# QUARTERLY

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# APPLIED MATHEMATICS



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*Seventh Midwestern Conference*

# FLUID MECHANICS AND SOLID MECHANICS

*Michigan State University*
*East Lansing, Michigan*
*September 6, 7, 8, 1961*

An attractive and broad program is being prepared for the Seventh Midwestern Conference on Fluid Mechanics and Solid Mechanics. Authors wishing to present papers should send in their abstracts preferably by January 1, 1961.

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# BOOK REVIEWS

*Mathematical programming and electrical networks.* By Jack B. Dennis. Technology Press of The Mass. Institute of Technology, John Wiley & Sons, Inc., New York, Chapman & Hall, Ltd., London, 1959. vi + 186 pp. $4.50.

This well-written and well-organized little volume, the third in the MIT Research Monograph Series, presents the results of the novel and recent researches of the author in establishing an equivalence between mathematical programming, particular linear and quadratic, and electrical networks composed of voltage and current sources, ideal diodes, resistors, and transformers.

The central idea is based on the extremum principle that in a circuit of current and voltage sources, resistors, and transformers the currents and voltages in the various branches of the network are such as to minimize the heating power losses. This is essentially a minimization of a quadratic form in the currents when resistors are present and a linear form when no resistors are present. Additional linear constraints are provided by the Kirchoff node laws. The introduction of diodes into the circuit imposes a condition of non-negativity on the currents. Thus, with resistors one has the electrical equivalent of a quadratic programming problem and without them, a linear programming problem.

The first two chapters and the Appendix, which occupies the last third of the book, provide some general mathematical notions for mathematical programming, local and global minima, concavity and convexity, Lagrange multipliers, the Legendre transformation, duality, and a number of pertinent theorems. In view of the greater advantages, in speed, economy, accuracy, and facility, of the use of large scale digital computers in solving these problems, as the author has pointed out, it is unlikely that anyone would use these techniques to solve a practical problem. Nevertheless, this research provides essentially a new outlook and insight into mathematical programming which has heretofore relied almost completely on economic, scheduling, and allocation examples as the vehicles for the introduction to mathematical programming. The minimum employment of the economic language and examples so common to other introductions to linear programming should be to most engineers' liking.

Chapter Three describes the properties of the various electrical devices of the networks, the network laws, and the equivalences of the various network problems. This possibly should appear a little earlier so that the non-engineer can see what the author is driving at. Chapter Four presents a diode-source algorithm for solving electrical models of certain network flow problems. For the maximum flow problem this is similar to the Ford-Fulkerson algorithm for transportation problems. The central notion here, useful throughout the remainder of the book, is that of a breakpoint curve, a curve of possible voltage-current combinations for a diode. The algorithm, elaborated further in Chapters Five and Six, consists in following this curve through in a manner analogous to the consecutive steps of the simplex method of Dantzig. Certain modifications to the algorithm and several other electrical algorithms called "valve" and "by-pass" algorithms are also presented and applied to different problems.

The final chapter proposes an iterative method for solving general equality- and inequality-constrained minimization problems based on a quadratic program. This method does not appear to differ widely from some methods in current practice. Errors are very few and relatively minor in consequences. The bibliography is adequate.

M. L. JUNCOSA

*An introduction to statistical mechanics.* By J. S. R. Chisholm and A. H. de Borde. Pergamon Press, Inc., New York, London, Paris, Los Angeles, 1958. ix + 160 pp. $6.00.

This book is designed as a text suitable for an Honours course in British Universities. The subject matter is limited to equilibrium phenomena and the applications are those which can be found in most books on statistical mechanics written during the last twenty-five years or so. The main virtues of the book are that it is short and clearly written. The scope, however, is very limited and there is no indication of the modern trends of the subject. The author claims to have a simplified version of the Darwin-

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Fowler method for obtaining the canonical distribution. The details of this, which are given in an appendix, are sound but the outline of it given in the text is deceptive. In essence the book is a condensed version of the books by Fowler.

G. H. NEWELL

*Theory of value, an axiomatic analysis of economic equilibrium.* By Gerard Debreu. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1959. xii + 114 pp. $4.75.

This book is a landmark in the development of mathematical economics. Coordinating the work that has gone into the central body of economic theory since Cournot and recasting it in the mold of contemporary mathematics, this slender volume gives the definitive treatment of the Theory of Value constructed rigorously from first axioms.

Chapter 1 assembles all the mathematical concepts and propositions to be used in the economic chapters. Evidently a great deal of thought has gone into such organizational matters as notation and layout, and this pays off handsomely in clarity and readability throughout the book. The departure from traditional calculus-bound methods is apparent in this box of tools: set and point set theory, ordering, metric spaces, and topology including fixed point theory. Chapter 2 clarifies all economic concepts employed and will be particularly useful to the mathematical reader as a grammer of the economist's language. Economists on their part will be delighted at the crystal clarity with which the basic concepts and underlying assumptions are here exposed. The analysis gets under way in chapter 3 with a discussion of the theory of production. The axioms on technology are presented and discussed in order of decreasing plausibility. These include the technology assumed in Linear Programming models as a special case. The implications of profit maximization are studied and "comparative statics" is analyzed. The place of derivatives and of Lagrangian Multipliers is taken here by supporting hyperplanes to convex sets. The 11 pages of this analysis are the most compact complete treatment of the general theory of production in existence.

In Chapter 3 the theory of consumption is constructed analogously on the basis of complete, transitive, continuous preference orderings on closed, connected consumption sets. The existence of (ordinal) utility functions is proved and the theory of consumers' choice subject to wealth constraints is developed. The high point of the book is reached in Chapter 5 where the existence of an equilibrium for a "private ownership economy" characterized essentially by non-increasing returns in production and insatiable consumption subject to non-increasing marginal utility (these are old-fashioned economic terms not used here) is proved from Kakutani's fixed point theorem. Uniqueness and stability are deliberately excluded from this theory, presumable because the former is essentially trivial but messy and the latter leads into the vast area of dynamic processes where no orderly system is as yet in sight. Chapter 7 treats briefly the optimality of the equilibrium. This is followed in Chapter 8 by an extension of the model to a situation of uncertainty which interestingly enough does not involve any probabilistic considerations—and hence has no need of the apparatus of expected (cardinal) utility. This final chapter gives in effect a condensed summary of the entire theory of this book.

One can only marvel at the precision, the elegance, and the perfection of this work. Throughout, it is an original, penetrating statement of the heart of neoclassical economic theory, the theory of value and equilibrium, and it brings out as never before the unity and architectural beauty of this body of thought. Its impact will be felt by every mathematical economist. Without question this book will be a standard reference. Beyond this it should prove a boon to all mathematically educated persons in search of a royal road to economic theory.

With this latest volume the series of Cowles Foundation monographs—including such classics as "Activity Analysis of Allocation and Production" and "Statistical Inference in Dynamic Economic Models"—has achieved a new high point. Alfred Cowles, to whom this book is dedicated has indeed been honored.

MARTIN BECKMANN

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*Applications of the theory of matrices.* By F. R. Gantmacher. Interscience Publishers, New York, London, 1959. ix + 317. $9.00.

This is a translation of the second part of the author's *A Theory of Matrices* which was published in 1954 in Russian. As such it constitutes a unique mathematical monograph. The applications alluded to in the title lie in other fields of mathematics. The reader should possess considerable maturity in these fields as well as in matrix theory in order to appreciate the elegance of the author's developments.

There are five chapters. The first two, treating the normal forms of complex matrices and singular pencils of matrices, are quite formal being little more than a sequence of lemmas and theorems. The third chapter treating matrices with non negative elements begins formally but becomes more readable when it turns to applications to Markov chains with a finite number of states and to oscillating matrices. The two final chapters contain the major applications. Chapter 4 is a study of systems of linear differential equations and Chapter 5 is devoted to the Routh-Hurwitz problem.

As an indication of the level, the chapter on differential equations includes Lyapunov's stability theory, the theory of singular points and works up to conclude that certain results of V. Volterra and G. D. Birkhoff concerning isolated singular points were erroneous due to insufficient examination of the degenerate cases involving nonlinear divisors.

The author's developments are elegant and worthy of study by experts in the fields mentioned. In one case they appeared overly elaborate to this reviewer. In Chapter 3 the properties of oscillating matrices are developed to show that certain vibratory systems (e.g., a stretched string carrying $n$ masses) possess the Sturm properties. Apparently the author was unaware that in 1884 E. J. Routh had demonstrated these same results in a much more direct manner by extending Sturm's argument to finite difference equations.

STEPHEN H. CRANDALL

*Matrix calculus.* By. E. Bodewig, Second Edition. North Holland Publishing Co., Amsterdam, and Interscience Publishers, Inc., New York, 1959. xi + 452 pp. $9.50.

Recent progress in digital computer techniques for the determination of the eigenvalues and -vectors of large matrices is reflected in this new edition by the inclusion of Lanczos's $pq$-algorithm, Rutishauser's $LR$-algorithm and Wilkinson's method. Several hitherto unpublished results by the author are also described. The work retains its character as an invaluable fund of information on matrix methods and their relative merits in different context.

WALTER FREIBERGER

*Mathematical methods and theory in games, programming, and economics.* By Samuel Karlin. Volumes I and II. Addison-Wesley Publishing Co., Inc., Reading, Mass., and London, 1959. x + 433 pp. (Vol. I), xi + 386 pp. (Vol. II). $12.50 each.

The last twenty years have seen the remarkable growth of a new branch of applied mathematics—the use of mathematics in decision making within both business and government. These two books present a systematic and elegant account of many of the mathematical methods which have been developed for this subject, written by one of its leading practitioners.

The first four chapters of Volume I are fairly standard material about matrix games. Then come linear programming—theory and computational methods—and a chapter on nonlinear programming. The various famous special types of problems (optimal assignment, transportation, network flow, etc.) are taken up in detail. Fenchel's theory of conjugate convex functions is developed from a new geometric viewpoint, and is applied to get a duality theorem for nonlinear programming. The rest of Volume I deals with theories of production and consumption, with welfare economics, and with the dynamics

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of a competitive economy, in particular, the convergence over time of the price vector to an equilibrium. For the latter problem a key condition is gross substitutability of commodities, which in mathematical terms says that a certain matrix is of Metzler type (i.e., all nondiagonal elements are nonnegative.)

Volume II is devoted to continuous games, in particular polynomial games, games with convex or bell-shaped kernels, games of timing (duels) and poker models. The main emphasis is on finding the specific structure of the optimal strategies for each player. The art of doing this is somewhat like that of solving differential equations. In each case one must judiciously choose a method which exploits the special features of the kernel; and quite often a good initial guess about the form of the solution is also needed. The methods involve, for example, the geometry of moment spaces, integral equations with positive kernels, and the Neyman-Pearson lemma.

These books are intended partly as textbooks for graduate students and mature undergraduates. They ought to have an important influence on the coming generation of economists and operations researchers. Numerous good problems have been included. In addition the reader is sometimes expected to fill in not entirely obvious details as he goes along, e.g., (p. 123, Vol. I) the fact that a linear function which is bounded above on a closed, unbounded, convex polyhedron $P$ attains its maximum in $P$. There are Appendixes (worth reading for their own sakes) dealing with vector spaces, matrices, convex sets, and convex functions, to fill possible gaps in the reader's background in these subjects. A knowledge of the rudiments of general topology and real function theory will prove helpful, especially in Volume II. The two volumes are independent of each other; in fact, both have the same first chapter (about matrix games) and the same Appendixes.

WENDELL H. FLEMING

*Hydrodynamics.* By D. H. Wilson. St. Martin's Press, New York, 1959. viii + 149 pp. $5.50.

The purpose of this book is best described by quoting a paragraph from its preface.

"This book is intended primarily for students specializing in mathematics or theoretical physics but will also be found useful by General Degree and Engineering students. Its object is to provide a brief and concise introduction to 'classical' hydrodynamics while at the same time providing the basis and acting as a starting point for the study of the many modern developments in fluid mechanics."

The author suceeds admirably in reaching his objectives. He uses vector calculus to derive, in a very lucid manner, the equations of fluid mechanics in Chapter I. His representation of the basic principles can be recommended for perusal to students who value a fortunate combination of physics and rigorous mathematics. Chapter II proceeds, in conventional fashion, with a study of two dimensional motion of the inviscid incompressible fluid. Chapter III consists of a careful study of vortex motion. Chapters IV and V are devoted to the conformal mapping technique for two dimensional potential flows and the stream function technique for axisymmetrical potential flows respectively. The book closes with a study of viscous motion in Chapter VI. It contains 147 pages.

The author is to be commended for this clear and concise presentation. The book is a worthwile addition to the library of any student of fluid mechanics.

P. F. MAEDER