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BOOK REVIEWS

Geometric algebra. By E. Artin. Interscience Publishers, Inc., New York and London, 1957. x + 214 pp. $6.00.

This incisive monograph leads one by rapid and brilliant steps through the author's own algebraic foundation of affine and projective geometry, into a bird's-eye view of orthogonal and "symplectic" geometries over general fields and, after a brief discussion of determinants over non-commutative fields, the structure of what modern algebraists now refer to as the "classical" groups associated with these geometries. The generality and penetration of the results obtained are truly remarkable. Thus, "fields" are not even assumed to be commutative; "metric structures" (generalized dot products) need be neither symmetric (though $\mathbf{x} \cdot \mathbf{y} = 0$ is assumed to imply $\mathbf{y} \cdot \mathbf{x} = 0$) nor positive. Another interesting feature is the systematic use of coordinate-free methods.

On the other hand, "applied" mathematicians primarily interested in the real and complex number systems will probably find the book unnecessarily abstract and sophisticated, and the author's antipathy to coordinates and matrices (p. 13) exaggerated. Therefore, it is perhaps well to recall that all mathematics is potentially applicable, and that the volume being reviewed is at least to be admired as an original *tour de force*, among the very best works of its kind.

GARRETT BIRKHOFF

On human communication. A review, a survey, and a criticism. By Colin Cherry. Published jointly by the Technology Press, M. I. T., Mass., John Wiley & Sons, Inc., New York, and Chapman and Hall, Ltd. London, 1957. xiv + 333 pp. $6.75.

This is a very attractive and useful book. On pages 2 and 3, the author explains his purpose, in a statement which we regretfully abbreviate, but using his own words: "At the time of writing, the various aspects of communication, as they are studied under the different disciplines, by no means form a unified study; there is a common ground which shows promise of fertility, nothing more./ We shall attempt a review, a survey and a criticism of the study as it is being developed./ This book is introductory./ The book is written for that curious person, the 'general reader'./ We are seeking to extract from/ different sciences—linguistics, phonetics, communication theory, semantics, psychology/ the common related concepts and ideas concerning communication."

The applied mathematician, being usually a man who likes to look over the fence, will derive both pleasure and profit from this very clearly written, conscientious, and unassuming survey. He will get a good idea of the work of a very interesting group of people, located at Harvard and M.I.T., experts in the various disciplines quoted above, who for the last fifteen years, in a remarkable cooperative effort, have built up this broad common field of communication. The present book is the first volume of a series, "Studies in Communication", published by M.I.T. Press and Wiley, and the editors of the series, William N. Locke, Leo L. Beranek and Roman Jakobson, are prominent members of the group. Claude Shannon, who has done so much in the mathematical part of the field, has recently come to Cambridge. Prominent visitors from England have been A. Tustin, D. Gabor and the author of this book. Their work and that of more than a hundred people, from Adrian to Zipf, is discussed in the book and referenced in a marvelous bibliography of 367 titles.

To the applied mathematician who, after reading *On Human Communication*, would like to go further into communication theory, we would recommend reading, in order of publication, the excellent books of Shannon and Weaver (Illinois Press, 1949) Woodward (Pergamon-McGraw-Hill, 1953) and Brillouin (Academic Press, 1956). Some knowledge of cryptography (a habit-forming pastime) is very helpful.

P. LE CORBEILLER

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BOOK REVIEWS

(*Continued from p. 340*)

Handbuch der Laplace-Transformation. By Gustav Doetsch. Volume II. Verlag Birkhäuser, Basel and Stuttgart, 1955. 436 pp. $13.15.

This book is the second volume of the author's trilogy on the Laplace transformation. Volume I, which appeared in 1950, was concerned with the theory of the Laplace transformation. Volumes II and III are dedicated to the applications of the Laplace transformation in various domains of pure and applied mathematics. The present volume is divided into three parts, entitled: Asymptotic expansions, Convergent expansions, and Ordinary differential equations. An introductory chapter collects together the main results and "rules" concerning the one sided Laplace transformation $f(s) = \int_0^\infty e^{-st}F(t)\,dt$ and the two sided Laplace transformation $f(s) = \int_{-\infty}^{\infty} e^{-st}\,F(t)\,dt$. The first part centers around the following general definition of an asymptotic expansion ("im Sinne von Poincaré"): a function $g(z)$ is said to have an asymptotic expansion $\sum_{j=0}^{\infty} c_j h_j(z)$ in a neighborhood U of z_0 provided that for each $n = 0, 1, \ldots$ one has $g(z) - \sum_{j=0}^{n} c_j h_j(z) = o(h_n(z))$ as $z \to z_0$. (It is supposed that $h_n(z) \neq 0$ in U.) Given any functional transformation T, (so that $T(F) = f$, say), a theorem which concludes something about the asymptotic behavior of the image function f from a knowledge of the asymptotic behavior of the original function F is designated as an "Abelsche Asymptotik". If the rôles of the image function and the original function are interchanged in the last statement, then such a theorem is known as a "Taubersche Asymptotik". The main portion of the first part is dedicated to the Abelsche Asymptotik of the one and two sided Laplace transformations, the Mellin transformation, and the complex inversion integral of the Laplace transformation (this last is of particular importance in technical applications, when the properties of a function defined by an inversion integral have to be determined). There is also a chapter on the Taubersche Asymptotik. The second part devoted to convergent expansions, consists of two chapters, the first on faculty series

$$\sum_{n=0}^{\infty} \frac{a_n n!}{s(s+1)\cdots(s+n)},$$

and the second on various series expansions, e.g. theta function, Laguerre polynomials, and others. The main concern here is with convergent series which are obtained by term by term application of the Laplace transformation. The first chapter of the third part deals with the ordinary differential equation with constant coefficients (which may be written in symbolic form thus: $p(d/dt)Y = F$, where $p(x) = x^n + c_{n-1}x^{n-1} + \ldots + c_1 x + c_0$ and $F = F(t)$) on the interval $0 < t < \infty$, the initial conditions being

$$Y_0 = \lim_{t \to +0} Y(t), \cdots, Y_0^{(n-1)} = \lim_{t \to +0} Y^{(n-1)}(t),$$

while the next chapter considers the same differential equation on the infinite interval $-\infty < t < +\infty$ with various boundary conditions at $-\infty$ and $+\infty$. There are detailed applications to the theory of wave filters and electrical networks, among others. The system of ordinary differential equations with constant coefficients

$$\begin{gathered} p_{11}(D)Y_1 + p_{12}(D)Y_2 + \cdots + p_{1N}(D)Y_N = F_1(t) \\ \cdots \qquad \cdots \qquad \cdots \\ p_{N1}(D)Y_1 + p_{N2}(D)Y_2 + \cdots + p_{NN}(D)Y_N = F_N(t), \end{gathered}$$

where

$$p_{jk}(s) = c_n^{jk}s^n + \cdots + c_1^{jk}s + c_0^{jk} \qquad (j, k = 1, \cdots, N)$$

is also treated (specially in the "Normalfall"). The remaining chapters develop the theory of ordinary differential equations with variable coefficients. As is his custom, the author has included a large number of valuable literary and historical remarks at the end of the volume. This book is remarkable for the wealth of detail and the clarity of the exposition.

J. B. Diaz

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enables us to interchange the roles of **X** and **x** and thus obtain an analogous interpretation for $\mathbf{c}^{-1}/\det \mathbf{c}^{-1}$.

These results are equivalent to a *second principle of duality*: Any proposition on changes of length expressed in terms of **C** and **c** yields a theorem on changes of area if **C**, **c**, and "length" be replaced by $\mathbf{C}^{-1}/\det \mathbf{C}^{-1}$, $\mathbf{c}^{-1}/\det \mathbf{c}^{-1}$, and "area", respectively.

Of the many theorems that may be derived in this way, I record only one: The elements of area suffering extremal changes are normal to the principal directions of strain, and the greatest (least) change of area occurs in the plane normal to the axis of least (greatest) stretch; in fact, if the principal stretches dx/dX satisfy $\lambda_1 \geqq \lambda_2 \geqq \lambda_3$ the corresponding ratios da/dA satisfy $\lambda_2\lambda_3 \leqq \lambda_3\lambda_1 \leqq \lambda_1\lambda_2$. While this theorem is geometrically plausible, the first part does not seem obvious.

BOOK REVIEWS

(*Continued from p. 394*)

The theory of linear antennas. By Ronald W. P. King. Harvard University Press, Cambridge, 1956. xxi + 944 pp. $20.00.

The basic material in this treatise on the linear antenna is founded on a graduate course given by the author, and also includes both the theoretical and experimental data of many other workers in the antenna field. Although much of the information has been published previously in various technical journals, the detailed and extensive compilation and evaluation of so much of this work may be regarded as a worthwhile contribution to the antenna specialist. It is the belief of this reviewer, however, that the general usefulness might have been greatly enhanced if the material had been separated into two or more books.

The stated purpose of this book, which is basically mathematical in its approach, is to provide a bridge from the mathematician to the practical antenna engineer. An introduction summarizing the highlights in the historical development of the linear antenna theory is followed by a short chapter on the essentials of electromagnetic theory.

The mathematical difficulties in treating the behavior of a single linear radiator as end-load for a two-wire line are considered in chapter II. Only antennas having a cross section which is small compared with the wavelength are investigated in this text. Particular attention is paid to the end effect complications and cross-coupling between the line and the antenna. An approximate method of compensating for these effects is developed which employs appropriate lumped reactive elements at the junction between the line and the antenna. The characteristics of the isolated antenna are then studied in detail using several different formulations of the problem. The antenna impedance and admittance calculations are presented in a number of different types of graphical plots as well as in useful tables of numerical values. Unfortunately, in this section of particular interest to the practical engineer, a number of typographical errors occur in identifying the graphs.

Comparisons are made between theoretical calculations and experimental measurements obtained from various sources, and the constructional difficulties involved in precise measuring systems are discussed in considerable detail. In one reference in which this reviewer participated, however, it is noted that the author was incorrect in his statements describing the procedure.

Chapter III is devoted to a general investigation of the mutual coupling between antennas in various geometric configurations. An analysis is made of some of the more common types of antennas such as coplanar arrays, parasitic elements, folded dipoles, V-antennas, asymmetrically driven antennas, etc.

Chapter IV is devoted to the general analysis of the essential properties of receiving and scattering antennas. The freespace patterns and gains of various types of linear radiators are taken up in chapter V. Chapter VI discusses the electromagnetic fields of various configurations of linear radiators in commonly-used arrays.

Chapter VII is devoted to a study of the primary electromagnetic field and radiation characteristics of antennas over a conducting earth. This portion could well have been omitted. The effect of the ionosphere is not taken up in this book.

In the last chapter, the antenna is formulated as a boundary-value problem with primary emphasis on the mathematical rather than the engineering approach. The radiation characteristics of hemispheroidal, conical, and cylindrical antennas are considered in detail. Recent works on large-angle conical antennas are not described in this book.

The appendix contains tables of generalized sine and cosine integrals useful in the theoretical calculations, a considerable number of problems based on the presented material, and a bibliography on antenna theory and measurements.

In general, the author seems inclined to favor the more complex approaches to the problems. For example, a simpler method of analysis for the ground plane antenna described in the literature* has not been included. Taken as a whole, however, the book provides a good background and reference for both the mathematician and applied scientist interested in this particular field. The book will be of relatively less usefulness to the general antenna engineer working with more complicated feed systems and antennas of large cross section which are beyond the scope of the simplified arrangements described.

O. M. Woodward, Jr.

Hydrodynamics. By Hugh L. Dryden, Francis D. Murnaghan and H. Bateman. Dover Publications, Inc., New York, 1956. 634 pp. $2.50 (paper-covered).

As is indicated on the reverse of the title page, this is a republication of Bulletin No. 84 of the National Research Council, which has long been out of print.

Deformation and flow of solids. Edited by R. Grammel. Springer-Verlag, Berlin, Gottingen, Heidelberg, 1956.

Work on plasticity of solids has developed in the past mainly along two lines:

1. Physical metallurgists and solid state physicists have attempted to interpret the basic plastic properties of materials in terms of their atomistic structure; the creation of the dislocation theory represents the most impressive example of this type of activity.

2. Mechanical engineers and applied mathematicians have attempted to establish certain general principles of a macroscopic nature for the deformation of solids; these principles permit, for example, computations of the plastic behavior of materials under more complicated cases of loading, if the behavior under simple loading conditions is known.

In both of these two branches of the theory of plasticity, much progress has been realized in the past ten years. However, there has been only little interaction between the two. Since it is felt by many that a close connection between the two branches would be fruitful for the development of either of them, the International Union of Theoretical and Applied Mechanics held a meeting in Madrid in 1955 on the deformation of solids, with the express purpose of including all aspects of the subject.

The book contains the lectures given at this meeting and the contributions to the discussion. The main topics are: theory of dislocations, theory of fracture, mathematical theory of plasticity, non-linear elasticity, viscoelasticity and relaxation. Some of the articles are in the form of surveys and others in the form of representative original papers. In this way the book easily provides a certain understanding of the present status of even those parts of the theory of plasticity with which the reader may be less familiar. It therefore should be of value to anyone who is interested in the deformation of solids.

K. Lücke

Handbuch Der Laplace-Transformation. By Gustav Doetsch. Band III. Birkhauser Verlag, Basel and Stuttgart, 1956. 300 pp. $9.35.

Volume 2 of this multi-volume work concluded with the treatment of ordinary differential equations and this third volume begins, appropriately, with a treatment of partial differential equations. After

*"An Ultra-High-Frequency Antenna of Simple Construction," G. H. Brown and J. Epstein, Communications, July 1940, p. 3.

a brief chapter defining the boundary value and initial value problems, a detailed treatment of the application of the Laplace transform to second order hyperbolic, parabolic, and elliptic equations with constant coefficients is given.

Briefly considered in the third chapter are the operational treatment of a variable coefficient equation in which (1) coefficients are independent of the transformation variable, and alternatively (2) the coefficients may be linear in the transformation variable. There follow chapters dealing with compatibility and uniqueness and with Huygens' and Eulers' principles.

Subsequent parts of the book are concerned with the application of the transform technique to difference equations and to integral equations whose kernel is necessarily of the convolution type, both for the finite and infinite domains.

The last chapter deals with the finite Laplace transform. Finally, an historical survey and a detailed bibliography are provided. The book should be of considerable interest and value to those who want a careful detailed spelling out of the use of the Laplace transform in connection with the problems mentioned above.

George F. Carrier

Theoretische Hydromechanik. By N. J. Kotschin, I. A. Kibel, and N. W. Rose. Translated from the Russian into German by K. Krienes. Akademie-Verlag, Berlin. Volume I, 1954, XII + 508 pp.; Volume II, 1955, VIII + 569 pp. $11.50 each volume.

The translation is of 1948 Russian edition. Volume I is concerned with the classical theory of the motion of an ideal fluid. The contents are as follows, the figures in brackets indicating the number of pages devoted to each topic; Chapter I, Kinematics of fluid motion: Deformation of a fluid drop (7): equation of continuity (15): kinematic character of irrotational and rotational flow (12). Chapter II, Basic equations of the hydrodynamics of an ideal fluid (36). Chapter III, Hydrostatics: Hydrostatic pressure (12): equilibrium of floating bodies (13). Chapter IV, the simplest motions of an ideal fluid: The integrals of Bernoulli and Cauchy (19): plane irrotational flow (14). Chapter, V, Vortex motion of an ideal fluid: The basic equations of vortex theory and the Helmholtz vorticity conservation theorems (27): the determination of the velocity field from a given vortex and source field (31): the Kármán vortex street (30). Chapter VI, The plane problem for the motion of a body in an ideal fluid (107). Chapter VII, The three-dimensional problem for the motion of a body in an ideal fluid (41). Chapter VIII, Wave motions of an ideal fluid: Basic equations of wave theory (7): plane waves (67): three-dimensional waves (21): long waves (31). The apparently disproportionate length of Chapter VI is explained by the fact that it treats also of free streamlines, thin aerofoils, and (in some detail) the planing of a plate at the surface of water. The plate here considered is of infinite breadth, although the case of the plate of finite breadth was solved by A. E. Green as long ago as 1936.

The subject is treated in great detail and there are some exercises which enhance the value of the work. On the other hand nothing like full use is made of vectors and the complex variable. Thus, for example, three pages are used to prove Kelvin's theorem on the constancy of circulation and Cartesian presentation is used throughout the section on plane waves. It is, however, gratifying to see convincing expositions of some of the physical deductions concerning such diverse matters as generation of vorticity, trade winds, breezes, and ocean currents due to variable salinity, which flow from the theorem of Bjerknes. This theorem expresses the rate of change of circulation in a circuit in terms of the number of unit tubes defined by isobars and isobulks which thread the circuit.

The contents of Volume II are as follows; Chapter I, Theoretical foundations of gasdynamics (by Kibel): The equations of gasdynamics (21): steady motion, the plane problem (143): steady motion, three-dimensional, including conical, flow (52): time dependent flow (24). Chapter II, The motion of viscous fluids (by Kotschin): Basic equations of motion of viscous fluids (46): exact solutions of the equations of motion (61): approximate solutions for small Reynolds number (42): approximate solutions for large Reynolds number, including boundary layer theory (86) and Oseen's theory of vanishing viscosity (24). Chapter III, Elements of turbulence theory (by Kibel): Turbulence and instability (27): fully developed turbulence (17): mean values of the hydrodynamic quantities (9).

In this volume also the treatment is detailed in some cases to the point of prolixity. Thus in Chapter II a good tensor notation could have reduced to about one the eleven pages which end up with the assumption that in a viscous fluid $3\lambda + 2\mu = 0$. It should be noted, and this applies to both volumes, that 8 years have elapsed since the publication of the work in Russian so that it may be inferred that the treatise describes the state of the subject as it existed at least 10 years ago. In the case of Volume II therefore the reader must not be disappointed to miss the great advances which have been made in recent years. With this cautionary remark, it may be stated that the book gives a good and readable account of the classical parts of the subject matter listed above.

L. M. MILNE-THOMSON

Spheroidal wave functions. By J. A. Stratton, P. M. Morse, L. J. Chu, J. D. C. Little, and F. J. Corbato. The Technology Press of M.I.T., Cambridge, John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1956. xiii + 613 pp. $12.50.

In the foreword to these tables of coefficients of expansions of spheroidal wave functions, P. M. Morse points out that while it is known that there are eleven systems of coordinates which permit separation of the scalar wave equation, only three can be conveniently used in practice since only in the cases of rectangular, circular cylindrical, and spherical coordinates have the corresponding functions been tabulated. The solutions of the wave equation for elliptic cylindrical coordinates lead to Mathieu functions which have been treated in some detail in the literature, and for which some tables are available. The book under review presents corresponding material for spheroidal wave functions.

A section by Chu and Stratton discusses various expressions for these functions and mathematical relations between them. This is a reproduction of an earlier paper from the Journal of Mathematics and Physics (*XX*, 1941). The application of this material to the determination of the coefficients tabulated is presented in a second section by Little and Corbato. They discuss the numerical methods utilized, and emphasize the influence of the application of a high speed digital computer in carrying out this work. The computer was programmed and the output arranged to print the material in the form in which it appears in the book. This system avoids errors of computation and errors due to transcriptions between the calculated result and the printed tables.

The tables give the coefficients of the expansions of both oblate and prolate spheroidal radial functions in series of spherical Bessel functions, and of the corresponding angular functions in series of associated Legendre functions. References to auxiliary tables of spherical Bessel and associated Legendre functions are given. Tables of the spheroidal wave functions themselves were not presented at this time due to the volume of material involved with the many subsidiary parameters. The coefficients tabulated permit numerical values of the functions to be obtained quite quickly.

E. H. LEE

Engineering analysis. A survey of numerical procedures. By Stephen H. Crandall. McGraw-Hill Book Co., Inc., New York, Toronto, London, 1956. x + 417 pp. $9.50.

This well-written volume, latest member of the Engineering Societies Monographs series, is concerned with the analysis of complex engineering problems and the methods appropriate to their numerical solution. It is addressed to engineers or engineering mathematicians at the graduate level, and is based on lecture courses given by the author at the Imperial College of Science and Technology and the Massachusetts Institute of Technology. By far the largest part of the book is devoted to a survey of important numerical techniques; however, the prior task of reducing a problem to mathematical form is also recognized and discussed.

Perhaps the outstanding feature of the book is its organization, not according to a classification of numerical methods, but according to a classification of the problems to which they may be applied. Thus a given computational scheme, for example, relaxation, may be treated in several places. Three classes of problems are considered: equilibrium, eigenvalue, and propagation problems. The author devotes two chapters to each class, one dealing with systems with a finite number of degrees of freedom, the other with continuous systems. Each chapter contains roughly the following: (i) a derivation of several typical problems; (ii) a review of the salient mathematical characteristics of the type of problem in question; (iii) a discussion of the most suitable techniques for obtaining numerical solutions, usually accompanied by examples worked out wholly or in part. Remarks concerning the convenience of particular methods for hand or machine computation are included, as are brief discussions of the possible errors involved.

In the opinion of the reviewer this book is a valuable contribution to the literature, particularly since it is written from the point of view of the engineer or applied mathematician who seeks to make use of numerical computations, rather than from that of the numerical analyst, who may be chiefly concerned with the refinement of the methods themselves.

Since this work is designed as a survey and not as an exhaustive treatise, the treatment may at times be somewhat condensed for the student entirely unfamiliar with the subject. Copious references to the existing literature, however, will guide the serious reader to more detailed accounts of those topics about which he requires more information. The book will be valuable as a reference to anyone concerned with the more practical aspects of numerical analysis, and should also be quite suitable as a text for a senior or graduate course in the field. Its worth in the latter connection is enhanced by the unusually large collection of exercises, some of which are drill problems, while others extend the ideas of the text. Most are accompanied by answers or hints as to their solution. The subject matter is well-indexed, and references to other works are provided by numerous footnotes, a selective bibliography, and an index of the authors cited. The physical appearance of the book is first rate, adhering to the standard set by previous members of the series. There are a number of minor errors, but the author has prepared a list of those which have come to his attention.

WILLIAM E. BOYCE

Symposium on Monte Carlo methods. Edited by Herbert A. Meyer. Held at the University of Florida. Conducted by the Statistical Laboratory. Sponsored by Wright Air Development Center of the Air Research and Development Command. March 16 and 17, 1954. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1956. xvi + 382 pp. $7.50.

The proceedings of the symposium contains twenty papers covering a fairly wide range of topics including the generation of random numbers, general theory, and applications of Monte Carlo methods. Several papers are reviews written in a simple language for the novice whereas other are more specialized. Unfortunately, the papers are printed in the order of presentation at the symposium rather than in a logical order based upon content or difficulty. A reader unfamiliar with the subject could, however, understand nearly all the papers if he were to read them in a certain order starting with the reviews.

The book also contains an excellent bibliography divided into three sections. Part I lists paper and abstracts on "Monte Carlo Proper" including many rather inaccessible reports published by various laboratories. Part II lists references and abstracts on random digits and is meant to be fairly complete. Part III contains a selected list of articles and books (with some abstracts) mostly on related topics such as sampling.

Despite the fact that there is no obvious attempt to organize the material as would be expected of a text book, the end result is actually better than many books supposedly written in an organized way. It is certainly a valuable and timely contribution to the literature.

G. F. NEWELL

Einführung in die mathematische Statistik. By Dr. Leopold Schmetterer. Springer-Verlag, Wien, 1956. vi + 405 pp. $11.65.

The author, noting the lack of any modern book on mathematical statistics in the German language, presents a fairly complete survey of developments of the last quarter century. Although labeled an introduction, it is not an elementary level book. The reader is assumed to be familiar with matrix theory, calculus and elementary set theory. With this as a starting point, the author concentrates on the subject at hand with a minimum of distraction.

About a third of the book is an introduction to probability theory, primarily an outline of the modern approach with many theorems stated without proof. This is followed by chapters on elementary sample theory, confidence regions, theory of parameter estimating, introduction to test theory, regression theory, introduction to nonparametric theory, and the classical Bayes method. There is also a German to English translation of most of the technical words.

The style of writing is concise, easy to translate, and orderly. Although certain important topics may be left out, no topic is treated at excessive length. The book is well suited as a text book even for English speaking students preferably at the graduate level in American universities.

G. NEWELL

Elasticity, fracture and flow, with engineering and geological applications. By J. C. Jaeger. Methuen & Co., Ltd., London; John Wiley & Sons, New York, 1956. viii + 152 pp. $2.50.

A thorough study of the mathematical theory of elasticity is often held to be the obvious first step in exploring the mechanics of deformable solids. In favor of this view speaks the fact that the mathematical theory of elasticity sets standards of mathematical rigor that have not yet been, and probably never will be, attained in other branches of solid mechanics. On the other hand, prolonged exclusive exposure to the theory of elasticity may foster patterns of thinking that will make it unnecessarily difficult for the student to absorb the differing viewpoints of other branches of mechanics of solids. In recognition of this danger, several schools are currently experimenting with the idea of a general introductory course in mechanics of continua that is to be followed by more specialized courses on the various branches of mechanics of solids and fluids. The lack of suitable textbooks has severely handicapped these efforts. The present volume fills this gap for solids.

The first chapter (48 pp.) is concerned with the analysis of stress, and finite and infinitesimal strain. In the second chapter (68 pp.), the author discusses the mechanical behavior of actual materials and introduces suitable mathematical models of elastic, viscous, plastic, and visco-plastic behavior. The third chapter (43 pp.) treats a number of typical elementary problems concerning equilibrium and motion of these solids.

The exposition is clear and easy to follow. Throughout the book, the emphasis is on the basic assumptions and the manner in which these affect the solutions.

W. PRAGER

Modern mathematics for the engineer. Edited by E. F. Beckenbach. McGraw-Hill Book Company, Inc., New York, 1956. xx + 514 pp. $7.50.

The chapters of this book correspond to a series of invitation lectures given at the University of California, Los Angeles. The book consists of three parts. Part I is entitled Mathematical Models and contains the following chapters: Linear and Nonlinear Oscillations (S. Lefschetz), Equilibrium Analysis: The Stability Theory of Poincaré and Liapunov (R. Bellman), Exterior Ballistics (J. W. Green), Elements of Calculus of Variations (M. R. Hestenes), Hyperbolic Partial Differential Equations and Applications (R. Courant), Boundary Value Problems in Elliptic Partial Differential Equations (M. M. Schiffer), The Elastostatic Boundary-Value Problems (I. S. Sokolnikoff). Part II is devoted to Probabilistic Problems; it contains the following chapters: The Theory of Prediction (N. Wiener), The Theory of Games (H. F. Bohnenblust), Applied Mathematics in Operations Research (G. W. King), The Theory of Dynamic Programming (R. Bellman), Monte Carlo Methods (G. W. Brown). Part III has the title Computational Considerations and consists of the following chapters: Matrices in Engineering (L. A. Pipes), Functional Transformations for Engineering Design (J. L. Barnes), Conformal Mapping Methods (E. F. Beckenbach), Nonlinear Methods (C. B. Morrey, Jr.), What Are Relaxation Methods? (G. E. Forsythe), Methods of Steepest Descent (C. B. Tompkins), High-Speed Computing Devices and Their Applications (D. H. Lehmer).

W. PRAGER