field for the last three decades and is still continuing at a rapid pace. These circumstances make a comprehensive treatment such as Professor Schlichting's book particularly valuable, for the research worker as well as the newcomer to the field. The subject matter is organized into four parts: Basic laws of viscous flow—Laminar boundary layers—Transition from laminar to turbulent flow—Turbulent boundary layers. In addition to the material indicated by the heading, the first part contains a valuable review of exact solutions of the Navier-Stokes equations. The second part presents the theory of the laminar boundary layer for plane steady flow, for axially symmetric, and three-dimensional flow, and for unsteady, in particular oscillatory flow. Temperature boundary layers and boundary layers in compressible flow are also discussed. The third part is rather brief, covering only thirty-one pages. The discussion of turbulent flows in the fourth part is based on the older hypotheses regarding mixing length or similarity rather than the more recent statistical concepts. The exposition is clear throughout and easy to follow.

W. PRAGER

Advanced dynamics. By E. Howard Smart. Macmillan and Co., Ltd., London, 1951. Vol. I, xi + 419 pp. Vol. II, xi + 420 pp. $12.00 per set.

One outstanding feature of this two-volume work is its tremendous scope. No previous knowledge of mechanics is assumed, and the subject of the dynamics of a particle and of rigid bodies is developed slowly, methodically, and in great detail from the very beginning through to the more advanced topics such as the Hamilton theory. Many topics, which in many books on mechanics are given only a cursory treatment, appear here in great detail. The text abounds in examples worked out at great length. There are extensive exercises for the reader, many of the problems being quite interesting. As for prerequisites, a first course in differential equations should, for the most part, suffice for volume I and the first half of volume II. However, for the last half of volume II considerable more mathematical maturity seems advisable.

Volume I deals with the dynamics of a particle, almost all of the material being on two-dimensional topics. The kinematics of a particle is first introduced, then Newton's laws of motion. There follow extensive treatments of the rectilinear motion of a particle, as well as the plane motion of a particle investigated by the use of acceleration components in the directions of rectangular cartesian coordinates and plane polar coordinates. Included here are broad treatments of the ballistic problem and central orbits. Among other topics are the motion of a particle on a curve or surface and the motion of chains. Volume I concludes with a chapter on the motion of a particle in three dimensions.

Volume II deals with the dynamics of a rigid body, and with certain advanced topics. Two-dimensional motions are first considered at some length. Included here is an extensive treatment of impulses. There is one chapter on three-dimensional motions. About the last half of volume II is occupied with more advanced topics including the top and gyroscope, Lagrange's equations of motion, the Hamilton theory, and the theory of small oscillations of conservative systems.

The reviewer has but two criticisms to make, both of which may be matters of opinion. First, the title of the work seems a bit misleading, since the subject of dynamics is developed in this work right from the beginning, and only the last half of volume II deals with what is commonly called "advanced dynamics". Secondly, almost no use of vectors is made. The notion of a vector is introduced after it has been thoroughly motivated by the velocity and acceleration of a particle. But the only operation introduced from the algebra and calculus of vectors is addition. Very many derivations and equations are thus expressed in terms of components, which is relatively awkward and space consuming. Also, no special notation is used for vectors, so that it is sometimes difficult to ascertain whether vector or scalar character is implied.

<div align="right">G. E. Hay</div>

Numerical solution of differential equations. By W. E. Milne. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1953. xi + 275 pp. $6.50.

As is stated in the preface, the book attempts to acquaint the reader with the principal techniques for the numerical solution of ordinary and partial differential equations.

Chapters 1 (Introduction), 3 (Analytical Foundations), and 9 (Linear Equations and Matrices) contain useful background material not directly connected with numerical techniques of integrating differential equations. Elementary methods of solving ordinary differential equations are discussed in Chapter 2 (point-slope formula, trapezoidal formula). Chapter 4 is devoted to methods based on numerical integration and Chapter 5 to the method of Runge-Kutta and methods involving higher derivatives. Systems of ordinary differential equations are treated in Chapter 6, and two-point boundary value problems in Chapter 7. Chapter 8 deals with explicit methods for parabolic and hyperbolic partial differential equations, and Chapter 10 with implicit methods for elliptic equations. Numerical methods of obtaining characteristic numbers are presented in Chapter 11. Appendices are concerned with round-off errors, large-scale computing machines, and the Monte-Carlo method.

The exposition is careful and clear. The detailed numerical examples will enable the practical computer to acquaint himself with the characteristic features of the various methods. Reflecting the considerable practical experience of the author, the book is a most welcome addition to the literature on this important field.

<div align="right">W. Prager</div>

Vorlesungen über die Theorie der Integralgleichungen. By I. G. Petrovskij. Translated from the Russian by R. Herschel. Physica-Verlag, Würzburg, 1953. 100 pp. $7.80.

The little volume contains an excellent exposition of the Fredholm-Hilbert-Schmidt theory of linear integral equations.

W. PRAGER

Thermionic vacuum tubes and their applications. By W. H. Aldous and Sir Edward Appleton. John Wiley & Sons, Inc., and Methuen & Co. Ltd., London, 1952. vii + 160 pp. $2.00.

This monograph treats the basic theory and applications of high vacuum tubes including velocity modulated tubes (klystrons, etc.). Written in a clear and concise manner it serves the authors' purpose as an introduction to electronic circuits while the chapter on limits to amplification is worth reading for the more advanced worker.

S. L. LEVY