The tabulated values state:

$$h'(1) = 1.25953 \pm 10^{-5},$$
$$h''(0) = 1.32823 \pm 10^{-5},$$
$$h''(5) = 0 \pm 10^{-5},$$
$$h(5) = 8.27922 \pm 10^{-5},$$
$$\frac{h''(5)}{h(5)} \leq \frac{1}{8} 10^{-5}.$$

Now $m^2 - 1 = \frac{1}{2}\epsilon(M) \leq \frac{1}{16}10^{-5}$ and $m \leq 1 + 10^{-6}$. Also $1 < k < m$.

To estimate $h(kM/m)$ and $h''(kM/m)$, use the crude estimation:

$$h(x) \geq h'(1)(x - 1)$$

and the representation

$$h''(x) = h''(0) \exp\left[-\int_0^x h(t)\, dt\right] \leq h''(0) \exp\left[-\int_1^x h(t)\, dt\right].$$

After certain manipulations, which may be far from the best, the author has arrived at the following result:

if $M \geq 1 + 2(N + 1)^{1/2}$, then

$$| g(x) - f(x) | < 10^{-N} \qquad \text{for } x \leq M.$$

Hence, $M = 8$ gives at least 10-place accuracy, and $M = 10$ gives at least 19-place accuracy.

These last estimates are all crude. Once M is chosen, however, and the tabulation is made, $Mg''(M)/g(M)$ can be calculated easily, and measures the accuracy well.

References

1. H. W. Emmons and D. Leigh, *Tabulation of the Blasius function with blowing and suction*, Harvard University, Division of Applied Sciences, Combustion Aerodynamics Laboratory, Interim Technical Report No. 9
2. John C. Martin, *On Magnus effects caused by the boundary layer displacement thickness on bodies of revolution at small angles of attack*, Ballistic Research Laboratory, Report No. 870
3. H. Schlichting, *Boundary layer theory*, Part I, Laminar Flows, NACA TM 1217, 1949

ON THE EQUATIONS OF LINEARIZED CONICAL FLOW*

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In an irrotational flow field with velocity potential function $\Phi(x, y, z')$ let u, v, w' be velocity components at the point with rectangular coordinates x, y, z'. If u, v, w' are constant on rays through the origin the field is said to be *conical* [1, 2]. Then u, v, w' are homogeneous of order zero in x, y, z', and Φ is homogeneous of order one if an unimportant arbitrary additive constant is neglected. Furthermore, if the flow field arises by slightly perturbing a uniform flow parallel to the z' axis at Mach number M, then

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Φ satisfies approximately

$$\partial^2\Phi/\partial x^2 + \partial^2\Phi/\partial y^2 + (1 - M^2)\, \partial^2\Phi/\partial z'^2 = 0.$$

The transformation $z = i(M^2 - 1)^{-1/2}z'$ takes this into

$$\partial^2\Phi/\partial x^2 + \partial^2\Phi/\partial y^2 + \partial^2\Phi/\partial z^2 = 0.$$

Thus the construction of linearized conical flow fields can be reduced to determination of homogeneous harmonic functions of order one. In this connection Poritsky [4, 5] has investigated homogeneous harmonic functions of order n, especially for $n = 0$ and 1, to obtain among other results well-known parametric representations of u, v, w' for linearized conical flows in terms of analytic functions of a complex variable [2]. This note develops an alternative derivation of these formulas that is implicit in some of Busemann's early work on conical flow [1].

Write $\Phi(x, y, z') = z\varphi(X, Y)$, where $X = x/z$ and $Y = y/z$. Then

$$u = \partial\Phi/\partial x = \partial\varphi/\partial X, \qquad v = \partial\Phi/\partial y = \partial\varphi/\partial Y,$$

$$w = \partial\Phi/\partial z = \varphi - uX - vY, \tag{1}$$

and Laplaces's equation becomes

$$(1 + X^2)\, \partial^2\varphi/\partial X^2 + 2XY\, \partial^2\varphi/\partial X\, \partial Y + (1 + Y^2)\, \partial^2\varphi/\partial Y^2 = 0. \tag{2}$$

It is easy to show that if u and v are functionally dependent $z\varphi$ must be linear in x, y, z. Hence with no real loss of generality one may assume that $u(X, Y)$ and $v(X, Y)$ are functionally independent. Then exactly as in Busemann's derivation of the equation for non-linearized conical flow [1], under the Legendre transformation (1),

$$w_u = \partial w/\partial u = -X, \qquad w_v = \partial w/\partial v = -Y, \tag{3}$$

and (2) becomes

$$(1 + w_v^2)\, \partial^2 w/\partial u^2 - 2w_u w_v\, \partial^2 w/\partial u\, \partial v + (1 + w_u^2)\, \partial^2 w/\partial v^2 = 0. \tag{4}$$

Now interpret u, v, w as rectangular coordinates in a "hodograph" space. By (4) *the "hodograph" surface $w = w(u, v)$ of a homogeneous harmonic function of order one is a minimal surface* [1]. In accordance with the Weierstrass parametrization of minimal surfaces [3]

$$u = Re\, \frac{1}{2} \int (F^2 - G^2)\, d\zeta, \quad v = Re - \frac{1}{2} i \int (F^2 + G^2)\, d\zeta, \quad w = Re \int FG\, d\zeta \tag{5}$$

for some analytic functions F and G of the complex variable $\zeta = \xi + i\eta$. Conversely, for any analytic $F(\zeta)$ and $G(\zeta)$ for which u and v are functionally independent (5) defines a minimal surface. The so-called *isothermal parameters* ξ, η map the surface conformally onto the ζ-plane.

To find the correspondence between the plane of $Z = X + iY$ and the ζ-plane note that (3) implies that the ray from (0, 0, 0) to (x, y, z) is normal to the "hodograph" surface. Hence by (5) and the Cauchy-Riemann equations

$$Z^*F^2 + 2FG - ZG^2 = 0,$$

where $Z^* = X - iY$. This has the root

$$V(\zeta) = F/G = Z/[1 + (1 + ZZ^*)^{1/2}] = (X + iY)/[1 + (1 + X^2 + Y^2)^{1/2}].$$

Then (1) and (5) imply

$$u = Re\,\frac{1}{2}\int (V - V^{-1})\,df, \qquad = Re - \frac{1}{2}i\int (V + V^{-1})\,df, \qquad w = Re\,f(\zeta),$$

$$\varphi = \frac{1}{2}Re\left[X\int (V - V^{-1})\,df - iY\int (V + V^{-1})\,df + 2f\right],$$

for some analytic $f(\zeta)$. These comprise the desired results in the form exhibited by Poritsky, from which u, v, w' can easily be obtained as functions of x, y, z'. It is customary to choose isothermal parameters such that $V = \zeta$, so that in the supersonic case the interior of the Mach cone of the origin will map onto $|\zeta| \leqq 1$.

References

1. A. Busemann, *Aerodynamischer Auftrieb bei Ueberschallgeschwindigkeit*, Luftfahrtforschung **12**, 210-220 (1935)
2. A. Busemann, *Infinitesimale kegelige Ueberschallstroemung*, Schriften Deutsch. Akad. Luftfahrtforschung **7B**, 105-122 (1943); also NACA Tech. Memo. No. 1100
3. R. Courant and D. Hilbert, *Methoden der mathematischen Physik*, vol. II, Berlin, 1937 pp. 133-135
4. H. Poritsky, *Homogeneous harmonic functions*, Quart. Appl. Math. **6**, 379-388 (1949)
5. H. Poritsky, *Linearized compressible flow*, Quart. Appl. Math. **6**, 389-404 (1949)

BOOK REVIEWS *(Continued from p. 168)*

Introduction to aeronautical dynamics. By Manfred Rauscher. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London. xv + 664 pp. $12.00.

This book, as its unusual title indicates, has an unusual aim. It includes particle dynamics and rigid-body dynamics as well as the theory of fluid motion. The author obviously wishes to give a very detailed and thorough grounding in the fundamental ideas and notions in these fields. To this purpose, he provides detailed derivations of the basic laws; figures and graphs usually omitted in more concise presentations are generously included; basic examples are worked out in full detail; many original and stimulating examples and discussions are included, to help the beginning student overcome the usual basic difficulties. Thus, this book apparently has a higher aim than to be just a textbook; namely, to replace the teacher as well. Its features, described above, make it truly suitable for self-study, although there is no doubt that it will also be valuable in the classroom.

As the various chapters proceed into more detailed and advanced matters, the author shows a certain preference for those special domains in which elegant solutions along classical lines can be obtained. For example, thin-airfoil theory is omitted in favor of an unusually complete presentation of airfoil generation by conformal mapping. Perhaps this is because otherwise the attempt to provide a book going into such detail would have led to a volume of unwieldy size and expense. At any rate, the result is a textbook of unusual character, reflecting, perhaps more than most books do, the special interests and predilections of its author.

N. Rott
W. R. Sears

Einführung in die Determinantentheorie einschliesslich der Fredhomschen Determinanten. By Gerhard Kowalewski. Walter de Gruyter & Co., Berlin, 1954. $7.16.

The first edition of this well-known book was published in 1909. This fourth revised edition was prepared by the author, although publication was delayed due to his death in 1950.

G. Newell