$$
\begin{array}{llllll}
-L_0pi & = i & -v_1 & -v_2 & -v_3 & -v_4 \\
-L_1pi_1 & = & -v_1 & & & \\
-C_1pv_1 & = i + i_1 & & & & \\
-L_2pi_2 & = & & -v_2 & & \\
-C_2pv_2 & = i & +i_2 & & & \\
-L_3pi_3 & = & & & -v_3 & \\
-C_3pv_3 & = i & & +i_3 & & \\
\cdot & & & & & \\
\cdot & & & & & \\
\cdot & & & & &
\end{array}
$$

This set of network equations in the variables $i, i_1, v_1, i_2, v_2, \cdots$ has the given characteristic determinant. The proof of the converse follows that of the previous theorem if the continued fraction expansion is replaced by a partial fraction expansion.

From the point of view presented here it follows that any general synthesis procedure will lead to a canonical representation of Hurwitz polynomials, e.g., for any Hurwitz polynomial $f(p)$ there is a constant K such that the rational function $K/f(p)$ can be synthesized as the transfer impedance of a lossless network operating between two one-ohm resistors[5]. The resulting determinant has the same form as Bückner's except that the last diagonal element has the form $1 + a_n p$.

NOTE ON A PROBLEM CONSIDERED BY TIFFEN*

By R. K. FROYD (*University of California, Los Angeles*)

In a paper published in the *Quarterly Journal of Mechanics and Applied Mathematics* [2], Tiffen considered the two dimensional elastic problem of determining the stress distribution in a semi-infinite plane with a parabolic boundary. Muskhelishvili [1] had previously given a general solution to this problem which yields the results much more easily than the method used by Tiffen. Muskhelishvili's method of solution avoids the rather cumbersome expressions obtained by Tiffen and effects a considerable saving in the work involved in obtaining the results. Since it may be that not everyone working in the field is familiar with Muskhelishvili's work, the solutions using his general result are given below.

Consider the region exterior to the parabola in the z-plane,

$$x^2 = a^2(a^2 - 2y).$$

This region is mapped conformally on to the upper half of the ζ-plane. $\zeta = \xi + i\eta$, by the mapping function

$$z(\zeta) = -\frac{i}{2}(\zeta + ia)^2.$$

[5]S. Darlington, *Synthesis of reactance 4-poles*, J. Math. and Phys. **18**, 257-353 (September 1939).

*Received January 24, 1955; revised manuscript received February 15, 1956.

The lines ξ = constant, η = constant, correspond to orthogonal, confocal parabolas in the z-plane. It is convenient to use this orthogonal net as our coordinate system. The stresses relative to this coordinate system can be written in terms of two functions of a complex variable as follows:

$$\tau_{\xi\xi} + \tau_{\eta\eta} = 2[\Phi(\zeta) + \bar{\Phi}(\bar{\zeta})],$$

$$\tau_{\eta\eta} - \tau_{\xi\xi} + 2i\tau_{\xi\eta} = \frac{2}{\bar{z}'(\bar{\zeta})}\,[\bar{z}(\bar{\zeta})\Phi'(\zeta) + z'(\zeta)\Psi(\zeta)].$$

Muskhelishvili has shown that these functions are given by*

$$\Phi(\zeta) = \frac{1}{2\pi i(\zeta + ia)} \int_{-\infty}^{\infty} \frac{(N - iT)(t + ia)}{t - \zeta}\,dt,$$

$$\Psi(\zeta) = \frac{-1}{2\pi i(\zeta + ia)} \int_{-\infty}^{\infty} \frac{(N + iT)(t - ia)}{t - \zeta}\,dt + \frac{\zeta - ia}{\zeta + ia}\,\Phi(\zeta) - \frac{(\zeta - ia)^2}{2(\zeta + ia)}\,\Phi'(\zeta),$$

where N and T are the normal and tangential surface tractions.

First consider the stress distribution

$$N(t) = -1, \quad |t| < b; \qquad N(t) = 0, \quad |t| > b; \qquad T(t) = 0, \quad \text{all } t.$$

Substituting into the above equations and integrating we obtain

$$\Phi_1(\zeta) = -\frac{1}{2\pi i}\left(\frac{2b}{\zeta + ia} + \log\frac{\zeta - b}{\zeta + b}\right),$$

$$\Psi_1(\zeta) = \frac{b}{\pi i(\zeta + ia)}\left[\frac{2ia}{\zeta + ia} + \frac{(\zeta - ia)^2}{2}\left(\frac{1}{(\zeta + ia)^2} - \frac{1}{\zeta^2 - b^2}\right)\right].$$

These equations correspond to Tiffen's results given by his Eqs. (42), (45), (63), and (68). The relationships between these functions and those used by Tiffen are:

$$\Phi(\zeta) = \frac{\Omega'(\zeta)}{4z'(\zeta)}, \qquad \Psi(\zeta) = \frac{1}{4z'(\zeta)}\,\frac{d}{d\zeta}\left(\frac{\omega'(\zeta)}{z'(\zeta)}\right).$$

Next consider the stress distribution

$$N(t) = -t^2, \quad |t| < b; \qquad N(t) = 0, \quad |t| > b; \qquad T(t) = 0, \quad \text{all } t.$$

In this case we find

$$\Phi_2(\zeta) = -\frac{1}{\pi i}\left(\frac{b^3}{3(\zeta + ia)} + b\zeta + \frac{\zeta^2}{2}\log\frac{\zeta - b}{\zeta + b}\right),$$

$$\Psi_2(\zeta) = \frac{b}{\pi i(\zeta + ia)}\left[\frac{2iab^2}{3(\zeta + ia)} + \frac{(\zeta - ia)^2}{2}\left(\frac{b^2}{3(\zeta + ia)^2} - 1 - \frac{\zeta^2}{\zeta^2 - b^2} - \frac{\zeta}{b}\log\frac{\zeta - b}{\zeta + b}\right)\right].$$

These equations correspond to Tiffen's Eqs. (48), (51), (63), and (70).

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*See [1], pp. 391-394. Muskhelishvili mapped to the lower half-plane. The necessary changes have been made to allow for the change in the mapping.

For the case of hydrostatic pressure,

$$N(t) = b^2 - t^2, \quad |t| < b; \quad N(t) = 0, \quad |t| > b, \quad T(t) = 0, \quad \text{all } t,$$

we can take

$$\Phi_3(\zeta) = b^2\Phi_1(\zeta) - \Phi_2(\zeta), \qquad \Psi_3(\zeta) = b^2\Psi_1(\zeta) - \Psi_2(\zeta),$$

or we can proceed as in the previous examples.

References

1. N. I. Muskhelishvili, *Some basic problems of the mathematical theory of elasticity*, 3rd ed., Moscow, 1949. (Translated by J. R. M. Radok, P. Noordhoff Ltd., Gronigen, Holland, 1953).
2. R. Tiffen, *Solution of two dimensional elastic problems by conformal mapping to a half-plane*, Quart. J. Mech. Appl. Math. **5**, 352-360 (1952)

ON THE INTEGRATION METHODS OF BERGMAN AND LE ROUX*

By J. B. DIAZ[1] and G. S. S. LUDFORD[2] (*University of Maryland*)

Introduction. In a previous note [7] a correspondence was found between the representation of solutions, $u(x, y)$, of the linear hyperbolic differential equation

$$L(u) \equiv u_{xy} + a(x, y)u_x + b(x, y)u_y + c(x, y)u = 0, \tag{1}$$

in the forms

$$u(x, y) = 2\int_0^1 E(x, y, t)f[\tfrac{1}{2}x(1 - t^2)]\,\frac{dt}{(1 - t^2)^{1/2}}\,, \tag{2}$$

and

$$u(x, y) = \int_0^x U(x, y, \alpha)g(\alpha)\,d\alpha. \tag{3}$$

The first representation is due to Bergman [3, 4, 6] and the second to Le Roux [1]. In Eq. (2), $E(x, y, t)$ is the even part of a solution for $-1 \leq t \leq 1$ of

$$(1 - t^2)(E_{yt} + aE_t) - \frac{1}{t}(E_y + aE) + 2xL(E) = 0, \tag{4}$$

such that, for $x \neq 0$,

$$\frac{(1 - t^2)^{1/2}(E_y + aE)}{xt} \tag{5}$$

is continuous for $t = 0$, and tends to zero for each (x, y) as t approaches $+1$; in Eq. (3), $U(x, y, \alpha)$ is a one-parameter family of solutions of Eq. (1) satisfying the characteristic condition

$$\frac{\partial U}{\partial y} + aU = 0 \quad \text{on} \quad x = \alpha. \tag{6}$$

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*Received February 18, 1956.

[1]The research of this author was supported by the United States Air Force through the Office of Scientific Research.

[2]The work of this author was sponsored by the Office of Ordnance Research, U. S. Army, under Contract DA-36-034-ORD-1486.