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STATISTICAL DECISION THEORY

By LIONEL WEISS, Cornell University. *McGraw-Hill Series in Probability and Statistics.* Ready in August, 1961.

A text describing and developing modern statistical decision theory at an intermediate mathematical level. The first four chapters develop the necessary probability theory. The next four chapters cover statistical decision theory, including linear programming as a computational tool and problems involving making a sequence of decisions over time. The final chapter develops the standard techniques of conventional statistical theory as special cases of statistical decision theory. Requires elementary calculus.

ELEMENTS OF QUEUING THEORY WITH APPLICATIONS

By THOMAS L. SAATY, Office of Naval Research. Ready in September, 1961.

This book presents a variety of queuing ramifications, methods of treatment, and in general provides a broad account of the rapid development in this challenging field. Most of the fundamental ideas of queues are discussed and developed. Many applications are described and discussed, in addition to a discussion of both Poisson and non-Poisson queues with different queuing disciplines. Bibliography of queues included.

MATRICES AND VECTORS

By JACOB T. SCHWARTZ, New York University. Ready in September, 1961.

An elementary, practical introduction to matrix algebra designed for the senior high school or early college student and intended to bring the relatively inexperienced student to a point where he can appreciate some sophisticated approaches to mathematics. Covers algebra of matrices; the minimal equation and its use in inverting matrices; systems of linear equations; geometry of vectors in 2, 3, and n-dimensions; and some special additional topics in the algebra and analysis of matrices.

Academician V. I. Smirnov's LINEAR ALGEBRA AND GROUP THEORY

By V. I. SMIRNOV, U.S.S.R.; revised, adapted, and edited by RICHARD A. SILVERMAN, Formerly of MIT and New York University. 464 pages, $12.50.

In this unique work, an internationally known authority offers several features of special interest to English-language readers. In addition to a detailed treatment of linear algebra, it also gives an excellent introduction to group theory and an extensive discussion of group representations.

SURVEY OF NUMERICAL ANALYSIS

Edited by JOHN TODD, California Institute of Technology. Ready in September, 1961.

The work of 14 nationally known authors, this book covers numerical analysis, both classical and modern, together with accounts of certain areas of mathematics and statistics which support it yet are not adequately covered in current literature. The first third of the book provides a basic training in numerical analysis and the remainder of the text is devoted to accounts of current practice in solving, by high speed equipment, special types of problems in the physical sciences, engineering and economics.

INTRODUCTION TO PARTIAL DIFFERENTIAL EQUATIONS

By DONALD GREENSPAN, Purdue University. *International Series in Pure and Applied Mathematics.* Ready in August, 1961.

Designed for a one-semester course at an advanced level, this text is particularly suited for undergraduate and graduate students who do not have a previous knowledge of ordinary differential equations, Fourier series, and complex variables. Requiring only a facility with advanced calculus, the text emphasizes second order equations, and explores both practical methods of solution and the unifying theory underlying the mathematical superstructure.

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BOOK REVIEWS

Solutions numériques des équations algébriques. By E. Durand. Volume 1. Masson et Cie., Paris, 1960. vii + 327 pp. $13.27.

Whereas the present volume is concerned with the numerical solution of a single equation, a forthcoming second volume is to treat systems of equations. Three introductory chapters (90 pp.) present methods that apply to transcendental as well as algebraic equations (series development, continued fractions, iteration, inverse interpolation, etc.). The principal part of the work, which is concerned with polynominal equations begin with a chapter on computing with polynomials (20 pp.). There follow chapters on transformations of polynomial equations (35 pp.), location of the roots in the complex plane (25 pp.), methods using the powers of the roots (34 pp.), consecutive determination of the roots by iteration (45 pp.), and simultaneous determination of the roots by iteration (14 pp.). A brief chapter on double precision and floating point arithmetic (14 pp.) concludes the volume.

Through out the book the exposition is pragmatical, making only very modest demands on the reader's mathematical background. A wealth of numerical examples illustrate all important features of the methods.

W. PRAGER

Qualitative theory of differential equations. By V. V. Nemytskii and V. V. Stepanov. Princeton University Press, Princeton, New Jersey, 1960. viii + 523 pp. $12.50.

This volume is a translation of an excellent two-volume Russian work on the qualitative and quantitative theory of differential equations. The first half of the book (Volume One of the original) is devoted to a study of the stability of solutions of linear and nonlinear differential vector equations of the form $dx/dt = A(t)x + f(x, t)$, while the second half covers topological dynamics and ergodic theory.

Both halves of the book are well-written and lucidly presented. The second part will be of most interest since the material of the first part is available in a number of contemporary books in the English language and in translations of the American Mathematical Society.

RICHARD BELLMAN

Theory and solution of ordinary differential equations. By Donald Greenspan. The Macmillan Co., New York, 1960. viii + 148 pp. $5.50.

This is an excellent outline for an introductory text in the theory of ordinary differential equations. The only serious weakness as far as the outline is concerned is the lack of matrix techniques in dealing with N-th order linear equations and linear systems.

The author attempts to present in 147 pages, everything from basic concepts of point sets to linear equations to existence theory; from power series solutions to approximate solutions to Sturm-Liouville theory. It can't be done.

RICHARD BELLMAN

Mathematical tables. Volume 4: Tables of Weber parabolic cylinder functions and other functions of large argument. By L. Fox. Her Majesty's Stationery Office, London, 1960. iii + 40 pp. $1.85.

The main table gives the Weber functions $W(a, \pm x)$, which occur in the solution of the two-dimensional wave equation in parabolic co-ordinates, in the range $x^{-1} = 0(0.005)\ 0.1$, $a = -10(1)10$ to an accuracy of 8 significant figures. In addition, the exponential integral, sine and cosine integrals, Airy integrals, the error integral and the first six factorial functions are tabulated to 10 significant figures for large values of the argument x. Full interpolation facilities are provided.

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BOOK REVIEWS

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Theoretical physics in the twentieth century. Edited by M. Fierz and V. F. Weisskopf. Interscience Publishers, New York and London, 1960. x + 328 pp. $10.00.

Pauli was considered by many physicists to have a deeper and keener insight into the fundamental problems of physics than anyone else in the past 50 years. This memorial volume to Wolfgang Pauli supplies the evidence for such a claim. There seems to be practically no field in physics to which Pauli has not contributed, and this account of his contributions written by experts makes extremely interesting reading.

ROHN TRUELL

Special relativity. By W. Rindler. Interscience Publishers, Inc., New York, 1960. x + 186 pp. $2.25.

This is intended to be a concise introduction to the subject of special relatively and it is. It is a text that can be recommended for seniors and first year graduate students. The reviewer has found the problems and exercises from the first four chapters useful for seniors studying wave phenomena and radiation physics. The book assumes only that the reader is familiar with calculus and vector methods. Tensor theory is developed, to the extent needed, in an appendix.

ROHN TRUELL

Annual review in automatic programming. Volume I: Papers read at the Working Conference on Automatic Programming of Digital Computers held at Brighton, 1-3 April 1959. Edited by R. Goodman. Pergamon Press, New York, Oxford, London, Paris, 1960. xi + 300 pp. $10.00.

Aside from papers on automatic coding systems for specific computers (mostly British) there are the following papers of more general character: Future trends in automatic programming (A. E. Glennie)—Some problems of a universal autocode (K. A. Redish)—The application of formula translation to automatic coding of ordinary differential equations (J. P. Cleave)—The philosophy of Programming (S. Gill)—Automatic programming and business applications (G. Cushing)—Auto-programming for numerically controlled machine tools (J. E. Meggitt).

Continuous geometry. By John von Neumann. Foreword by Israel Halperin. Princeton University Press, New Jersey, 1960. xi + 299 pp. $7.50.

This is a carefully edited and corrected reprint of lecture notes multigraphed in 1935–7; it provides a definitive technical account of a fascinating generalization of projective geometry, invented by von Neumann. This generalization has applications to operators on Hilbert space, and hence it sheds some light on the foundations of quantum mechanics.

No discussion of these applications is given in this book. However, as a display of brilliant algebraic techniques and incisive axiomatic analysis, by one of the most versatile pure *and* applied mathematicians of this century, the book is still extremely stimulating. In particular, it is one of the most penetrating studies available of the abstract algebra of subspaces of function space under intersection and union, and of the related algebra of semisimple and "regular" rings.

GARRETT BIRKHOFF

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(*Continued from p. 136*)

Classical mechanics. By H. C. Corben and P. Stehle. John Wiley & Sons, Inc., New York, 1960. xi + 389 pp. $12.00.

The first edition of this widely used text was reviewed on p. 334 of volume 9 of this *Quarterly*. The present edition contains many changes, the most important of which are as follows. A chapter on time-dependent forces and non-conservative motion has been added containing sections on the inverted pendulum, rocket motion, atmospheric drag, the Poynting-Robertson effect, and the damped oscillator. The presentation of the Hamilton-Jacobi theory has been made more concise and a chapter on special applications of this theory has been added containing sections on non-central forces, spin motion, variational principles in rocket motion, the Boltzmann and Navier-Stokes equations. A chapter on continuous media and fields has been added and the chapter on orbits of particles in high energy accelerators has been considerably expanded. The new edition has four appendices dealing with Riemannian geometry, linear vector spaces, group theory and molecular vibration, and quaternions and Pauli matrices; the second of these replaces a chapter of the earlier edition.

W. PRAGER

Theoretical hydrodynamics. By L. M. Milne-Thomson. Fourth edition. The Macmillan Co., New York, 1960. xxviii + 660 pp. $11.00.

According to a statement in the preface, this edition differs from the preceding one primarily by the addition of articles concerned with the following subject: the formulas of Plemilj, flows under gravity with a free surface, an exact treatment of the surface wave of constant form and the resulting "exact linearized theory," and comparison theorems including Serrin's "under-over" theorem.

Theory of thermal stresses. By B. A. Boley and J. H. Weiner. John Wiley & Sons, Inc., New York, 1960. xv + 586 pp. $15.50.

Interest in thermal stresses and their importance in engineering science have increased rapidly during the last decade. The book presents the extensive material in a clear manner. Written in a broad style, it is divided into two main parts. In the first part, the mechanical and thermodynamical foundations of the theory are laid. Although the laws of thermodynamics are introduced in a very general form, their application to the thermal stress problem is restricted from the outset to small deformations and to linear elastic and viscoelastic media. A number of uniqueness proofs for various cases is given in considerable detail. The—in general negligible—influence of inertia effects and of the coupling between the heat equation and the stress-strain equations is thoroughly and clearly discussed. Illustrative examples are given. Of course, most of the particular problems treated are of the uncoupled, quasi-static type.

A long chapter—more than one hundred pages—is devoted to a presentation of the theory of heat conduction. In the opinion of the reviewer this inclusion makes the book unnecessarily bulky. The excellent treatise by Carslaw and Jaeger will have to be consulted anyway by everyone interested in thermal stress and, hence, in heat conduction.

The second part of the book is, apart from a few cross-references, completely independent of the first. The basic equations are developed once more; this time, however, from a more intuitive point of view. First, only purely elastic systems are considered, coupling effects and, with two exceptions, inertia effects being neglected. Beams, built-up structures and plates are treated in detail. A chapter on thermoelastic stability is included. The final three chapters deal with inelastic systems and present a very good introduction to linear and nonlinear viscoelastic and to plastic stress analysis.

It seems that the book, which to a considerable extent is based on original work of the authors and their coworkers, has been written with two different types of readers in mind: the scientist who wants

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to be informed about the present state of the art, and the practicing engineer who has to solve a given concrete problem. The latter will find a wealth of useful material in the form of tables, graphs, formulas and references. To the first the limitation of the general chapters to linearized theory will probably be somewhat disappointing. For both groups, however, the book will prove to be an indispensable tool.

H. PARKUS

Stochastic population models in ecology and epidemiology. By M. S. Bartlett. John Wiley & Sons, Inc., New York, and Methuen & Co. Ltd., London, 1960. x + 90 pp. $2.00.

This monograph discusses models for biological populations in ecological and epidemiological contexts where a full stochastic formulation is advisable. It is noteworthy that even such simplified models as are presented imply complex consequences. First various models to determine the frequency distributions for single species are presented; the Poisson model is the simplest of these; birth-and-death processes and the problem of extinction are next discussed. Growth and interaction, and competition, are complications leading to more complicated models. In the concluding chapters, epidemiological models are considered for the vicissitudes of animal or human populations subject to infection by some disease transmitted from individual to individual. The restriction to a closed population is relaxed to permit recurrent epidemics and in the last chapter, epidemiological problems are noticed in which the spatial or topographical factor is relevant.

The work will appeal to biometricians with the necessary background, to theoretically minded ecologists and other interested research workers in biology and medicine.

W. FREIBERGER

Automatic language translation. By Anthony G. Oettinger, with a Foreword by Joshua Whatmough. Harvard University Press, Cambridge, Mass., 1960. xix + 380 pp. $10.00.

About a dozen projects in mechanical language translation are being currently conducted in the United States alone; the great majority of them have, as their ultimate aim, a reasonably accurate and smooth translation from Russian into English, mostly in scientific and technical fields. This is also the case with the research being conducted at the Computation Laboratory at Harvard University of which Mr. Oettinger is the principal investigator and this book a thorough and informative progress report.

The book under review here deals with only one of the major problems which have to be resolved in machine translation research: the compilation of an adequate bilingual dictionary in a form which would make it readily accessible to the computer (stored on magnetic tape in the Harvard project), and the devising of programming procedures through which the words of a running text could be accurately matched with entries in this dictionary, thus achieving one of the essential steps of translation, the finding of lexically equivalent or near-equivalent elements in two languages. Once this step has been mastered, a rough approximation of the original text in another language, a so-called word-by-word translation, can be achieved. Word-by-word translations are of some use but are basically unsatisfactory because they contain no information about the grammatical properties of words or their syntactic relationship (except what may be communicated through lexical characteristics or the preservation of the original order of items), not even speaking of their complete disregard of such major problems as multiple meanings of words, idiomatic and other non-equivalent constructions, correct word-order, etc. Nevertheless, the compilation and operation of an automatic bilingual dictionary is a prerequisite to research in the other essential aspects of translation by data-processing machines, be it the mechanical analysis of the grammar and syntax of the source langugage, or the generating of grammatically and syntactically acceptable strings in the language into which the translating is being done. Oettinger discusses these other aspects only in general terms, pointing out that the Harvard project has not

progressed beyond the initial stages in such research. However, the fact that this book deals primarily with the compilation and operation of an automatic dictionary, possibly one of the more easily manageable of the major steps in mechanical translation, does not detract from its importance. In spite of the occasional optimistic newspaper reports, the time when a good translation can be obtained entirely by automatic means is not exactly around the corner. Thus, the thorough treatment which Oettinger gives to the work already accomplished at Harvard and his sober discussion of the limitations and difficulties which still must be overcome in the future will be appreciated by those researchers interested in automatic linguistic analysis and translation, who are aware of all the complexities of this field of research.

The book is divided into ten chapters, which are interspersed with numerous tables, diagrams, appendices, and reproductions of actual machine-produced lists. Some chapters also contain special bibliographical information in addition to the sizeable bibliography at the end of the volume. Although the author tried to make the discussion of information-processing machines and their programming as general as possible, the book is unavoidably strongly oriented toward the Univac I computer which the Harvard project has been using, and some of the special characteristics as well as the limitations of this machine have determined the approach to individual problems.

The first chapter is a general introduction to the organization and programming of data-processing machines, with the Univac system serving to illustrate the individual points. The chapter begins with a very elementary outline of the principles of such machines, obviously intended for the complete beginner in the field, but the speed with which the subsequent complex information is introduced is such that one may doubt whether a reader who has never studied a computer manual would be able to follow the explanation for more than a few pages. The first chapter also contains several sample problems and a number of exercises (without solutions supplied). The exercises, in the later chapters, get progressively rarer until, toward the end of the book, their occasional appearance in the midst of the discussion cannot but impress the reader as a distinct afterthought. I found this apparent hesitation about the format of the book, and the sporadic and not very effective attempts to give it a bit of a textbook character, rather distracting.

The second chapter, which deals with the structure of signs and pays particular attention to the properties of sets, is well organized and informative but still comprehensible to the non-specialist. It also contains a description of the Cyrillic Unityper, a specially adapted typewriter used for transcribing characters onto magnetic tape. The machine serves, in lower-case, for encoding Russian characters while, in upper-case, the usual layout of Roman letters is maintained. Chapter three discusses flow charts and automatic programming, and includes an illustrative flow chart and programming information for the transformation of the computer representation of Russian characters into one of the commonly used transliteration systems of Russian through letters of the English alphabet. Chapter four investigates the various aspects of a translation process and states the assumptions which make mechanical translation a realistic, even if complex, possibility. The bulk of the book, chapters five to nine, are devoted to the compilation and operation of a dictionary. The Harvard Automatic Dictionary contained, at the time this book was written, some 22,000 stem entries (it has apparently been enlarged to 30,000 since then). One stem entry does not necessarily correspond to a word in the conventional sense; one word may be represented by one or more stem entries, depending on the degree and complexity of the inflection which it undergoes. However, the Harvard Automatic Dictionary is not a "paradigm dictionary" which would list all the inflected forms of a word separately, but rather something of a "canonical dictionary", containing only those stems which are necessary for the successful matching of the strings in actual text and, of course, a list of affixes, so that strings of the text can be factored before they are matched with the dictionary entries. The dictionary is compiled in several steps, some of which are automatic. The appropriate words are selected for inclusion from conventional dictionaries and other sources, and then manually classified into their various word-classes (nouns, verbs, etc.) and inflectional sub-classes. This classificatory information is included when the words are transcribed onto magnetic tape. The computer, supplied with a list of inflectional endings and a program for applying them, generates the complete paradigm of each word (if that word can indeed be inflected); after this has been accomplished, all the forms now in the list are automatically factored into stem and affix, duplications eliminated, and the final list of distinct stems entered in the dictionary, together with the English equivalents which have been assigned to them by a human translator. The running text which is to be analyzed is processed in a similar fashion, with words factored into stems and affixes. Since the same algorithm is used in the generation of stems which become entry keys in the dictionary and in

the operation to factor strings of the analyzed text, there is a guarantee that a matching stem will be found for each word of the text, provided, of course, that such a word had been originally processed for inclusion in the dictionary.

The final format of an entry in the automatic dictionary consists of 30 machine words (of 12 characters per word) which include, besides the Russian stem and one or more of its English equivalents, the class designation of the Russian item and various other information, such as the specification of verbal aspect or syntactic government by verbs, which are intended for use in the subsequent automatic grammatical analysis.

Mr. Oettinger also gives an actual example of a word-by-word translation (which he calls a "trot"), containing all the possible English meanings included in the dictionary for every Russian word. Through manual post-editing which supplies some grammatical information and selects the appropriate English alternative, the "trot" can result in a useful, even if inelegant, translation.

The last chapter, with the descriptive title "From dictionary to translation", discusses the future plans of the Harvard project which will hopefully lead to an acceptable mechanical translation of Russian texts.

The book is well organized, with some minor exceptions. The numerous tables and diagrams, which are scattered throughout, are referred to in the text by their numbers rather than by pages on which they occur; this results in a tedious search when they are mentioned in various places in the book. The index which completes the volume leaves something to be desired.

As a whole, this book is a competent and informative contribution to a complex field of research. As mentioned previously, it deals, in a detailed manner, only with one of the essential aspects of automatic translation, the lexical one, and it is careful to avoid any implications that the other major steps which still must be tackled are about to be resolved. After reading the book, one is left with the impression that research in automatic translation has made considerable strides in the last few years but that, all the same, the day when one could have a secretary copy a Russian article onto magnetic tape and get, in a few minutes, a good English translation from the computer is not yet within sight.

The foreword by Joshua Whatmough, which is promisingly announced on the title page and on the dust-jacket, is only a brief statement of somewhat whimsically expressed praise for the book.

HENRY KUCERA

A guide to mathematical tables. By A. V. Lebedev and R. M. Fedorova. English edition prepared from the Russian by D. G. Fry. Pergamon Press, New York, 1960. xlvi + 586 pp. $15.00.

The tables listed are arranged under the following headings: Powers, rational and algebraic functions—Trigonometric functions. Various values connected with the wide and the sphere—Exponential and hyperbolic functions—Common and natural logarithms—Factorials, Euler integrals and related functions—Sine and cosine integrals, exponential and logarithmic integrals and related functions—Probability integrals, probability distribution functions, and related functions—Elliptic integrals and functions—Legendre functions and polynomials—Cylinder functions—Certain special functions and integrals—Solutions of certain equations—Sums and values connected with finite differences—Mathematical constants—Primes, factors and products, quotients and fractions. While the text has been translated all tabular matter has been directly reproduced from the Russian original. This involves minor deviations from the customary notations (e. g. the use of tg for tan) but has the great advantage of excluding typographical errors that might have arisen if this material had been reset.

Stationary processes and prediction theory. By Harry Furstenberg. Princeton University Press, New Jersey, 1960. 283 pp. $5.00.

The question of the "predictability" of a sequence is examined in this study. Specifically, given a sequence of observations $\cdots, \xi(-n), \cdots, \xi(-1), \xi(o)$ (possibly nonnumerical), the notion of prediction of future possible values is considered. The discussion is highly motivated by the statistical theory of prediction (see the work of A. N. Kolmogorov, N. Wiener and their followers) but is formulated in rather abstract language. The sequence of observations can be thought of as the realization of a random

process in the past and present. If the realization is consistent with stationarity of a potential random mechanism generating it, a prediction in terms of a probability distribution on possible futures is proposed. The presentation rests heavily on the theory of commutative C*-algebras.

M. ROSENBLATT

The potential theory of unsteady supersonic flow. By J. W. Miles. Cambridge University Press, New York, 1959. xii + 220 pp. $8.50.

This monograph is a purely theoretical one, essentially on the unsteady aerodynamics of slender bodies in supersonic flow. The treatment is limited to frictionless potential flows and, almost entirely, to linearized flows. Within these bounds of subject matter the author has prepared a comprehensive and well-organized monograph. In the book are evident not only the results of the author's own extensive research in the field but also his thorough knowledge of the literature.

The first three chapters introduce the subject, by presenting the basic linearized equations and discussing their transformations. Chapter 4, treating reduction of the equations to those for steady flow, includes the reductions of Magnaradze and of Garner. Succeeding chapters treat in detail two-dimensional problems, simple planforms, the rectangular wing, the quadrilateral wing, slender wing methods, the delta wing, the low aspect ratio rectangular wing, and non-planar bodies. A final chapter on nonlinear problems and an appendix on reverse flow theorems close the monograph. A bibliography and three indices are provided.

This excellent monograph is one of a series of monographs on quite specialized topics in mechanics and applied mathematics. The availability in organized treatments of such material is of great value to those not already specialists in a field. Few indeed could easily get an understanding of the state of the art in the theory of unsteady supersonic flow from sources other than Miles' treatment. The editors are to be commended.

WALLACE D. HAYES

Digital computers and nuclear reactor calculations. By Ward C. Sangren. John Wiley & Sons, Inc., New York, London, 1960. xi + 208 pp. $8.50.

This book falls naturally into two parts: the first four chapters constitute an introduction to digital computers, programming, numerical analysis and reactor problems, and the last four discuss in some detail a particular reactor problem, namely fission-product poisoning, as a typical example of the application of computers to reactor physics.

The style is leisurely and most readable, the argument on the whole elementary. The book serves as an excellent survey for those without prior knowledge or specialised interest, or as an introduction with methods for generating random numbers obeying given probability distributions. In "Fourier to more advanced works, such as "*Numerical methods for nuclear reactor calculations*" by G. I. Marchuk (see Quart. Appl. Math., Vol. 18, p. 36), for anyone intending to work in the field.

WALTER FREIBERGER

Contributions to probability and statistics. Essays in honor of Harold Hotelling. Edited by I. Olkin, S. G. Ghurye, W. Hoeffding, W. G. Madow, H. B. Mann. Stanford University Press, California, 1960. x + 517 pp. $6.50.

This book is a collection of forty-two papers on probability and statistics, mostly mathematical statistics, dedicated to Harold Hotelling on his sixty-fifth birthday. Most of the papers are results of new research rather than expository papers.

G. F. NEWELL