BOOK REVIEWS

Näherungsmethoden der höheren Analysis. By L. W. Kantorowitsch and W. I. Krylow. VEB Deutscher Verlag der Wissenschaften, Berlin, 1956. xi + 611 pp. $11.25.

This book is a German translation from the third Russian edition (1950), the latter being only a minor revision of the second Russian edition (1941). It presents a wealth of methods for the approximate solution of partial differential equations, the main interest being centered about elliptic equations, especially Laplace's and Poisson's equations, and the biharmonic equation, with the usual boundary conditions. In Chapter I methods based on the representation of the solution as an infinite series are treated with emphasis on Fourier series and orthogonal functions. A section of this chapter is devoted to infinite systems of linear equations. Chapter II contains methods for the approximate solution of the Fredholm integral equation together with an application to the problems of Dirichlet and Neumann. In Chapter III the differential equation is approximated by a corresponding difference equation and methods for solving the latter are studied. Chapter IV is one of the most interesting of the book and is devoted to variational methods. There is given a clear account of the methods of Ritz and Galerkin; in contrast to many other presentations of this subject the question of convergence is carefully examined and estimations of error are given for some simple but important cases. The remainder of the book, Chapters V-VII, deals with various questions concerning conformal mapping, application of conformal mapping to boundary problems for harmonic functions, and the alternating method of Schwarz.

The book presents an excellent survey of different analytical tools of importance in numerical analysis, the level being somewhat more advanced than is ordinarily the case in text-books on this subject. Theorems are proved in a stringent way and much emphasis is laid upon estimations of error. Several theorems can, however, be shown to be true under more general conditions than are stated here, but the generality of the assumptions is entirely sufficient for the applied mathematician. Every new method is illustrated by one or two well-chosen problems, which are worked out in detail. There are now exercises for the reader.

Since different methods mentioned in the book can often be applied to the same problem, it would have been valuable if the advantages and disadvantages of the respective methods had been discussed. As is usual in Russian mathematical text-books references are made almost exclusively to Russian papers. Since the book except for minor changes is the same as the 1941 edition, the development of numerical analysis during the last fifteen years has not influenced the contents, for instance the unified treatment of many methods and theorems which can be made by means of functional analysis is not used and the possibility of applying a computer is not discussed. These minor objections are, however, in no sense important. As a whole, the book is a very interesting and useful contribution to the literature on numerical analysis.

<div align="right">C. G. ESSEEN</div>

Experimental designs. By William G. Cochran and Gertrude M. Cox. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1957. xiv + 611 pp. $10.25.

This book is "intended to serve as a handbook which is consulted when a new experiment is under consideration." The first edition was highly successful in this aim and the second edition is an augmentation of the first edition to take care of advances in the design of comparative experiments in the last few years.

This is the best book in existence on the down-to-earth practice of experimental design. There are aspects of the second edition which are unsatisfactory and these arise because the material of the first edition was transferred to the second edition *in toto.* Some of the material of the first edition could have been improved and the continuity of old and new material improved.

The book will be of very high value to experimenters and experimental statisticians and one can also surmise that workers in mathematical statistics will obtain some benefit in knowledge and outlook by perusing the book, since statistics is a subject which has in the past gained so much from understanding the needs of research workers.

<div align="right">O. KEMPTHORNE</div>

BOOK REVIEWS

(Continued from p. 334)

The molecular theory of solutions. By I. Prigogine, with the collaboration of A. Bellemans and V. Mathot. Interscience Publishers, Inc., New York, and North-Holland Publishing Co., Amsterdam, 1957. xx + 448 pp. $13.25.

It has long been recognized that the lattice theories of liquid mixtures are inherently inadequate: even the most elaborate versions fail to allow for changes in lattice parameters with composition, and more important, neglect the effect of composition upon the free volumes of the constituent molecules. Furthermore, no lattice model will yield an equation of state for the mixture. On the other hand, empirical relations such as the Benedict-Webb-Rubin equation of state, although lacking a proper theoretical basis, have been extremely successful in predicting the properties of certain mixtures from the properties of the pure components.

Professor Prigogine and his co-workers have combined the best features of both approaches into a treatment in which the mixture obeys the same reduced equation of state as the pure components, but with composition-dependent reduced variables. The theoretical justification of this is given in terms of the cell model for liquids, the difficult calculation of the free volume being avoided through the hypothesis that the reduced free volume in the mixture is equal to that of the pure components at the same reduced pressure and temperature. Although calculations made by this method seem somewhat more cumbersome than those involved in the simpler lattice theories which are most commonly used to discuss solution thermodynamics, there is no doubt that the results obtained are in considerably better agreement with experiment.

In addition to the detailed exposition and application of his own work on mixtures, Prof. Prigogine has given an excellent review of alternative approaches, including one dimensional solutions and the concept of conformal mixtures introduced by Longuet-Higgins. Quantum effects, especially in isotopic mixtures, are also treated.

This is probably the most original and stimulating discussion of the statistical mechanics of solutions to appear within the last several years. It should be useful both to those doing research in the area and to those who wish to obtain a general acquaintance with the recent literature.

STEPHEN PRAGER

Ordinary difference-differential equations. By Edmund Pinney. University of California Press, Berkeley and Los Angeles, 1958. xii 262 pp. $5.00.

Mr. Pinney has written a detailed account of practical methods of obtaining quantitative information on the solutions of difference-differential equations. He has directed his book towards the reader interested in applications, and the care with which the methods are described, and numerous illustrations make the book a valuable reference. Although nonlinear equations are discussed in the latter part of the book, these equations are nearly linear, and the book is primarily a study of a highly specialized linear analysis applicable to linear systems with constant coefficients.

Integral transform and integral representation methods are described and illustrated in Chapter I. The Euler-Laplace transform is used in Chapter II to obtain infinite series solutions of a general system of linear difference-differential equations with constant coefficients. The solution of the characteristic equation is studied in Chapter III. Nyquist's stability criterion is properly called Cauchy's index theorem. A great number of specialized first and second order linear equations with constant coefficients are studied in detail in Chapters IV through VIII. Chapter IX is described as the "first practical theory" of nonlinear difference-differential equations. A method is given for obtaining approximate solutions of a nearly linear difference-differential equation. It is a method of successive approximation that begins with a solution of the nearby linear system. The method is illustrated in Chapters X and XI. Existence and uniqueness theorems for a class of linear integro-differential systems are given in an appendix.

J. P. LaSalle

BOOK REVIEWS

Statistical analysis of stationary time series. By Ulf Grenander and Murray Rosenblatt. John Wiley & Sons, Inc., New York, and Almqvist & Wiksell, Stockholm, 1957. 300 pp. $11.00.

The basic probability model considered in this monograph consists of a discrete parameter stochastic process $y_t = x_t + m_t$, where x_t is a stationary stochastic process and m_t is a linear combination of known regression vectors. Often in the book x_t is specialized to a linear process, which can be described roughly as an infinite linear combination of indepent identically distributed random variables. Since x_t is stationary, the convariance sequence $r_n = E(x_t, x_{t+n})$ depends only on an n. The hermitian matrix (r_{m-n}) is non-negative definite, so by a well-known theorem in Fourier series there exists a nondecreasing function $F(\lambda)$ which has $r_0, r_1, \cdots$, as its Fourier-Stieltjes coefficients. This function is called the spectral distribution function (sdf) of x_t. The central problem of the book is to study the distributions of certain statistical estimators of the sdf and of its derivative, the spectral density (sd). These estimators are quadratic functions of a finite partial realization of the process y_t. The estimation of the regression coefficients and tests of hypotheses concerning them are also considered. The continuous parameter case is also discussed in several places. Some of the material in the later chapters is here published for the first time.

The first three chapters of the book contain background material. The first chapter presents the basic spectral theory of stationary processes. The second chapter contains a discussion of the least squares theory of prediction, interpolation, and estimation of regression constants, when the spectrum is known. The language of linear manifolds and function spaces is used to introduce the fundamental ideas. In the next chapter some examples of the older approaches to time series are given. The models considered are finite parameter ones such as the model in the first paragraph above but with independent, identically distributed stochastic elements x_t, and also the autoregressive case in which a finite moving average of the elements y_t is set equal to a succession of independent, identically distributed random variables. A feature of this chapter consists of a number of carefully polished proofs of some of the leading classical results including Slutzky's theorem.

The authors start taking up the new approaches in the fourth chapter, in which statistical estimators of a continuous sd are introduced and their asymptotic biases and variances are derived. Emphasis is placed on quadratic estimators which are weighted averages of the periodogram ("spectrograph estimates"). For linear processes they are shown to be as good in a certain sense for estimating the sd at a particular point as any of the more complicated types of quadratic estimators. There is a rigorous discussion of the asymptotic bias and variance of a certain estimator of the sd in the time-continuous case. This is one of the few major demonstrations presented for that case in the book.

In the next chapter some sketches of the role of spectral analysis in the study of electronic devices, turbulence, and water waves are presented. In the sixth chapter, distribution theory is again taken up. The discussion there centers upon a linear process and upon an estimator of the sdf which consists merely of the indefinite integral of the periodogram. The major result can be described roughly as follows: Under certain regularity conditions, the asymptotic distribution function, say, $H(a)$, of the maximum distance (properly scaled) between the estimator $F^*(\lambda)$ and the sdf $F(\lambda)$ for $0 \leq \lambda \leq \pi$ is equal to the probability that a certain normal process $z(\lambda)$ stays inside $(-a, a)$ for $0 \leq \lambda \leq \pi$. The covariance function of $z(\lambda)$ unfortunately depends upon the sdf, so it is difficult to use the result to set up asymptotic confidence bands except in special cases. Extensions to more general estimators and to the case where a regression is present are considered. The chapter concludes with numerical illustrations in which the estimation techniques are applied to some artificially generated numerical series.

Various questions arising in regression analysis are considered in the seventh chapter. There is a comparison of the asymptotic efficiency of the least square estimator and the minimum variance estimator. This is followed by a careful study of least square estimation. The exposition is somewhat intricate and depends upon a device for handling the regression vectors all at once which the authors call the regression spectrum. The last chapter deals with miscellaneous problems. Perhaps the most interesting consists of the problem of prediction when the spectrum must be estimated. The authors present a consciously non-rigorous derivation of the mean square error of prediction in this case. The book con-

cludes with some problems for the reader and an appendix which presents some of the classical results connected with the names of Hardy and Littlewood on functions analytic in the unit circle.

The stated prerequisite for reading this book is "a knowledge of statistics and basic probability theory equivalent to that contained in H. Cramer, "Mathematical Methods of Statistics." " With this understanding, the exposition is highly self-contained. However, the student who has reached the level of mathematical competence at which he can just comfortably read Cramer's book, but nothing more difficult, will find some rough going here. To feel really at home, he will need to have had a good deal of experience with the methods of rigorous mathematical analysis, and in particular he should have a reasonably close acquaintance with the modern theory of trigonometric series. The proofs which the authors give for previously known results are often elegant but tend to be unmotivated and highly condensed.

The study of time series has always presented many challenging and urgent problems. The authors claim, with considerable justification, that the highly specified models used in the past have often been unsatisfactory. They embark upon a program designed to investigate whether the spectral methods which have been so useful in the engineering sciences and in physics can be developed in such a way as to give a better theory for general purposes. The investigation is undoubtedly of great importance, but as with every other known approach to time series analysis, there are some discouraging facts to be faced. For example, knowledge of the spectrum is an end in itself in many engineering and physical problems, and this is not always true in other contexts. The statistical estimation of the spectrum is somewhat tricky technically, particularly if not too many data are at hand. (For instance, the relation between the sample size and the maximum lag of the products used in the estimator must be carefully adjusted.) Accurate point estimates of the sd are elusive. The spectrum of a stationary process does not uniquely determine the process without further assumptions, so even if one could estimate the spectrum of a given series exactly, not all questions about its structure would be cleared up. The random elements of the processes encountered in the social sciences may be generally non-stationary, and the adaptation of spectral methods to non-stationary series is still somewhat problematical. The analytical derivation of exact sampling distributions in the spectral approach may be very difficult. (The statistical results in the present monograph are almost all of an asymptotic nature.)

These problems are all openly recognized by the authors, and they are careful to point out the tentative character of parts of their monograph. They state in the preface that one of their main purposes is "to stimulate research in time series analysis which will lead to practically useful and theoretically sound methods." The reviewer feels that the publication of this book is an important milepost in the development of the mathematical theory of time series. The book is rigorous and accurate, and at the same time it is full of interesting results and techniques which will probably insure its continuing value as a reference whichever way the research in the field turns.

There are the usual minor misprints which characterize a first edition. Perhaps the only accident which might confuse the reader takes place on the bottom line of page 86 and page 87, where the x's should all be y's.

J. H. CURTISS

Vector spaces and matrices. By Robert M. Thrall and Leonard Tornheim. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1957. xiii + 318 pp. $6.75.

This book presents the theory of vector spaces and matrices simultaneously at two levels, one concrete and one axiomatic, the concrete approach sometimes preceding, sometimes following, the axiomatic one. The authors have succeeded in providing a text that aims to develop in the student conceptual grasp rather than mere manipulative skill, as well as to aid him in the applications of the subject to various branches of pure and applied mathematics. The exposition, throughout clear and concise, is more detailed in the chapters forming roughly the first half of the book (often more so than is usual), and more advanced in the later ones. Each chapter contains a good number of selected exercises which frequently supplement preceding theory. Among the features generally not found in texts on this subject is a development of the general theory of simple algebraic extensions of a field (chapter 8) and an introduction to linear inequalities and game theory (chapter 11).

Table of contents: 1. Vector spaces. 2. Linear transformations and matrices. 3. Systems of linear

equations. 4. Determinants. 5. Equivalence relations and canonical forms. 6. Functions of vectors. 7. Orthogonal and unitary equivalence. 8. Structure of polynomial rings. 9. Equivalence of matrices over a ring. 10. Similarity of matrices. 11. Linear inequalities. There are appendices on mathematical induction, relations and mappings, and a bibliography.

<div align="right">H. A. Antosiewicz</div>

Water waves—the mathematical theory with applications. By J. J. Stoker. Interscience Publishers, New York and London, 1957. xxi + 567 pp. $12.00.

The theory of water waves as a branch of mathematical hydrodynamics may be said to begin with Laplace's work on tides, and during the nineteenth century it attracted the attention of many of the most eminent physicists. Their work is described by Lamb who devotes nearly one-third of his book to it. After 1900 the subject became unfashionable, interest in wave motion shifted to electromagnetism and atomic physics, while interest in the application of hydrodynamics shifted to aerodynamics; only a few mathematicians kept working on the numerous problems left unsolved in this field. Some outstanding theoretical work was done during this period which is only now receiving its merited recognition. New activity began during the recent war, and is still continuing. Much new and interesting mathematics was discovered, although much still remains to be done. The progress in theoretical knowledge has been accompanied by advances in hydraulics. A leading part in these developments has been played by the Institute of Mathematical Sciences of New York University where the well-known author of the present work is Professor of Mathematics.

This book is the first connected account of the subject since Lamb's *Hydrodynamics*, but so rapid has been the expansion of the subject that a book of nearly 600 pages cannot now cover all topics, and the author has been faced with the difficult problem of selection. The contents of the book are as follows:

Part I   1. Basic hydrodynamics.
          2. The two basic approximate theories (small-amplitude and long-wave)
Part II Subdivision A. Waves simple harmonic in time.
          3. Simple harmonic oscillations in water of constant depth.
          4. Waves maintained by simple harmonic surface pressure in water of uniform depth. Forced oscillations.
          5. Waves on sloping beaches and past obstacles.
         Subdivision B. Motions starting from rest. Transients.
          6. Unsteady motions.
         Subdivision C. Waves on a running stream. Ship waves.
          7. Two-dimensional waves on a running stream in water of uniform depth.
          8. Waves caused by a moving pressure point. Kelvin's theory of the wave pattern created by a moving ship.
          9. The motion of a ship, as a floating rigid body, in a seaway.
Part III
         10. Long waves in shallow water.
         11. Mathematical hydraulics.
Part IV 12. Problems in which force surface conditions are satisfied exactly. The breaking of a dam. Levi-Civita's theory.

As is explained in the introduction, the problems included in the book are chosen mostly from among those on which the author or his colleagues at New York University have worked, and this gives the book a personal character. Completeness is not aimed at, even in the problems treated here, and the literature cited is confessedly incomplete, even as regards recent work. The mathematical and physical equipment needed by readers differs in different parts of the book, and some of the most interesting sections (e.g. on ship wave patterns) may be read with little previous knowledge of hydrodynamics. There is great variety in the problems. "The physical problems range from discussion of wave motion over sloping beaches to flood waves in rivers, the motion of ships in a sea-way, free oscillations of enclosed bodies of water such as lakes and harbors, and the propagation of frontal discontinuities in the atmosphere, to mention just a few. The mathematical tools employed comprise just about the

whole of the tools developed in the classical linear mathematical physics concerned with partial differential equations, as well as a good part of what has been learned about the non-linear problems of mathematical physics. Thus potential theory and the theory of the linear wave equation, together with such tools as conformal mapping and complex variable methods in general, the Laplace and Fourier transform techniques, methods employing a Green's function, integral equations, etc. are used. The non-linear problems are of both elliptic and hyperbolic type." But, in spite of the diversity of the material, a thread of continuity runs through the book. Friction is neglected except in the chapter on mathematical hydraulics.

In the space allowed for this review it is not possible to discuss in detail any section of the book. and short comments on a few topics must suffice. One-half of the book treats small-amplitude waves, for which the non-linear free-surface condition is replaced by a linearized approximation. In the division dealing with time-periodic waves, the section on sloping beaches does not claim to describe adequately the behavior of real waves on beaches, the interest lies in the mathematical boundary-value problem, and in the behavior of solutions near the corner. The literature on this problem alone is now so big that only a few of the new methods could find a place in the book (There is a section on breaking waves in Chapter 10). Among other problems treated in this part of the book is Sommerfeld's problem of diffraction by a half-plane for which a new and very pretty solution is given. Many other methods and applications of time-periodic linearized waves receive brief mention. The next subdivision, devoted to transients, contains among other things a discussion of boundary conditions at infinity (radiation conditions) and of the principle of stationary phase. The next subdivision, on waves in a running stream, gives a treatment of unsteady waves, leading to the appropriate radiation condition, and goes on to describe ship wave patterns due to pressure points moving along straight lines and circles, with photographic illustrations. The last chapter in this subdivision treats ships in a seaway in a systematic manner. The infinitely thin ship (Michell ship) and its limitations are more clearly explained here than anywhere else in the literature, and there is reason to believe that this will be the opening of a new phase of development in this neglected interesting and technically important subject. A similar treatment for planing ships and other ships of thin section is also given. (For a recent extension of this work see A.S. Peters and J. J. Stoker, Comm. Pure Appl. Math, 10, 1957, 399-490.) The third part of the book deals with long-wave theory; in Chapter 10 an interesting variety of problems is treated; in Chapter 11 friction is regarded as dominant, and the theory is applied to flood routing on the Mississippi with success, using parameters determined from earlier floods and progressively corrected. Such calculations can be made quickly enough to give useful predictions about the later stages of a flood. Comparison with other methods of calculation would have been interesting.

The last part of the book discusses exact solutions, and gives a new proof of the existence of periodic progressive waves on deep water, first established by Levi-Civita.

The reviewer believes that this book has completely achieved its aim. A picture emerges of the present state of the subject: many problems have been solved, but some of the most interesting major problems remain unsolved, including the following:

(1) In the linearized theory of time-periodic waves (Laplace's equation with the simplest mixed-type boundary condition) the uniqueness theory is still missing.

(2) The theory of unsteady non-linear wave motion is still very incomplete, with corresponding difficulty in the interpretation of experiments.

(3) The development of experimental techniques, and the detailed comparison of theory and experiment still leave much to be desired

F. Ursell

Turbolenza di Parete. By Carlo Ferrari. Levrotto & Bella, Torino, 1957. 116 pp.

It is now generally recognized that the inner and outer parts of the turbulent boundary layer have quite different turbulent structure, and as Professor Ferrari shows in his lecture notes, acceptance of this single premise is almost sufficient for rejuvenating the mixing-length theory. Indeed, the inner sublayer is dominated by the wall shear to such an extent that dimensional argument and Taylor series yield quite detailed information, and the assumption of complete self-preservation in the outer layer (critically examined by Townsend, "Structure of Turbulent Shear-flow," Cambr. Univ. Press, 1956)

then suffices for deducing the logarithmic distributions of mean velocity and skin friction. On the further assumption of a turbulent shear diffusivity constant across the outer layer (cf. again Townsend), the mean velocity distribution can be completed, and this is carried out in detail for channel flow and the boundary layer without, and finally with, pressure gradient.

Of course, little understanding of turbulent structure is gained, but on the other hand, it is possible to make the theory appear almost independent of assumptions derived from experiments. In fact, no experiments are mentioned, beyond the Laufer-Townsend energy balance of the boundary layer without pressure gradient. An advantage of this approach is that the calculations can be extended in a natural way also to heat transfer and compressibility, in the manner pioneered by Reichardt and Ferrari, and this extension is presented for the case without pressure gradient.

<div align="right">R. E. Meyer</div>

Quantum mechanics. By F. Mandl. Academic Press, Inc., New York, and Butterworths Scientific Publications, London, 1957. x + 267 pp. $6.50.

This is a second edition of a justly popular book. A final chapter on the Dirac equation, requiring a rudimentary knowledge of the special theory of relativity, has been added to the first edition, published in 1954.

Apart from this, there are no essential changes. The exercises, together with hints for their solution, remain a most valuable part of the text.

<div align="right">Richard Bellman</div>

The numerical solution of two-point boundary problems in ordinary differential equations. By L. Fox. Oxford at the Clarendon Press, 1957. xi + 371. $9.60.

This book presents the most complete treatment of the two-point boundary problem by numerical methods which has so far appeared. The nearest rival is Collatz's "Numerische Behandlung" which devotes about 100 pages to boundary problems for ordinary differential equations. The points of view of Collatz and Fox differ considerably, the former leaning more toward analytical, the latter more toward purely numerical methods.

The scope of Fox's book is shown by an examination of the contents. Chapter 1 gives a brief introduction to the general problem, and emphasizes the distinction between two-point and initial-point problems, a distinction which is far more serious for numerical methods than for purely analytical methods. Chapter 2 presents finite differences and difference equations, supplying the basic computational background needed in later chapters. In Chapter 3, the author studies the solution of algebraic equations, both linear and non-linear, by direct methods, by iteration, and by relaxation. The method of relaxation is then applied to simultaneous finite-difference equations.

Up to this point the material has been preparatory. In Chapter 4 the author takes up second order differential equations with two-point boundary conditions. Various types of boundary conditions are treated by a variety of methods. Chapter 5 is entitled "First-order equations," and the reader may wonder what goes on, since the solution of a first-order equation cannot satisfy two independent conditions. A somewhat over-simplified explanation is the following. Obviously $(y_{n+1} - y_{n-1})/2h$ is in general a better approximation to the first derivative y_n' than is $(y_{n+1} - y_n)/h$, where h is the step interval. But when the first expression above replaces the derivative in the differential equation we are led to a *second order* differential equation. This differential equation has an extraneous oscillatory solution which must be suppressed if possible. Nevertheless it may be easier to solve the second order difference equation by relaxation than to solve the original differential equation by step-by-step methods. Techniques for suppressing the oscillation are discussed.

In Chapter 6, equations of higher order are examined. If central differences are used (as is often desirable because of greater accuracy for a given number of terms) the same problem of extraneous oscillatory solutions bobs up whenever the order of the equation is odd. This phenomenon is related to questions of stability for certain methods of solving differential equations step-by-step.

Chapter 7 is devoted to eigenvalue problems associated with ordinary differential equations. When

a linear differential equation is replaced by a difference equation the eigenvalue problem is approximated by the problem of latent roots and vectors of a matrix. Hence Chapter 7 contains a rather full treatment of latent roots and vectors. In Chapter 8, Dr. Fox discusses various ways to use initial-value methods in order to solve two-point problems and eigenvalue problems, pointing out some advantages and some dangers of this procedure. In Chapter 9 he considers accuracy and precision of boundary value methods. In linear problems or "linearized" problems this involves essentially an examination of the latent roots of an appropriate matrix. Also involved are error estimates for the various formulas of finite differences used in the solution. The book concludes with miscellaneous methods and comments.

The text abounds in interesting worked examples which not only illustrate the procedures being explained but also reveal the author's skill and long practical experience in computational methods. As stated in the preface, the author has concentrated on methods "which are suitable for paper-and-pencil and desk-machine computation. Most of the methods are adaptable to automatic high-speed machines."

It is possible that the real strength of the book, namely its wealth of computational ideas, may be a disappointment to some readers who want to find a simple cut-and-dried procedure which can be immediately programmed for a high-speed digital computer. In fact the reviewer would like to have seen at least some space given to solutions on high speed machinery, even though he agrees with the author that a good grasp of paper-and-pencil methods should precede electronic methods.

All in all Dr. Fox has made a welcome and significant contribution to the literature of numerical methods.

W. E. MILNE

Progress in semiconductors. By Alan F. Gibson, R. E. Burgess, and P. Aigrain. John Wiley & Sons, Inc., New York, 1956. Volume I, viii + 220 pp. $8.00. Volume II, vii + 280 pp. $10.50.

This consists at present of a two volume summary (Vol. I, 1956 and Vol. II, 1957) or review of work in semiconductor research and applications. It is intended that additional volumes be published annually. Each volume contains review articles written by specialists in the field. One of the outstanding features of these two volumes is the very large reference list accompanying each article. In view of the thousands of papers on semiconductor work published thus far, such a venture as this seems worthwhile even if only for the reference lists. The articles are, however, excellent whether for specialist or non-specialist. These annual volumes are particularly recommended for anyone who wants to be aware of the main features of semiconductor work without reading hundreds of papers.

R. TRUELL

An introduction to probability theory and its applications. Volume I. By William Feller, John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1957. xv + 461 pp. $10.75.

The first edition of this book, which is well known as a standard reference and popular textbook in probability theory, has been modified somewhat for this second edition. Although the style and the general outline of topics remain nearly the same, the detailed wording has been changed in many places. There is more emphasis on suggested topics for first reading and changes in wording appear to be motivated by a desire to improve the book's usefulness as a text. Despite the fact that many sections have been made more concise, the second edition is somewhat larger than the first as a result of the addition of two new chapters on "Fluctuations in coin tossing and random walk" and "Compound distributions, Branching processes." This edition also puts greater emphasis on waiting times and the connection between recurrent events and Markov chains. The high quality of the first edition is certainly maintained if not, in fact, considerably improved. The book's many admirers, however, continue to wait patiently for Volume II.

G. NEWELL

Mathematics and wave mechanics. By R. H. Atkin. John Wiley & Sons, Inc., New York, 1957. xv + 348 pp. $6.00.

This book consists of eight introductory chapters (198 pages) dealing with mathematical preparation for the study of quantum mechanics. The remaining six chapters deal with quantum mechanics and some of the more important applications. The form of the book should make it particularly valuable to seniors in physics, applied mathematics or chemistry, who want to begin the study of quantum mechanics. It will also be valuable to graduate students who lack mathematical background for the study of quantum mechanics.

R. Truell

Principles and techniques of applied mathematics. By Bernard Friedman. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1956. ix + 315 pp. $8.00.

This book has the laudable intention "to show how the powerful methods developed by the abstract [mathematical] studies can be used to systematize the methods and techniques for solving problems in applied mathematics." More specifically, the abstract theory of linear spaces and linear operators on these spaces is used extensively in an attempt to systematize the use of Green's function and eigenfunction expansions in the solution of ordinary and partial differential equations.

The first two chapters contain an excellent presentation of the theory of abstract linear spaces and operators which is required in the remainder of the work, and in this sense the book is self-contained. After introducing the Dirac δ-function in Chapter Three, the author introduces Green's function for a second order linear ordinary differential equation and expresses solutions in terms of the Green's function. The eigenvalue problem for ordinary differential equations is considered in Chapter Four, and Chapter Five discusses cursorily Laplace's equation, the diffusion equation, and the wave equation.

The reviewer feels that the strong emphasis on the theory of linear operators in the development of the theory of the last three chapters is obtained largely at the expense of rigour. For example, the proof of the completeness of the eigenfunctions $u_n = \sin nx$ $(n = 1, 2, \cdots)$ promised (on page 197) to be given later could not be found in the book by the reviewer. The formal approach, particularly in the last chapter, seems to obscure the underlying physical nature of the solutions to the equations considered. The reviewer also feels that the scope of the book is too limited for it to warrant such a broad title.

R. T. Shield

Proceedings of the Fifth Midwestern Conference on fluid mechanics. Edited by A. M. Kuethe. University of Michigan Press, Ann Arbor, 1957. viii + 388 pp. $8.00.

The Volume collects 26 papers, among which a discussion of low-density Couette flow by H. T. Yang and L. Lees stands out as clearly deserving careful reading by any students of the slip-flow regime. T. Y. Li calculates the hypersonic boundary layer corresponding to a prescribed pressure distribution, and W. N. MacDermott describes early operational experiences with an arc-heated blow-down hypersonic tunnel of about 0.03 sec. running time. H. H. Korst and W. Tripp predict the base pressure at the blunt trailing edge of a supersonic aerofoil at incidence; experiments confirm again the validity of the Korst model and suggest that an appreciable change in base pressure can arise from external pressure waves incident on the base region. A simplified model for the head-on interaction of shocks with rigid grids is developed by H. O. Barthel, and the characteristics systems of the equations governing interaction of a magnetic field with one-dimensional unsteady flow of a conducting gas are classified by S. I. Pai. R. W. Truitt proposes that the flow near the nose of a body in hypersonic flight be approximated by that generated when the body moves slowly towards a fixed concentric shell.

Some effects of chemical reactions on inviscid hydrodynamic stability are discussed by S. F. Shen, and Görtler draws attention to H. Oser's thesis on the vortex sheets arising in the low-frequency perturbations of rotating inviscid fluids, and to the analogous effect in fluids with stable density stratification. M. Lessen gives the compressible perturbation equations for flow with vortex-symmetry, and

A. de Neufville gives solutions for vortices decaying due to viscous shear diffusion. Some functions useful for the calculation of two-dimensional, incompressible laminar heat transfer from bodies with variable pressure and wall-temperature are tabulated by C. R. Guha and C. S. Yih, J. M. Robertson reviews data on turbulent flow in rough pipes, while E. R. G. Eckert and T. F. Irvine present some measurements on entrance length, transition and friction in non-circular pipes. D. W. Smith reports floating-element measurements of turbulent skin friction on a flat plate, V. A. Sandborn proposes a two-parameter family of mean profiles differing from Coles' (which reviewer prefers), and W. K. Mueller discusses why slow pulsations in pipe flow should reduce the average turbulent heat transfer slightly below that for steady mean flow.

P. Kaplan generalises Stokers work on the surface waves generated by pressure distributions to the case of uniformly moving oscillatory disturbances and shows that permanent advancing waves are formed also far ahead of the disturbance when the product of its frequency and speed is less than $g/4$. A. G. Strandhagen and G. R. Seikel calculate surface wave effects on the performance of a submerged hydrofoil of infinite span by the lifting line model, J. P. Breslin presents a simplified calculation of the suction exerted on a very slender body by a propeller at its stern, and F. E. Stelson and J. P. Murtha measure the virtual mass of some bodies moving in water. An ellipse-fitting approximation for calculations of two-dimensional impact of bodies on water by the "added-mass" model is proposed by G. Fabula. E. Silberman reports experiments confirming Rayleigh's prediction of the instability of gas jets in liquids, and W. H. Guilinger and E. Saibel discuss the programming of the approximate one-dimensional equations of adiabatic slider bearings. A little philosophical accompaniment is provided by M. M. Munk, M. Z. v. Krzywoblocki and C. G. Whittenbury.

R. E. Meyer

Differential equations: geometric theory. By S. Lefschetz. Interscience Publishers, Inc., New York and London, 1957. x + 364 pp. $9.50.

This book is strongly to be recommended to anyone entering the domain of differential equations, either as a pure or applied mathematician. The author has presented in a very readable and flowing style the fundamentals of the theory of the existence and stability of equilibrium and periodic solutions of nonlinear ordinary differential equations.

The title does not do the contents justice. In addition to presenting the geometric theory, in the sense of trajectories, critical points, and periodic solutions, the author gives a very lucid and detailed discussion of the analytic stability theory of Poincaré-Lyapunov. Starting with the ideas of these authors, he continues to the more recent work of Perron, Bellman, Levinson, Persidiski, Malkin, Massera and Antosiewicz.

Remarkably, in addition to a quite detailed discussion of systems of n-th order, and then of the second order, he presents a survey of various approximate techniques devised by Van der Pol and extensively developed by Krylov-Bogoliubov, and of the more recent stroboscopic method of Minorsky. These methods are illustrated by examples.

Richard Bellman

Integraltafeln zur Quantenchemie. By H. Preuss. Volume II. Springer-Verlag, Berlin, Göttingen, Heidelberg, 1957. 143 pp. $8.60.

This volume consists of two parts, in the first of which the integrals and the method of calculation are described along with the details of the one electron wave function used in the calculation of molecular energies. The second part of this volume is concerned with the numerical tabulation of the results of the calculation of the interaction integrals for the K and L Shells as a function of the screening parameters used in the normalized Slater functions.

These tables constitute the second volume of such integrals for quantummechanical calculations. This work is part of a program of the Max Planck Institute for Physics in Göttingen to prepare such tables for the more important types of calculation.

Rohn Truell