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CONTENTS

CONTENTS

SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{\ }$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\,[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos\,(\pi x/2b)}{\cos\,(\pi a/2b)} \text{ is preferable to } \frac{\cos\dfrac{\pi x}{2b}}{\cos\dfrac{\pi a}{2b}}$$

BOOK REVIEWS

Group theory and its application to the quantum mechanics of atomic spectra. By Eugene P. Wigner. Academic Press, New York and London, 1959. xi + 372 pp. $8.80.

This is an English translation of the original well known German edition (1931) together with the addition of three new chapters. The book still deals entirely with the quantum mechanics of atomic spectra. The translator points out that the translation was motivated by the lack of a good English work on the subject of group theory from "the physicist's point of view."

There is a particularly striking comment by the author in his preface to the effect that von Laue remarked that the most important result of the original edition was the "recognition that almost all rules of spectroscopy follow from the symmetry of the problem."

While this book is written for the mathematical and theoretical physicist it has sections (such as chapter 17) which are written in a descriptive fashion for the purpose of showing the reader how to use results of group theory without becoming any more involved than necessary with the mathematical details. Such sections will certainly appeal to anyone not thoroughly familiar with the subject. At the same time the book should appeal to the advanced student of the subject particularly in view of the recent additions to the work—additions such as the time inversion transformation. The transformation $t \rightarrow -t$ provides an additional symmetry element which transforms a given state into one where all velocities (and spins) are reversed in direction.

The translation seems to have been very worth while. It should make a considerable difference in the number of physicists in the country who learn something about group theory.

ROHN TRUELL

Sampling inspection tables. Second edition. By Harold F. Dodge and Harry G. Romig. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1959. xi + 224 pp. $8.00.

In this handsome, enlarged, edition of their much used tables the authors have made many new and useful additions. Perhaps the most important is the inclusion of "Operating Characteristic (OC) Curves". These curves give the probability of accepting a lot, when using a chosen sampling scheme, as a function of the percentage of defectives in the lot. Such information is useful in choosing a scheme as it is often important to know how a scheme behaves for a range of percentage defectives.

The introduction to the tables has been expanded so that the procedure and theory of setting up single or double sampling schemes are clearly explained. Two general types of consumer protection are considered: that based on Average Outgoing Quality Limit (AOQL) and that based on Lot Tolerance Precent Defective (LTPD). AOQL schemes ensure that the average quality delivered will, almost invariably, be above some chosen limit. LTPD schemes limit the percent of defectives in each sampled lot. All those concerned with quality control should find this new edition invaluable.

W. FREIBERGER

An introduction to advanced dynamics. By S. W. McCuskey. Addison-Wesley Publishing Co., Inc., Reading, Mass., 1959. viii + 263 pp. $8.50.

In the author's words "The purpose of this introductory book is to familiarize advanced undergraduate students in science and mathematics with a few ideas of classical dynamics not ordinarily treated in their courses in elementary mechanics .... The emphasis is on the underlying principles and a few simple and familiar applications for illustrative purposes only."

The end result is quite a readable account of a rather broad range of topics as indicated by the following list of chapter headings: Fundamentals of Newtonian Dynamics, Hamilton's Principle and Lagrange's Equations, Central Force Motion, Dynamics of a Rigid Body, Oscillatory Motion, Hamilton's Equations

(Continued on p. 360)

BOOK REVIEWS

(Continued from p. 348)

and Phase Space, The Hamilton Jacobi Equation. However, the main purpose of the book, i.e. to place the emphasis on the underlying principles, cannot be regarded as fulfilled. In fact the underlying principles receive a rather cursory treatment and the presentation of some of them lacks clarity and precision and may be misleading. It may suffice to cite one instance. It is stated on p. 44: "Suppose there are n particles in the system and that $\vec{F}_j$ represents the *resultant force* acting on the jth particle. Then the principle of virtual work states that the system will be in equilibrium when

$$\sum_{j=1}^{n} \vec{F}_j \cdot \delta\vec{r}_j = 0. \tag{2-5}$$

Here δr_j is the virtual displacement of the jth particle and is arbitrary except for the constraint conditions." Of course, if $\vec{F}_j$ is indeed the resultant force, then $\delta\vec{r}_j$ need not be compatible with constraints. By $\vec{F}_j$ the author undoubtedly meant the resultant *applied* force. But nowhere in the book are the applied forces distinguished from the forces of constraint. In fact, constraint forces such as string tensions, surface supports are listed as applied forces on p. 48 of the book.

E. T. ONAT

Mathematics in physics and engineering. By J. Irving and N. Mullineux. Academic Press, New York and London, 1959. xvii + 883 pp. $11.50.

The following table of contents indicates the scope of the work:

1. Introduction to partial differential equations (68 pp.).
2. Ordinary differential equations: Frobenius' and other methods of solution (58 pp.).
3. Bessel and Legendre functions (80 pp.).
4. The Laplace and other transforms (45 pp.).
5. Matrices (59 pp.).
6. Analytical methods in classical and wave mechanics (55 pp.).
7. Calculus of variations (70 pp.).
8. Complex variable theory and conformal transformations (64 pp.).
9. The calculus of residues (82 pp.).
10. Transform theory (73 pp.).
11. Numerical methods (65 pp.).
12. Integral equations (56 pp.).

There is an appendix (84 pp.) of miscellaneous elementary topics in pure mathematics which supplements the text.

The work is suitable as a basis for a first year graduate course on mathematical methods for physicists and engineers. It is clearly and concisely written, exceptionally well printed and contains a wealth of worked and unworked examples with solutions taken from many areas of applied mathematics; the emphasis in both text and the examples is not on mathematical niceties but on applications to, for instance, elasticity, supersonic flow, electromagnetism, wave mechanics and heat flow. There is a useful list of general references at the end of each chapter.

WALTER FREIBERGER

Fachbegriffe der Programmierungstechnik. Edited by J. Heinhold. R. Oldenbourg Verlag, Munchen, 1959. 34 pp. $1.05.

There has been no field in recent years in which questions of terminology have been as confusing as in digital computer science. The present booklet is published by the Gesellschaft für Angewandte

(Continued on p. 374)

BOOK REVIEWS

(Continued from p. 360)

Mathematik und Mechanik which is engaged, with the Association for Computing Machinery in the United States, in constructing an international algebraic language, to serve as a digital computer source language for mathematical problems. An agreement on terminology is a prerequiste for such an undertaking, and this work provides an acceptable basis, for translating programming terms between the English, French, Swedish, Dutch, and Russian languages. A cursory check revealed only a few omissions, e.g. object language, source language, mask, table look-up, assembly program.

WALTER FREIBERGER

Computational methods of linear algebra. By V. N. Faddeeva. Dover Publications, Inc., New York, 1959. x + 252 pp. $1.95.

This textbook on numerical methods consists of three chapters:

I. Basic material from linear algebra (62 pages).
II. Systems of linear equations (84 pages).
III. The proper numbers and proper vectors of a matrix (96 pages).

It is a translation of a noted Russian work and is unique in its clarity, elegance and scope. There is a wealth of useful numerical examples which, with the numerical tables, have been checked by the translator. The emphasis is on methods useful with desk calculators.

The first chapter presents the mathematical background, assuming knowledge of elementary determinant theory but not of matrices. Most proofs are shown and the author works in the complex field. The Jordan canonical form is given without proof, as is the theory of elementary divisors. There is a section on vector and matrix norms and on limits.

The second chapter starts with Gauss' elimination method in its various forms, recommends the square root method for symmetric systems and demonstrates partitioning and escalator methods. A welcome feature is a careful discussion of convergence criteria for the ordinary and the single-step (Seidel) iteration methods, with numerical illustrations, but gradient methods are omitted.

The third chapter treats, firstly, of various ways of determining the characteristic polynomial, methods associated with the names of A. N. Krylov, Samuelson, Danilevskii, Leverrier, Faddeev, and of the escalator and interpolation methods for obtaining the eigenvalues. The classical iteration method for finding the largest eigenvalue, with and without accelerated convergence, is described, with λ-differencing chosen as the method for determining the following eigenvalues.

The author has unfortunately not included a discussion of how automatic computing equipment affects the choice of techniques. If she had, she would no doubt have presented Jacobi's method for eigenvalue analysis. There is a bibliography of 40 titles and a useful index. The translation is readable and the book is invaluable for everyone with any occasion to analyse linear systems.

WALTER FREIBERGER

Operations research—methods and problems. By M. Sasieni, A. Yaspan, and L. Friedman. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1959. xi + 316 pp. $10.25.

The purpose of this book is largely to provide a collection of illustrative problems for use in courses in operations research. The problems (many of which are in the form of worked examples) are primarily applications of elementary probability theory; they are classified by background topic under chapter headings of sampling, inventory, replacement, waiting lines, competitive strategies, allocation, and sequencing. A short general discussion is given at the beginning of each chapter.

The keynote of the book is simplicity, and it is refreshing to note the author's statement that "Although a great deal of elegant mathematics appears in the literature, far more problems have been

(Continued on p. 408)

BOOK REVIEWS

(Continued from p. 374)

solved by the intelligent application of simple arithmetic". This policy has led to a choice of problems which require only a direct application of the discussed methods and formulas, and which therefore serve as useful exercises in probability algebra. Unfortunately, there is a corresponding artificiality to the problems, and there are very few which are either of a really imaginative nature or which lead to thought concerning their deduction from a real situation. On the other hand, this feature would not detract too much from the use of the book as a companion text in a "case-work" kind of course.

The first chapter provides a survey of probability concepts; although the authors do not hesitate to discuss "sample spaces" or "interaction and union of derived events", they rather paradoxically omit the derivation of the Poisson distribution formula to be used throughout the book. Also, the occasional dark comments as to the inherent mathematical difficulty of probability do not seem pedagogically useful. In the chapter on Allocation, the simplex method is presented rather clearly, but with some mathematical awkwardness (cf. the incorrect statement of the key theorem of linear programming on p. 226.) As a final minor objection, the point of the "principle of optimality" (p. 272) in the chapter on dynamic programming is quite lost on the reviewer.

CARL E. PEARSON

Instationäre Wärmespannungen. By Heinz Parkus. Springer-Verlag, Vienna, 1959. v + 165 pp. $9.05.

This book is a sequel to a previous volume on steady-state thermal stresses by E. Melan and the present author. It supplements the earlier treatise by providing a fairly extensive and up-to-date account of available investigations of thermal stresses and deformations due to time-dependent temperature fields. While most of the material selected here belongs to classical elasticity theory, the book ends with an excursion into the fields of viscoelastic and elasto-plastic solids.

The opening chapter contains certain elements of the linear theory of thermoelasticity for homogeneous, isotropic media. As in the preceding volume, however, the primary emphasis is not so much on a systematic and complete exposition of basic theory as on a careful and detailed discussion of solutions to particular problems. Chapter II deals with investigations of transient temperature stresses pertaining primarily to an unbounded domain, the half-space, the sphere, and the circular cylinder. Chapter III is devoted to the effect of periodic temperature changes, whereas problems involving moving heat sources are treated in Chapter IV. These quasi-static analyses are followed by a chapter on inertia effects in transient thermoelasticity which presents a resumé of recent studies in this area. Thermal stresses in viscoelastic and elasto-plastic media are taken up in two concluding chapters.

Some of the results included by the author are in the form of improper integrals. It is difficult to appreciate the usefulness and significance of such solutions to boundary-value problems in the absence of adequate numerical evaluations which are missing in several important instances. On the other hand, integral representations of singular solutions which correspond to thermoelastic nuclei are bound to be of limited interest in the further development of thermoelasticity theory.

A brief discussion of thermoelastic coupling effects might well have been appropriate in an enterprise of this kind. Further, the chapters on temperature stresses in elastic materials are rather sketchy and reflect the ritualistic character of our present approach to such problems—an approach that is for the most part safely removed from the complexities of the actual physical situation at which it aims. A more realistic treatment of thermo-inelasticity cannot escape the essential nonlinearities attached to this topic and must come to grips with the temperature-dependence of the physical parameters which govern the thermal and mechanical behavior of such materials at elevated temperatures.

It is hardly fair, however, to blame the author, who has made a very welcome contribution to a very timely subject, for inadequacies in the present state of this subject. The book is clearly written, carefully edited, and admirably printed. The comprehensive bibliography appearing at the end of the volume will be greatly appreciated by every interested reader.

ELI STERNBERG

BOOK REVIEWS

Analysis of industrial operations. Edited by Edward H. Bowman and Robert B. Fetter. Richard D. Irwin, Inc., Homewood, Illinois, 1959. xi + 485 pp. $7.95.

A collection of 27 papers (published, in the main, within the last four years) describing actual applications of quantitative methods to the analysis of industrial operating problems. This material was collected for use in a graduate course in the MIT School of Industrial Management. There is a bias in the selection of papers towards discussions of the fitting of mathematical models to actual problems, rather than the development of new models or new methods.

The reader is presumed to have a working knowledge of differential calculus and analytical statistics, and some understanding of linear programming, waiting line theory and the techniques of Monte Carlo simulation.

The papers are divided into five classifications: Applications of linear programming; Other programming applications; Waiting line applications; Applications of incremental analysis; Total cost and value models.

BRUCE CHARTRES

Elementary decision theory. By Herman Chernoff and Lincoln E. Moses. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1959. xv + 364 pp. $7.50.

The day of determinism is past, even in the field of engineering, and it is rather amusing to remark that the determinist has inevitably brought it on himself. By constructing more and more realistic models of the physical universe, of greater and greater complexity, he has slowly forced himself into a position where the only escape from stalemate lies in the introduction of new rules into the game. The result is that we must coat ignorance of true cause and effect and inability to handle the resultant equations, even if we knew them, by the soothing, albeit sophisticated, salve of probability theory.

The modern engineer who is interested in feedback control processes, in automation, in processes involving adaptive control, and in any number of significant uses of digital and analogue computers, must acquaint himself with the mathematical theory of decision processes. He must understand the difficulties and advantages of uncertainty, and be familiar with the various tricks that are used to construct precise mathematical models of imprecise situations.

Unfortunately, many of the standard works in the field of decision-making start from a high mathematical level and continue upward. The result is that they are not suited to the needs of the beginner.

The book by Chernoff and Moses is a book designed specifically for the scientist interested in decision processes, but possessing only a rudimentary mathematical background. The authors have succeeded admirably in what they have set out to do. Carefully and slowly, they illustrate how mathematical models of decision-making under uncertainty are formulated. They discuss the choice of a criterion function, the choice of possible policies, the description by means of state variables, and so on.

All of this is done in a very pleasant, readable style, with numerous examples, and much important discussion. Most important, they are honest with the reader in constantly pointing out pitfalls and limitations. Their presentation illustrates very clearly that fundamental ideas can be transmitted without the pseudo-abstraction that befogs the Bourbaki and their devotees.

In the second half of the book, they give an introduction to classical statistics. Throughout, their aim is to follow the guiding idea of Wald who conceived of statistics as decision-making under uncertainty. In their presentation, they have followed the teaching of the late M. A. Girshick.

The book is recommended to mathematicians, physicists, engineers, and so forth, who wish to obtain a relatively painless introduction to this modern field, either for their own studies, or merely to keep abreast of current intellectual activity.

RICHARD BELLMAN