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BOOK REVIEWS

Linearized theory of steady high-speed flow. By G. N. Ward. Cambridge at the University Press, 1955. xv + 243 pp. $6.00.

This excellent monograph is a revised version of an essay submitted in competition for the Adams Prize for 1949–50 in the University of Cambridge. The published version has been brought up to date so that it contains most of the important work in the field up to, and including 1953. The book is divided into three parts; in the first part the linearized equations of motion are derived, general solutions of these equations for subsonic and supersonic flow are presented, and the boundary conditions, aerodynamic forces, as well as uniqueness and flow reversal theorems are discussed. The second part deals with special methods of solution for two-dimensional and three-dimensional wing problems for both subsonic and supersonic flow, although much more emphasis is placed on the latter case. This portion contains sections on subsonic flow past thin bodies; supersonic flow past nearly plane wings, with full use made of the more sophisticated techniques such as "finite part of an integral"; conical fields in supersonic flow where the methods of complex function theory are utilized; and finally a section on the application of operational methods to problems of supersonic flow. The third and final part deals with slender-body theory, a field with which the author's name is most intimately connected. The monograph concludes with an inclusive bibliography of nearly 200 references.

Vectorial notation has been used extensively throughout and proves to be very useful in many instances, although this reviewer cannot wholly agree that it results in a more satisfactory approach than to work in terms of potentials. The reviewer agrees with the author that a knowledge of the elements of compressible fluid theory is required of the reader. Certainly, the book recommends itself highly for applied mathematicians, theoretical aerodynamicists, and research workers engaged in the study of problems of high speed flow. Moreover, there is no reason why it should not also make an excellent text for a graduate course in linearized compressible flow theory. One can only conclude by pointing out that this monograph continues to maintain the high standards in content, as well as in format and printing, which have already been established in this series of Cambridge Monographs in Mechanics and Applied Mathematics.

RONALD F. PROBSTEIN

Mathematical foundations of quantum mechanics. By John von Neumann. Translated from the German edition by Robert T. Beyer. Princeton University Press, New Jersey, 1955. xii + 445 pp. $6.00.

This is a translation of von Neumann's well-known book, *Mathematische Grundlagen der Quantenmechanik* which first appeared in 1932. The translator's preface is dated 1949, the long interval which has elapsed before publication not being explained. Since the original edition appeared, there have been important advances in Quantum Mechanics, notably in connection with the quantization of wave fields. No attempt has been made to bring the book up to date in this regard, and the translation sticks very closely to the original German edition. Even the references to text-books have not been brought up to date.

The translation is somewhat too literal in many places, and the translator often employs phrases and words which, if not absolutely incorrect, have at least a strange and somewhat "foreign" flavor to them. Nevertheless, the meaning is always clear. One slight difference with the original which most people will probably consider an improvement, is that the footnotes have been placed in the body of the text instead of at the end of the book.

The translator and publisher have performed a service in making this classic available to a wider circle of English-speaking readers. It remains indispensable to those who desire a rigorous presentation of the foundations of the subject. The price seems rather high in view of the quality of the printing and the paper binding.

A. F. STEVENSON

BOOK REVIEWS

Approximations for digital computers. By Cecil Hastings, Jr. Princeton University Press, New Jersey, 1955. viii + 201 pp. $4.00.

This is a book with a new and refreshing approach to problems of approximation. Perhaps the word "new" is misleading, since the author makes much use of the classical orthogonal polynomials of Chebyshev, but the newness consists in the imagination with which old methods are applied. The book is the outgrowth of work done by the author at the Rand Corporation in connection with its program of research for the United States Air Force.

Part I, dealing with best fit, weights, iteration, solution of equations, Chebyshev polynomials, and other illustrative material, serves as an introduction to the set of specific approximations contained in the second part. The concepts are clearly and neatly presented by means of aptly chosen figures and graphs, while the written text is compressed to a few lines of explanation for each diagram.

Part II consists of a list of seventy-four completely worked out approximating expressions for twenty-three transcendental functions of a single real variable. The approximating expressions are either polynimials or rational fractions except in a few cases where logarithms also are involved. Several different approximations are given for many of the functions, so that there are more than three times as many approximating expressions as there are functions.

The data for any given approximation are uniformly presented according to the following scheme (e.g. for arctan X)

Function:

$$\arctan X$$

Range:

$$-1 \leq X \leq 1$$

Approximation:

$$\begin{aligned} \arctan {}^*X &= C_1X + C_3X^3 + C_5X^5 \\ C_1 &= .995354 \\ C_3 &= -.288679 \\ C_5 &= .079331 \end{aligned}$$

Error Curve (The graph of the error is drawn to scale, from which the error for any X can be seen at a glance).

Comments: (Other approximations or pertinent information are often given under this heading.)

In the reviewer's opinion this book will be an invaluable piece of equipment for every large scale computer.

W. E. Milne

Asymptotische Entwicklungen mittels der Methode der stationären Phase. By Joachim Focke. Akademie-Verlag, Berlin, 1954. 48 pp. $1.14.

The asymptotic development in β of the function defined by $\iint g(x, y)e^{-\beta\varphi(x,y)}\,dx\,dy$ is studied by the method of stationary phase. The discussion opens with a consideration of the problem of evaluating $\int g(x)e^{-\beta\varphi(x)}\,dx$ where, at the stationary point, $g(x)$ behaves like $(x - \xi)^{-\lambda}$, $0 \leq \lambda < 1$. The Bessel function $J_0(\beta)$ and $\Gamma(\beta + 1)$ are discussed as examples. The double integral discussion includes consideration of several possibilities for the behavior of φ in the neighborhood of its stationary point but $g(x, y)$ is required to be analytic in both x and y everywhere in the integration domain. No examples are treated for the two dimensional case.

G. F. Carrier

BOOK REVIEWS

Vector and tensor analysis. By Nathaniel Coburn. The MacMillan Co., New York, 1955. xii + 341 pp. $7.00.

The book is divided into three parts: vector analysis (Chpts. 1-4), tensor analysis (Chpts. 5-8), and application of tensor analysis (Chpts. 9-12). The scope of the book is perhaps best summarized by giving the list of chapter headings: 1. The fundamental operations, 2. Differentiation theory, 3. Integration theory, 4. Vector analysis in applied mathematics, 5. Tensors in Cartesian orthogonal coordinates, 6. Tensors in general Cartesian coordinates, 7. Tensors in general curvilinear coordinates, 8. The theorems of Gauss and Stokes in N-dimensional Euclidean space, 9. The differential geometry of surfaces, 10. Introduction to the theories of elasticity and viscous fluids. 11. The theory of compressible fluids, 12. The theory of homogeneous statistical turbulence.

The reviewer feels that the book is a poor one, and this for two reasons. The first is that some of the author's statements are careless and misleading. The second and more subjective reason is that the author's approach to the subject is often one with which the reviewer finds himself out of sympathy.

The first point is illustrated by the author's statement in Sect. 8(a) that the triple vector product is equal to the volume of the parallelopiped spanned by the three vectors. This statement is not qualified by a discussion of the orientation of a vector triad or of the sign of the triple product. The author's proof of the identity $\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = \mathbf{A} \times \mathbf{B} \cdot \mathbf{C}$ can easily be generalized by the unwary reader to give the false result $\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = \mathbf{B} \cdot \mathbf{A} \times \mathbf{C}$.

The second point is illustrated by the following examples.

The author defines a vector as a quantity which (1) may be represented by a directed line segment and which (2) adds according to the parallelogram law; he carefully checks the second part of this definition in each concrete case. This old fashioned appeal to an a priori physical concept of addition seems both vague and unnecessary. Thus, in the space-time of the special theory of relativity, there is no simple physical interpretation for the vector sum of two 4-velocities, but the vector sum of the 4-momenta of two particles is the total momentum of the system. In the author's view then, the 4-momenta would be vectors and the 4-velocities not. Yet both 4-momentum and 4-velocity transform alike under Lorentz transformations, and one is a scalar multiple of the other.

The reviewer was unable to find any general discussion of contraction as a tensor operation, except in the special case of inner multiplication. The word "contraction" is not listed in the index.

The author essentially restricts himself to Euclidean space of three or more dimensions and, in Chapter 9, to curved surfaces in ordinary space. Riemannian space is mentioned several times, but there seems to be no definition or explanation of the term, nor does it appear in the index. Thus the discussion of the Riemann curvature tensor seems rather trivial in the general context of the book, since the tensor reduces to a single independent component for a 2-surface and vanishes completely for a Euclidean N-space.

A. Schild

Machine translation of languages. Edited by William N. Locke and A. Donald Booth. The Technology Press of the Massachusetts Institute of Technology, Cambridge, John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1955. xii + 243 pp. $6.00.

Although this book contains no mathematics, many mathematicians will find it to be very stimulating and enjoyable reading. It contains a collection of fourteen essays by various authors including experts both in languages and in machine computers. As a result of the organization of the book there is a great deal of repetition of ideas but the book serves a very useful purpose in bringing these ideas to the attention of a wider audience at a time when the subject is still in an early stage of development.

G. F. Newell