QUARTERLY

OF

APPLIED MATHEMATICS

VOLUME XVII JULY • 1959 NUMBER 2

QUARTERLY OF APPLIED MATHEMATICS

This periodical is published quarterly by Brown University, Providence 12, R. I. For its support, an operational fund is being set up to which industrial organizations may contribute. To date, contributions of the following industrial companies are gratefully acknowledged:

AVCO RESEARCH LABORATORY; EVERETT, MASS.,
BELL TELEPHONE LABORATORIES, INC.; NEW YORK, N. Y.,
THE BRISTOL COMPANY; WATERBURY, CONN.,
CALIFORNIA RESEARCH CORPORATION; SAN FRANCISCO, CAL.,
CURTISS WRIGHT CORPORATION; AIRPLANE DIVISION; BUFFALO, N. Y.,
EASTMAN KODAK COMPANY; ROCHESTER, N. Y.,
GENERAL ELECTRIC COMPANY; SCHENECTADY, N. Y.,
GULF RESEARCH AND DEVELOPMENT COMPANY; PITTSBURGH, PA.,
HUGHES AIRCRAFT COMPANY; CULVER CITY, CAL.,
INTERNATIONAL BUSINESS MACHINES CORPORATION; NEW YORK, N. Y.,
LEEDS & NORTHRUP COMPANY; PHILADELPHIA, PA.,
LOCKHEED AIRCRAFT CORPORATION; BURBANK, CAL.,
PRATT & WHITNEY, DIVISION NILES-BEMENT-POND COMPANY; WEST HARTFORD, CONN.,
THE RAMO-WOOLDRIDGE CORPORATION; LOS ANGELES, CAL.,
REPUBLIC AVIATION CORPORATION; FARMINGDALE, LONG ISLAND, N. Y.,
UNITED AIRCRAFT CORPORATION; EAST HARTFORD, CONN.,
WESTINGHOUSE ELECTRIC AND MANUFACTURING COMPANY; PITTSBURGH, PA.

The QUARTERLY prints original papers in applied mathematics which have an intimate connection with application in industry or practical science. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

Manuscripts submitted for publication in the QUARTERLY OF APPLIED MATHEMATICS should be sent to Professor W. Prager, Quarterly of Applied Mathematics, Brown University, Providence 12, R. I., either directly or through any one of the Editors or Collaborators. In accordance with their general policy, the Editors welcome particularly contributions which will be of interest both to mathematicians and to engineers. Authors will receive galley proofs only. The authors' institutions will be requested to pay a publication charge of $5.00 per page which, if honored, entitles them to 100 free reprints. Instructions will be sent with galley proofs.

The subscription price for the QUARTERLY is $6.00 per volume (April-January), single copies $2.00. Subscriptions and orders for single copies may be addressed to: Quarterly of Applied Mathematics, Brown University, Providence 12, R. I., or to Box 2-W, Richmond, Va.

Second-class postage paid at Providence, Rhode Island, and at Richmond, Virginia.

WILLIAM BYRD PRESS, INC., RICHMOND, VIRGINIA

CONTENTS

SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{\ }$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$\exp\,[(a^2 + b^2)^{1/2}]$ is preferable to $e^{(a^2+b^2)^{1/2}}$

BOOK REVIEWS

Switching circuits and logical design. By Samuel H. Caldwell. John Wiley & Sons, Inc., New York, 1958. xviii + 686 pp. $14.00.

This book is needed by every serious student of the logical design of switching circuits. The author has performed a much needed service by gathering most of the significant material on this subject into a single volume, reworking it into a coherent and well-written textbook.

The jacket blurb about Professor Caldwell is unnecessarily modest about his qualifications to have written this book. The two most important advances in the field of logical design were papers by Shannon and by Huffman, and each of these papers was written as an M.I.T. thesis for which Caldwell was a thesis adviser. Understandably he gives much space in his book to exposition of the ideas of these and later publications by Shannon and Huffman, and of significant unpublished material by these two men. The book also contains unpublished material original with Caldwell, and various items due to other M.I.T. men. The author takes so much advantage of the excellent resources of M.I.T. that he errs somewhat on the side of provincialism, tending to ignore some of the important papers written elsewhere, such as Gilbert's work on frontal switching functions, and Muller's work on complexity in electronic switching circuits. He completely omits all work on logical design done outside the English-speaking world. The books by Gavrilov, Higonnet and Grea, and Plechl are never mentioned, and the papers by Lunts, Povarov, Shestakov, Cardot, and others are similarly ignored.

But many useful methods are treated, some of which had not been published or widely known before. This book gives a much more full and thorough coverage of the subject of logical design than any of the other books on the field, including those mentioned above, and the English language ones by Keister, Ritchie, and Washburn, by the staff of the computation laboratory, by Richards (all cited in the book under review), by Phister (Logical design of digital computers, Wiley, 1958), and by Culbertson (Mathematics and logic for digital devices, Van Nostrand, 1958).

This book gives an intuitive rather than a formal approach to its subject, and depends on many well explained illustrations and many tricky exercises to appeal to the ingenious puzzle-solver type of student, rather than attempting to prove theorems or set up formidable mathematics. The book frequently goes out to the limits of what is known in a given part of the subject, and explicitly or implicitly states many problems which are of current research interest.

A strong point of this book is the emphasis that it gives to asynchronous circuit design. The other recent books on logical design cover this topic very incompletely, despite its practical importance, and the many interesting theoretical problems which arise in dealing with it.

There are two minor difficulties which may stand in the way of the wide use of this book as a textbook. The first of these is that most of it (except Chapters 9, 14, and 15) is written about relay circuits, but that most electrical engineers who will be teachers or students of courses in this subject are prejudiced against relays, as compared to the much faster, more modern, and more glamorous electronic switching elements. From both the puzzle solving and the mathematical points of view, relay circuits have many interesting properties which are not to be found in the electronic circuits, although much of the skill or knowledge gleaned in either area is applicable in the other. Perhaps the advent of the cryotron (a recent electronic switching element which is functionally equivalent to a relay) will make this prejudice obsolete.

The second of these difficulties is the close knit way in which the book is integrated. This would be an advantage when teaching a two-semester course just like the one Caldwell teaches. However, he has not only failed to supply asterisks to show which topics may be omitted without loss of continuity, but has interwoven these topics closely with the absolutely essential ones. Anyone wishing to teach a shorter course or to make room in a two-semester course for other topics not treated in this book, such as computer programming, system design, or arithmetic units, will have a major editing job in store for himself.

But neither of these difficulties as a textbook will impair the lasting value of this book as a reference book, explaining in clear and heuristic language the main methods of logical design or its current value as a source of orientation about the frontiers of the field, with its frank comments on many of the kinds of problems we do not know how to solve.

EDWARD F. MOORE

(*Continued on p. 146*)

BOOK REVIEWS

(*Continued from p. 128*)

The gyroscope—theory and applications. By James B. Scarborough. Interscience Publishers, New York, London, 1958. xii + 257 pp. $6.50.

This is an introduction to the mechanics of the gyroscope, written on an elementary level and covering a fair amount of theory, but with emphasis on applications. Part I (theory) contains a chapter on vector calculus, one on the fundamental principles of mechanics and two chapters on the theory of the gyroscope, treating the free gyro, forced precession and the heavy gyro. Part II (applications) is devoted to all kinds of gyros on vehicles: gyro-compass, spherical pendulum, stabilizers of ships and of monorail cars. The last chapter is devoted to astronomical applications.

One of the features of this book is its simplicity of reasoning. With very few exceptions everything follows from two simple sets of equations, (16.4) and (25.2). The treatment of the gyro-compass is particularly elegant, also the representation of the pseudo-regular precession by means of a Taylor expansion. The part of the book dealing with applications is very complete. Some numerical examples are helpful for the appreciation of the importance of various factors. Frequent references to modern developments are particularly valuable.

In some instances, especially in the part of the book devoted to theory, the simplicity mentioned above is carried too far. The definitions of the concepts "gyroscope" (p. 37), "heavy gyro" (p. 65), "spherical pendulum" and "gyroscopic pendulum" (p. 161) are rather vague. The pseudo-regular precession of the heavy gyro is discussed under very restrictive initial conditions (p. 70). A gyro having two degrees of freedom should not be referred to as a gyro of one degree of freedom (p. 143). The stability investigations on pp. 55 and 61 start from incorrect assumptions; the results are either incorrect or mere first approximations. Equations as (22.2) through (22.4) or (27.2) through (27.4) where one side is a scalar, the other one a vector, should be avoided.

Though this list might be continued, the value of the book as a whole is beyond question.

A suggestion concerning terminology: the term "precession" for a certain partial motion of a gyro stems from the verb "to precede" (lat.: praecedere); one should not derive a new verb "to precess" from this noun but rather say that a gyro precedes if it moves in a precession.

H. ZIEGLER.

A course in multivariate analysis. By M. G. Kendall. Hafner Publishing Co., New York, 1957. 185 pp. $4.50.

Until recently multivariate analysis has not been well covered by the text books in theoretical statistics. At almost the same time three monographs appear now in this field: the one by M. G. Kendall, another called Introduction to multivariate statistical analysis by T. W. Anderson and a third by S. N. Roy: Some aspects of multivariate analysis. The two latter ones are published by John Wiley & Sons.

Dr. Kendall's book is meant to be an introduction to the subject, perhaps especially for a reader interested more in applying these statistical methods than in their mathematical derivation. Indeed, one attractive feature of the book is the emphasis given to model construction and the interpretation of numerical results. The validity of the available techniques is critically examined, e.g. the centroid method in the first chapter, and alternative methods are examined and compared. A great number of detailed numerical examples illustrate the theoretical discussion and add a good deal to the value and usefulness of the book. The exposition is clear and the book reads very well.

The book begins with a brief introduction describing what multivariate analysis is and to what fields it has been applied. The second chapter is devoted to component analysis and contains some elementary but basic results, which are used throughout the rest of the book. A brief discussion of factor analysis follows, and the fourth chapter is concerned with functional relationships with stochastic elements. This topic is often treated in an obscure way in the literature and often causes difficulties to the student

(*Continued on p. 164*)

BOOK REVIEWS

(*Continued from p. 146*)

of statistics. Canonical analysis is discussed and illustrated by examples in Chapter 5, while Chapters 6 and 8 deal with distribution problems and tests of multivariate hypotheses. Among other topics treated in this book are tests of homogeneity, discriminant functions and some history of multivariate analysis. An extensive bibliography and a set of exercises end the book.

ULF GRENANDER

The analysis of multiple time-series. By M. H. Quenouille. Hafner Publishing Co., New York, 1957. 105 pp. $4.75.

The last decades have witnessed rapid progress in statistical time series analysis, as far as a single series is concerned. This was made possible by the results in probability theory describing the probability structure of time series of various types. For multiple (vector valued) time series the progress has been slower, one of the reasons perhaps being that we know less about the basic properties of vector valued stochastic processes. It is typical that the linear prediction problem solved in the early forties for scalar processes, is still not quite settled in the vector case, although recent work of Wiener and Masani may change this.

In this book the author collects available methods and develops new ones for analyzing multiple time series. His starting point is the assumption that the underlying process is of the finite parameter type: a moving average, an autoregressive scheme or a combination of both. This restriction may be reasonable in the present preliminary stage of development of the theory.

After a brief discussion of these schemes and of their covariance properties and identification problems, the author studies the numerical behavior of five artificially generated series. The next chapter, Practical complications, contains a discussion of how various factors, may influence the anaylsis. Such situations occur e.g. when 1) the errors are serially correlated 2) the sample size is too small, so that large sample approximations are inadequate or 3) there are unknown trends or seasonal variation. After two chapters on estimation and testing the author investigates an econometric example in detail trying to apply the methods described in the book.

The emphasis of the book is on the formal development of statistical techniques, and one would have appreciated a more complete critical examination of the validity of the methods used. However it is clearly too early to ask for definitive and convincing general methods of analysis of multiple time series, and the need for further work is obvious. At present we will have to use the tools of analysis available to us, even if they leave a good deal to be desired. When applied with some caution they should enable us to extract some information from the data.

This book is the first in the new series "Griffin's statistical monographs and courses" in which has also appeared the book on multivariate analysis reviewed elsewhere in this journal.

ULF GRENANDER

Applied mathematics for engineers and physicists. By Louis A. Pipes. McGraw-Hill Book Co., New York, Toronto, London, 1958. xi + 723 pp. $8.75.

This second edition of a book first published in 1946 remains essentially the same as the first edition as regards subject matter, style and purpose. New material has been added to most chapters making the book somewhat larger than its predecessor. Particularly there is a greater emphasis on methods of numerical analysis some of which have been developed during the last decade as a result of the stimulus furnished by the high speed computers.

The sections on theory and applications of matrices and of Laplace transforms have been revised and contain much material not usually found in a text of this type. The book continues, however, to give recipes for solving specific problems with considerable disdain for sound mathematics.

G. F. NEWELL

(*Continued on p. 172*)

BOOK REVIEWS

(*Continued from p. 164*)

Logical design of digital computers. By Montgomery Phister, Jr. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1958. xvi + 408 pp. $10.50.

This book contains a development of a methodology of functional design of digital circuits and gives examples of application of the techniques to design of digital computer components. Discussion is limited to synchronous circuit design. The book begins with a brief discussion of the system design problem and the logical designers role, and of circuit components and the binary number system. Next, the fundamentals of Boolean Algebra are presented, followed by a description of various methods of simplifying Boolean functions. The general methods of application of Boolean Algebra in sequential circuit design are developed and utilized in a detailed discussion of the design of the major digital computer components; memory unit, input-output unit, arithmetic unit and control unit. Examples of complete computer design are included.

The book provides a compilation of the techniques available in the field of logical design and, because of both content and organization, should be useful to the logical design student and to the practicing designer. The references to literature in the field and the supply of exercises for the student are comprehensive.

The book serves as an excellent compilation of techniques, particularly in its exposition of the methods of reduction of Boolean functions and the general methods of application. However, motivation for the specification of characteristics of the digital computer components is somewhat weak. This fact is implicitly recognized by the author in the discussion of the logical designer's role as one who is responsible for developing the implementation of a set of requirements provided by the system designer: development of characteristics is not fundamentally in the logical design province. The result of the lack of motivation and discussion of the prescription of system parameters limits the usefulness of the book to those who have some experience in the computer field. The book would not be useful, for example, as a text in a first course on digital computer organization and use, nor in a course on computer system design, but should be reserved for use in an advanced course where the objective is solely logical design.

DEAN GILLETTE

Introduction to difference equations. By Samuel Goldberg. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1958. xii + 260 pp. $6.75.

This volume was motivated by a monograph written at the invitation of the Social Science Research Council Committee on the Mathematical Training of Social Scientists. It is intended as an introduction to the theory and application of recurrence relations of the form $u_{n+1} = f(u_n)$ for students and research workers in the social sciences who have only a rudimentary knowledge of mathematics.

It is a pleasure to say that the author has been eminently successful in his task. The volume is not only easily intelligible, but full of important applications which illustrate the significance of this field. The book should certainly be useful as a basis of a course following after one patterned along the lines of the recent book by Kemeny, Snell and Tompson.

Furthermore, the book would be equally useful as an introductory junior or senior course for physical science majors. In these days of narrow specialization, it is essential that research workers in one domain get a glimpse of how scientists in other fields construct their mathematical models.

RICHARD BELLMAN

(*Continued on p. 184*)

BOOK REVIEWS

(*Continued from p. 172*)

Fundamentals of gas dynamics. Edited by H. W. Emmons (Volume III of high speed aerodynamics and jet propulsion). Princeton University Press, 1958. xiii + 749 pp. $20.00.

It is a pleasure to record the appearance of another volume of this monumental series. The title is perhaps not too informative, and the first duty of the reviewer is to give a short list of contents and authors: The equations of gas dynamics (Tsien); One-dimensional treatment of steady gas dynamics (Crocco); One-dimensional treatment of nonsteady gas dynamics (Kantrowitz); The basic theory of gasdynamic discontinuities (Hayes); Shock wave interactions (Polachek and Seeger); Condensation phenomena in high speed flows (Stever); Introduction to combustion (v. Kármán); Gas dynamics of flame fronts (Emmons); Gas dynamics of detonations (Taylor and Tankin); Flow of rarified gases (Schaaf and Chambré).

The author of a "handbook" article always has to face the choice between the restricted aim of providing an introduction to his subject and guide to the literature, and the ambitious aim of giving a definitive and exhaustive account. Tsien has chosen the more modest alternative and contributed a lucid description of the equations underlying the "classical" part of Gas Dynamics, with indications of where to look for the "modern" departures. The stress is less on the derivation of the Navier-Stokes equations than on the discussion of the derived systems of general equations, such as the circulation and entropy theorems, the stream-function approach and the variational formulation for potential flow.

Crocco has not only chosen the other alternative, but indeed interpreted the title of his article in its full, literal generality. There are no steady compressible flow processes in ducted machinery so complicated that useful one-dimensional models could not be inferred from a physical understanding of the essential mechanisms—and here the models are displayed in a sweep never attempted before, over the whole gamut from the ideal nozzle to the ducted rocket motor. The article is a book in its own right, with a number of original papers thrown in for good measure, and one surmises that only illness prevented Professor Crocco from adding a handbook of chemical engineering! The approach is broad and versatile—physical discussion of complex viscous, thermodynamical and chemical interactions alternate with engineering charts—and the number of topics caught in the Crocco net defies enumeration here. No doubt, this article by itself will ensure a heavy demand for the volume in the aeronautical, mechanical and chemical engineering industries.

A particular trait of the "one-dimensional" theory has always been that its conceptual basis is vague, and even this problem is attacked in the article. The starting point is the recognition that uniformity of physical quantities, or even physical mechanisms, over any cross-section of the duct is not a necessary premise. Large parts of the theory deal with the interaction between quite different streams flowing side by side. The outline of a general mathematical theory of the application of the conservation principles to mean values representing such flows is sketched and elaborated in the direction of a classification of processes.

Kantrowitz' choice, on the other hand, is a very personal introduction to the propagation of plane-wave pulses and the stability of transonic duct flow. The treatment is physically illuminating, even if it lacks the unambiguous lucidity of Tsien. The article also contains a brief review of the procedures of the numerical method of characteristics and a sketch of a few results relating to strong shock propagation.

After the leisurely style of Kantrowitz, Hayes' telegram on the theory of shock, detonation and deflagration fronts comes indeed as a shock. The first part is devoted to a thorough discussion of the system of equations representing the conservation principles and the additional relations which make the system determinate—one might call it the general theory of the Hugoniot curve. In the second part, the ordinary differential equation governing the structure of the front on the basis of the Navier-Stokes equations for one-dimensional steady motion is discussed, both for the shock and the front with one-parameter exothermic reaction. For weak shocks, the description is extended to the case of slow variation in time. As with all telegrams, the onus of deciphering is on the reader—the reviewer passes on with the feeling that his appreciation of this article is rather incomplete.

The discussion of the systems of equations governing the reflection and refraction of shocks follows

(*Continued on p. 219*)

BOOK REVIEWS

(*Continued from p. 184*)

naturally on the theory of the Hugoniot curve. Polachek and Seeger's article is notable for the balance of its coverage and for the felicitous presentation of experimental records illustrating the theory.

Stever's discussion of condensation phenomena from the point of view of the wind-tunnel man includes both a guide through the kinetic theory of condensation and a description of Oswatitsch and Wegener's theories of shocks and gradual condensation. It is rounded off by a discussion of the experimental evidence and its relation to the theory.

After an illuminating introduction to one-parameter combustion by v. Kármán, Emmons discusses present knowledge on flames. No scope here for a definitive account, so Emmons concentrates instead on inviting the reader to join in the fray! The few topics singled out—flame structure, normal and oblique discontinuous fronts, vorticity production, steady flame geometry and beginnings of a stability theory—are treated with elegance, and the gaps in present knowledge are vividly underlined.

The chapter on detonations by Taylor and Tankin is distinguished from the others by the extensive use of numerical examples to support the physical argument. After a disucssion of the detailed computation of a Hugoniot curve, the viewpoint foreshadowed in Hayes' chapter is taken up that the Hugoniot relation for the detonation front needs to be complemented by a consideration of the unsteady motion of the burned gas behind the front, the possible types of stable front being determined in many cases by boundary conditions far behind the front. The same idea is applied to the case of a shock followed at some distance by a flame front, to explain one of the possible mechanisms of transition from deflagration to detonation, and the discussion of both cases is extended from the plane front to the spherical front moving at a constant speed. The chapter closes with a brief description of the theory of spinning detonation and its experimental support, and of the tentative extension of the gas-dynamical theory to explosions in liquids and solids.

The chapter on rarefied gas dynamics describes the situation at the time of writing and some of the reasons why the situation will be very different very soon. A brief survey of the method and main results of free molecule flow calculations is followed by one of the derivation of the Thirteen Moment and Burnett equations, but Schaaf and Chambré make no secret of their feeling—so well confirmed since—that their study should be postponed in favor of that of the good old Navier-Stokes equations. To give this chapter at least maximum short-term usefulness, they add an extensive survey of present experimental results, and a bibliography which surpasses even those given by the other authors.

With two exceptions, this collection of ten articles meets the specification—that it should not only be, but also remain for many years, the foremost reference work in its field.

R. E. Meyer

Jets, wakes, and cavities. By G. Birkhoff and E. H. Zarantonello. Academic Press, Inc., New York, 1957. xii + 353 pp. $10.00.

This book is a noteworthy essay in bringing together the known theories and facts concerning the flows described in the title, all of which are characterized by the presence of free streamlines or streamlines of discontinuity. For this, if for no other reason, the authors are to be congratulated in filling a real gap in the existing literature of fluid motion.

To quote from the preface "our book draws on the resources of pure and applied mathematics and on experimental physics, and it sheds light on numerous problems of hydraulics and aeronautics. Therefore, it will perhaps have the greatest interest for readers whose scientific curiosity spans all the fields just mentioned. However, we hope that others will also find it a useful and stimulating reference in connection with many special questions".

Owing to the development of high-speed computing methods which enable many calculations to be performed which hitherto would have taken a prohibitive amount of time, approximations are now the order of the day. It may therefore be permissible to say that the present work falls, using the authors' delightful phrase (page 220), "into two roughly equal halves"; the first concerned with exact mathematical methods applied to the two-dimensional motion of an inviscid liquid, the second with the sig-

nificant theory of axisymmetric jets and cavities which has developed in the last two decades, and the reinterpretation of the fundamental facts about vortex trails, and turbulent jets and wakes.

The exact two-dimensional theory of Chapters II-IX proceeds naturally by use of the complex variable and is well-classified to permit the application of uniform methods, but one would like to have seen at least one problem involving gravity flow pursued to a conclusion instead of stopping (page 204) at the point where it becomes interesting. The most valuable contributions of these chapters are contained in Chapter IV where there are brought together accounts of recent work on comparison of flows originated by Lavrentieff and simplified in a series of papers by Gilbarg and Serrin with applications to uniqueness and minimum cavity drag, and in Chapter VII on existence and uniqueness.

The remaining chapters concern (X) axially symmetric flows, (XI) unsteady potential flows, (XII) steady viscous wakes and jets, (XIII) periodic wakes, (XIV) turbulent wakes and jets, (XV) miscellaneous experimental facts. These chapters treat generally increasingly complicated situations. At each stage of complication the authors try to give a picture of what is known, organized as rationally as possible. Here rigorous mathematical theory is not always forthcoming and hypothesis and empirical methods play a large part. Nevertheless, these chapters constitute an admirable and absorbing part of the book.

L. M. MILNE-THOMSON

Elements of gas dynamics. By H. W. Liepmann and A. Roshko. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1957. xv + 439 pp. $11.00.

Many of us engaged in teaching high speed aerodynamics have eagerly awaited the publication of this book for some time now. It is the opinion of this reviewer that a book as excellent as this one was well worth waiting for. The author's aim was to provide a comprehensive text for senior-first-year graduate study in modern gas dynamics, and in this respect they could not have achieved greater success. The presentation is almost always novel; the material is discussed in the clearest physical and mathematical terms; there is never any loss in rigor; and yet, there is a clarity which makes it ideal for student use. If anything, the student may be led to believe that many of the topics covered are too elementary, however this feeling should be dispelled by the fine set of problems which require much more than just "putting in numbers" to obtain a solution.

Perhaps one of the best features of the book is that it is as modern as the latest research in the field of gas dynamics. Thus, in Chapter 1 the concepts of thermodynamics are introduced including the thermodynamics of reacting and dissociating gas mixtures, a knowledge of which is so essential for the study of hypersonic flow. Chapter 2 considers steady one-dimensional gas flow, and Chapter 3 unsteady one-dimensional wave motion along with a discussion of shock tubes. As throughout, experimental results and photographs are included to give the reader a vivid illustration of the "comparison of theory with experiment." Chapter 4 is primarily concerned with wave and wave interaction phenomena in steady two-dimensional supersonic flow, although more general problems such as the detached shock wave are also considered. Flows in ducts and wind tunnels are considered in Chapter 5, while Chapter 6 outlines methods of measuring supersonic flow phenomena. In addition to discussing the now conventional techniques such as schlieren photography, pitot measurements, etc., the more unconventional and modern X-ray absorption method, direct skin friction technique, and hot-wire methods, are also presented. In Chapter 7 the equations of frictionless flow are derived, along with general flow theorems, and a discussion of the character of the equations of motion. The small perturbation equations and boundary conditions are derived in Chapter 8 for subsonic, transonic, supersonic, and hypersonic flow, and Ackeret's two-dimensional thin airfoil theory is presented. Chapter 9 considers supersonic linearized potential flow over bodies of revolution and a clear distinction, so usually lacking, is made between first-order and slender body theory. All of the similarity laws for the high speed flow regimes are derived in Chapter 10. Chapter 11 is devoted to the elements of transonic flow and, as in the rest of the book, a very clear physical picture is given of the problems involved in this difficult field. In Chapter 12 the method of characteristics is developed by using intrinsic or natural coordinates, so that the discussion is unencumbered by the usual mass of algebra associated with a presentation of the method in cartesian coordinates. An extremely concise but nonetheless clear chapter is 13 which considers the effects of viscosity and conductivity both for laminar and turbulent flow. Here the authors introduce the problems associated with boundary layers, including the effects of dissociation, by the use of the simple and

instructive Couette flow. The most recent work in boundary layer-inviscid flow interaction phenomena are also considered. In the closing Chapter 14 the concepts of gas kinetics of importance to aerodynamics are considered, starting with the fundamental idea of probability concepts, and including such topics as the flow of highly rarefied gases, relaxation phenomenon, and continuum theory limits.

As is always true with a book as comprehensive as this one, there are some minor criticisms. For example, the book would probably have been enhanced by more adequate referencing. In addition, this reviewer would take exception with some of the views expressed in the topics which are now actively being studied, such as boundary layer-induced interactions and continuum theory limits. However, such comments can indeed be considered trivial in the light of the excellence of the book, including, by the way, the format and clarity of printing. Apart from a "must" text in any theoretical aerodynamics course, this reviewer would also recommend the book for anyone involved in high speed aerodynamics research, as well as for those persons with a suitable background who wish to obtain an overall picture of modern gas dynamics. The authors say that they hope this book will be followed by one of a more advanced and specialized nature—we can only look forward to its publication with the greatest interest.

RONALD F. PROBSTEIN

The calculation of atomic structures. By Douglas R. Hartree. John Wiley & ons, Inc., New York, and Chapman & Hall, Ltd., London, 1957. xiii + 181 pp. $5.00.

This book is based on a series of lectures given by the author at Haverford College in 1955. The practical methods of calculating atomic structures are presented in a way to be of immediate help to anyone concerned with such calculations.

Unfortunately, the results of the calculations on atomic structures are not given but a list of references to such results is given in an appendix. Recent work in calculations of this type has been appreciable with thirty or more papers appearing during the past seven or eight years. The fact that no really adequate discussion of these calculation methods has previously appeared in one place should make this book most welcome.

ROHN TRUELL

Rheology, theory and applications. Edited by Frederick R. Eirich. Academic Press, Inc., New York, 1956. xiii + 761 pp. $20.00.

This volume, the first of three intended to provide "integrated surveys" of "well-demarcated areas of rheology", described as "the science of deformation and flow," contains seventeen articles on an impressive range of topics. Most of the work is of an experimental, physical, or chemical nature and need not be described here. Furthermore, most if not all of the articles concerning mechanics or applied mathematics consist in material easily available in other surveys, often by the same authors, and it is possible that the main virtue of the volume is to put between two covers a number of different conceptions of this fluid science.

"Phenomenological macrorheology" by M. Reiner and "Dynamics of viscoelastic behavior" by Turner Alfrey, Jr., and E. F. Gurnee deal with what seems to be the same topic from the mechanical point of view and differ in that the former emphasizes terminology, the latter, the consequences of linearity and the associated operator techniques. While both articles contain passages filled with tensor equations, the methods and problems are essentially one-dimensional, and prominence is given to the devising of complicated materials by imagined couplings of simpler materials.

Only three articles approach the mathematical theories of genuine large deformation and flow from a standpoint that seems tenable in view of recent researches in general mechanics. J. G. Oldroyd's "Non-newtonian flow of liquids and solids" deals mainly with rectilinear flows. As is now well known, specialization of correct three-dimensional equations to such flows does not yield the same results as were assumed in the purely one-dimensional treatments of the older studies on rheology. The article of Oldroyd, however, concentrates upon its author's researches of some years ago and cannot be regarded as giving a just view of the field today. While the general equations for an incompressible fluid are the author's equations (46), viz.

$$p_{ik} = \eta D_{ik} + \tfrac{1}{2}\psi \sum D_{ij}D_{jk} - p\delta_{ik},$$

virtually all of the article is restricted to the case $\psi = 0$; that is, to a fluid which might be described as devoid of cross-viscosity though possessed of variable shear viscosity. Rivlin's general solutions for $\psi \neq 0$ are alluded to but not described. Moreover, more recent researches of Ericksen[1] and Stone[2] indicate that rectilinear flow generally fails to exist if $\psi/\eta \neq$ const.; in a special case, Green and Rivlin[3] have obtained an approximate solution in which a circulating cross flow is superimposed upon the flow down the tube.

William Prager's "Finite plastic deformation" is an interesting exception to the usual literature of plasticity in that it attempts to select those theoretical problems which may be interpreted as involving large deformation. Except for a few pages at the end, only the case of a body rigid up to yield and perfectly plastic thereafter is considered; even here, "finite" must be qualified, since the inertial terms in the equations of motion are neglected. This article is very clearly written and is illuminating for its well chosen discussion, examples, and figures. Most of it concerns plane flows, and the complicated calculations are omitted in favor of explanation of the results in a form comprehensible to those who are not specialists in plasticity.

"Large elastic deformations" by R. S. Rivlin attempts a task less ambitious than those set themselves by the other authors, since it is devoted to a definite and well poised problem of mechanics. Its value lies principally in explaining to those who might wish to apply it the general theory of elasticity in the form and status a considerable body of specialists now see for it. The article presents only the parts of the theory where the author has made notable contributions; this results in a certain lack of appreciation of other aspects, and in particular it seems to me that the work of Signorini does not deserve to be dismissed along with that of Seth on p. 353, and that the general approximation process initiated on pp. 382–383 was given, substantially, twenty years ago by Signorini. The reader of Rivlin's article will come away with an accurate summary of certain definite predictions of finite elasticity, but if he has no more mathematical apparatus than the author requires of him, it is unlikely that he can read further in the literature of the subject.

Another exceptional article, of a very different kind, is "The statistical mechanical theory of irreversible processes in solutions of macromolecules," by J. Riseman and J. G. Kirkwood. This is a condensed extract from one aspect of the group of statistical theories proposed by Kirkwood and his students. A much higher level of mathematical and physical preparation is needed for the reader of this article, which concerns statistical mechanics of chains and employs a Riemannian geometry for the subspace obtained by imposing constraints on the motions in the molecular configuration space. Approximation procedures are introduced to get definite results for special models.

Looking back at the volume as a whole, while we find much overlap of qualitative discussion, especially regarding names and symbols, at the same time this heavy tome seems to have found no space to present the recent exact studies of visco-elastic and time rate processes. The fundamental memoir of Oldroyd[4] is cited thrice (pp. 51, 657, 676), but in such a way that one not already familiar with it could scarcely divine its subject; the memoirs of Noll[5] and of Rivlin and Ericksen[6] perhaps appeared too late for mention. However, it seems unlikely that fundamental problems of large deformation can ever be described successfully, even *a posteriori*, at the mathematical level most of the surveys in this volume strive to maintain.

[1] Q. Appl. Math. **14**, 318–321 (1956).
[2] Q. Appl. Math. **15**, 257–262 (1957).
[3] Q. Appl. Math. **14**, 299–308 (1956).
[4] Proc. Roy. Soc. London A **200**, 523–547 (1950).
[5] J. Rational Mech. Anal. **4**, 3–81 (1955).
[6] J. Rational Mech. Anal. **4**, 323–425 (1955).

C. TRUESDELL

Experimental designs in industry. Edited by Victor Chew. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1958. xi + 268 pp. $6.00.

During recent years experimental designs have been employed at an increasing rate in industrial research and development work. The various classical designs as Latin squares, balanced incomplete block designs, etc., although they have their place in industrial research are quite inadequate for many problems that arise in industry where often the number of factors and the number of levels of each

factor are far too large to work with complete replications. The fractional replications introduced by Finney in 1945 proved particularly useful in this situation. The methods of fitting response surfaces first introduced by Box and Wilson in 1951 seem to have proven their value in situations where the factors may be applied in continuously varying levels.

It was thought that these designs had been used for a long enough time to exchange information on experiences with their use. To this purpose a symposium was held at North Carolina State College on November 5–9, 1956. The present volume contains a selection of papers presented at this symposium. The first paper by V. Chew gives a general survey of the basic ideas of the analysis of variance. The next paper by R. L. Anderson deals with complete and fractional factorials and the technique of confounding. This is followed by a paper on multiple regression analysis by R. J. Hader and A. H. E. Grandage. An exposition by J. E. P. Box and J. S. Hunter of the methods of fitting response surfaces introduced by Box concludes the theoretical part of the volume. The second part reports on experiences in the application of designs in industry in papers by W. S. Connor, W. H. Horton, M. B. Carroll and O. Dykstra Jr., De Baun and A. M. Schneider and concludes with a report by C. A. Bicking on experiences with designs and needs for designs in ordnance experimentation.

Although all papers presented will be of interest to anyone in the field of mathematical statistics, the exposition of the methods of fitting response surfaces seems of particular significance. It seems to the reviewer that these methods introduce ideas that are not only basically new but have also arisen from a very urgent practical need.

It is of course not the purpose of the papers presented in this volume to give a complete account of the mathematical foundations of the methods presented. The authors therefore restrict themselves to a presentation of the calculation procedures and a description of the situations in which the methods apply. Numerous applications to problems that have actually arisen in industrial research are presented. For those who are interested in a more thorough study a large index of references is given at the end of each paper.

H. B. MANN

Principles of noise. By J. J. Freeman. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1958. x + 299 pp. $9.25.

This book is designed to introduce graduate students, principally in electrical engineering, to the fundamentals of noise analysis. Thus, the mathematical tools—Fourier series and integrals, probability theory, stochastic processes—are developed in sufficient detail for the purpose and the physical sources of noise are described before proceeding to their analysis. The work may also serve as an elementary introduction to several recent more advanced treatises on random signals and noise. There are many well chosen examples and problems for the reader.

W. F. FREIBERGER

The preparation of programs for an electronic digital computer. By Maurice V. Wilkes, David J. Wheeler, and Stanley Gill. Addison-Wesley Publishing Co., Reading, Mass., 1957. xiv + 238 pp. $7.50.

When the first edition of this work was published in 1951 it represented the first effort to make the art of programming accessible in bookform and formed an invaluable aid in spreading knowledge of a relatively new field. Since then, developments have been rapid and books on computers are rolling off the presses. Thus, this second edition of a pioneering treatise inevitably seems somewhat outdated. The language in which the programs are presented is still the order code of the EDSAC and although material on other machines is included, it is this code the reader will have to master to understand the book. One of the most striking developments of recent years, at least in America, has been the increasing use of automatic coding systems, and there is no discussion of this interesting and important subject in the book.

This is, thus, strictly a book for the specialist who will make the effort of mastering it for the sake of the innumerable hints on methods and techniques strewn throughout its pages; the authors' wide experience vouchsafes their effectiveness.

W. F. FREIBERGER