QUARTERLY

OF

APPLIED MATHEMATICS

VOLUME XXI OCTOBER • 1963 NUMBER 3

QUARTERLY OF APPLIED MATHEMATICS

This periodical is published quarterly by Brown University, 180 George St., Providence, R. I. 02912. For its support, an operational fund is being set up to which industrial organizations may contribute. To date, contributions of the following companies are gratefully acknowledged:

AVCO RESEARCH LABORATORY; EVERETT, MASS.,
BELL TELEPHONE LABORATORIES, INC.; NEW YORK, N. Y.,
THE BRISTOL COMPANY; WATERBURY, CONN.,
CALIFORNIA RESEARCH CORPORATION; SAN FRANCISCO, CAL.,
CURTISS WRIGHT CORPORATION; AIRPLANE DIVISION; BUFFALO, N. Y.,
EASTMAN KODAK COMPANY; ROCHESTER, N. Y.,
GENERAL ELECTRIC COMPANY; SCHENECTADY, N. Y.,
GULF RESEARCH AND DEVELOPMENT COMPANY; PITTSBURGH, PA.,
HUGHES AIRCRAFT COMPANY; CULVER CITY, CAL.,
INTERNATIONAL BUSINESS MACHINES CORPORATION; NEW YORK, N. Y.,
LEEDS & NORTHRUP COMPANY; PHILADELPHIA, PA.,
LOCKHEED AIRCRAFT CORPORATION; BURBANK, CAL.,
PRATT & WHITNEY, DIVISION NILES-BEMENT-POND COMPANY; WEST HARTFORD, CONN.,
THE RAMO-WOOLDRIDGE CORPORATION; LOS ANGELES, CAL.,
THE RAND CORPORATION; SANTA MONICA, CAL.,
REPUBLIC AVIATION CORPORATION; FARMINGDALE, LONG ISLAND, N. Y.,
UNITED AIRCRAFT CORPORATION; EAST HARTFORD, CONN.,
WESTINGHOUSE ELECTRIC AND MANUFACTURING COMPANY; PITTSBURGH, PA.

The QUARTERLY prints original papers in applied mathematics which have an intimate connection with application in industry or practical science. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

Manuscripts submitted for publication in the QUARTERLY OF APPLIED MATHEMATICS should be sent to Professor W. Prager, Quarterly of Applied Mathematics, Brown University, Providence 12, R. I., either directly or through any one of the Editors or Collaborators. In accordance with their general policy, the Editors welcome particularly contributions which will be of interest both to mathematicians and to engineers. Authors will receive galley proofs only. The authors' institution will be requested to pay a publication charge of $5.00 per page which, if honored, entitles them to 100 free reprints. Instructions will be sent with galley proofs.

The subscription price for the QUARTERLY is $8.00 per volume (April-January). Single issues can be purchased at $2.50, and back volumes at $8.00 per volume as far as they are available. Subscriptions and orders for single issues may be addressed to: Quarterly of Applied Mathematics, Box 1885, Brown University, Providence 12, R. I.

Second-class postage paid at Providence, Rhode Island, and at Richmond, Virginia

WILLIAM BYRD PRESS, INC., RICHMOND, VIRGINIA

CONTENTS

BOOK REVIEWS

Frame analysis. By Arthur S. Hall and Ronald W. Woodhead. John Wiley & Sons, Inc., New York and London, 1961. xv + 247 pp. $8.50.

This book presents the essential ideas of the analysis of general frame structures. Linear behavior under loads is assumed (Hooke's Law, infinitesimal deflections). A "frame" is defined as a structure consisting of an assemblage of one-dimensional members connected at joints, and the concepts are used of deformation of a general one-dimensional member having arbitrary shape and properties. The first half of the book deals with "action methods" in which the basic equations state conditions of geometric compatibility, the unknowns are redundant forces of moments, and the properties of the structure are flexibility coefficients. The second half presents "displacement methods," wherein the basic equations express static equilibrium of joints, the unknowns are displacements of joints, and the structural properties are stiffness coefficients. The relations between the two types of methods are pointed out. Elementary matrix terminology and operations are used throughout, and many illustrative problems are presented. The book is clearly and concisely written, and seems an excellent instruction to the subject. It would especially provide good preparation for the solution of frame problems using electronic computers, although the use of computers is not specifically discussed in the book.

P. S. SYMONDS

Russian-English mathematical dictionary—words and phrases in pure and applied mathematics with roots and accents, arranged for easy reference. By L. M. Milne-Thomson. The University of Wisconsin Press, Madison, 1962. xiv + 191 pp. $6.00.

A special feature of this dictionary of over 10,000 entries, is the indication of the root of each noun, verb, or adjective, of Russian origin, and the provenance of other words. While this information is not without intrinsic interest, it has been primarily included for its mnemonic value, which should greatly assist the user in keeping in mind the meaning of a given word and in classifying words that derive from the same root. The dictionary also indicates the stressed syllable for each word listed. An outline of Russian grammar (28 pp.) and a list of Russian abbreviations conclude the useful work.

W. PRAGER

Numerical mathematical analysis. By J. B. Scarborough. Fifth edition. The John Hopkins Press, Baltimore, 1962. xxi + 594 pp. $7.00.

The most important additions made in this latest edition of the well-known text concern divided differences, the derivation of all central-difference interpolation formulas by means of divided differences, and methods of investigating the errors in the solution of a single equation or a system of linear equations when the coefficients are subject to errors.

Mathematics and industry. By John Crank. Oxford University Press, London, New York, Toronto, 1962. vi + 91 pp. $2.00.

According to a statement in the preface, the book attempts to bridge the gap between mathematics as an abstract discipline and its use in real life. Three introductory chapters discuss the use of mathematics in industry, the building of mathematical models, and the advantages of dimensionless variables. The remaining five chapters are devoted to a selection of typical problems in industrial mathematics.

(Continued on p. 253)

BOOK REVIEWS

(Continued from p. 188)

Introduction to space dynamics. By William T. Thomson. John Wiley & Sons, Inc., London, New York, 1961. xiv + 317 pp. $11.50.

The enormous advances in astronautics in the last few years and the consequent upsurge of interest in classical mechanics has led to the introduction of university courses for which this book is written. One of the most interesting aspects of the book is the wealth of problems that are of considerable practical importance and which not very long ago would have been considered artificial textbook exercises. The number of relatively new or insufficiently explored problem areas in what is considered the most highly developed of all sciences is also a point of significance.

The book is uncommonly well written for an undergraduate text in dynamics. It is intended for an advanced undergraduate or a beginning graduate course. The presentations are clear and direct. The vector approach is fully utilized as it should be in this field. The chapters that relate specifically to space dynamics are on the dynamics of gyroscopic instruments, space vehicle motion, and on the performance and optimization of rocket flight. What is particularly gratifying in this text is the direct development of the principles from the basic Newtonian equations and the avoidance of the concept of "inertial forces" which has managed to confuse so many engineering students in the past.

S. R. Bodner

Selected translations in mathematical statistics and probability. Volume I. American Mathematical Society, Providence, R. I., 1961. v + 306 pp. $4.80.

Translations of 25 articles from Russian journals, published between 1949 and 1958. Six are by Gnedenko, two by Dynkin, and the longest (over 50 pages) is by Rozanov on the spectral theory of multi-dimensional stationary random processes with discrete time.

Walter Freiberger

Tables of elementary functions. By Mieczyslaw Warmus. Pergamon Press, New York, London, Oxford, Paris, 1960. vi + 564 pp. $15.00.

These tables comprise all the important elementary functions. Trigonometric functions are given both for degrees and radians and all tables are given for the functions and their inverses. Linear interpolation is used, the differences are given to three digits and the function values generally to six significant figures. Every page includes estimates of the errors in the approximations used and of the errors to be expected by linear interpolation.

Walter Freiberger

Tables for the distribution and density functions of t-distribution. Edited by N. V. Smirnov. Translated from the Russian by Prasenjit Basu. Pergamon Press, New York, Oxford, London, Paris, 1961. 129 pp. $12.50.

These tables give the distribution function of "Student's" distribution for arguments up to 6.5 and degrees of freedom up to 24, and arguments from 6.5 to 9.0 and degrees of freedom up to 10. Related tables give the probability density function, quadratic interpolation coefficients and other useful parameters. Some of the problems for which these tables are useful are: testing of the significance of the deviation of the sample mean from the center of the normal distribution in a given normal population with unknown variance; construction of confidence intervals for the center of the normal distribution for an unknown sample variance; testing of the significance of the divergence of the means in two independent samples from normal populations with the same variance; testing of the significance of the deviation of the sample regression coefficient; and many others.

Walter Freiberger

Mathematical tables—tables of generalized exponential integrals. By G. F. Miller. Volume 3. British Information Services, New York, 1960. iii + 43 pp. $1.45.

The generalized exponential integral

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n}\,dt$$

plays an important part in many branches of theoretical physics, for example in the theory of radiative equilibrium in astrophysics, in the theory of neutron diffusion in nuclear physics and in the evaluation of integrals occurring in problems of molecular structure. Values of the auxiliary function

$$F_n(x) = (x + n)e^x E_n(x)$$

are given to eight places for $x = 0(0.01)1$ and $n = 1$ to 8; $x = 0(0.1)20$ and $n = 1, 2, \ldots 24$; $1/x = 0(0.001)0.05$ and $n = 1, 2, \ldots 24$. The tables overlap somewhat with the more recent and extensive "Tables of the Exponential Integral" by V. I. Pagurova, published, in translation, by Pergamon Press in 1961.

WALTER FREIBERGER

Integrals of Bessel functions. By Yudell L. Luke. McGraw-Hill Book Co., Inc., New York, Toronto, London, 1962. xv + 419 pp. $12.50.

This book deals with the evaluation of definite and indefinite integrals involving Bessel funtions. It covers such topics as the representation of integrals in terms of Lommel functions, integrals involving Airy functions, incomplete Gamma functions, repeated integrals of Bessel functions, integrals involving Strove functions, and Schwarz functions. Also many miscellaneous indefinite integrals involving Bessel functions are treated.

M. E. GURTIN

Combinatorial chance. By F. N. David and D. E. Barton. Hafner Publishing Co., New York, 1962. ix + 356 pp. $10.25.

The authors of this volume are statisticians who have encountered a variety of combinatorial problems in their investigations. This has led them to look into the earlier literature quite often . They have apparently written this volume as an attempt at a synthesis of the old and new. Some idea of the topics considered is given by the following enumeration of the chapters: 1. Elementary combinatorial method. 2. Definitions and Probability - generating Functions. 3. Moments and Moment-generating Functions. 4. Expectation Techniques. 5. Limiting Distributions and their properties. 6. Distribution Theory of Runs. 7. Derangements and Matching. 8. Multiple Runs in Line and Ring. 9. Multivariate Moments and Cumulants. 10. Randomization Problems. 11. Ranking Tests. 12. Variants of the Matching Problem. 13. Combinatorial Extreme-Value Distributions. 14. Occupancy. 15. Generalized Bernouilli Numbers. 16. Approximations to Differences of Zero. 17. Symmetric Functions and Polykays. 18. Noncentral Distributions. The exposition often consists of a succession of illustrative examples. Even though a number of the problems arose in a statistical context, this is not indicated and they are stated in combinatorial language. The references to the earlier literature are often interesting.

M. ROSENBLATT

Elasticity, fracture and flow. By J. C. Jaeger. Methuen & Co. Ltd., London, and John Wiley & Sons, Inc., New York, 1962. viii + 208 pp. $3.00.

This revised edition is pointed more directly at structural geology and rock mechanics through the addition of a chapter on applications. Elastic and plastic solutions and criteria of failure are discussed in some detail for homogeneous material with and without cracks or underground openings.

D. C. DRUCKER

The dynamics of automatic control systems. By E. P. Popov. Pergamon Press Ltd., London, Paris, Frankfurt, and Addison-Wesley Publishing Co., Inc., Reading, Mass., 1962. xiii + 761 pp. $10.75.

Nowhere is the rapid advance of engineering knowledge and technology more in evidence than in the field of automatic control. The pressures of industrial and military requirements have forced engineers to use ever more sophisticated mathematics for control systems analysis and design. In this country and, to judge by this book, in the Soviet Union as well, this rapid advance in knowledge has presented many engineers with the necessity of learning new techniques after their formal education is over. Mr. Popov's book is primarily intended for assisting this self-instruction.

The book is divided into five parts, titled: "General Information about Automatic Control Systems," "Ordinary Linear Automatic Regulation Systems," "Special Linear Automatic Regulation Systems," "Non-Linear Automatic Regulation Systems," and "Methods of Plotting the Regulation-process Curve."

In the first part, the control problem is introduced with many pictures, diagrams and words. Some familiarity with differential equations is assumed, but operational notation, singularity functions, and functional block diagrams are introduced. The use of integral control to reduce static errors and derivative control to improve transient response are discussed and illustrated with many physical examples.

In part II, the analysis and design of control systems described by ordinary linear differential equations with constant coefficients are discussed. The uses of algebraic and graphical stability criteria are illustrated by their application to five physical examples of practical control problems including speed, voltage, and pressure regulation. The treatment is approximately at the level of the standard introductory control course in most engineering schools in the United States. The treatment in this book is distinguished by the detail with which specific examples are included in the text, and by the omission of the root-locus technique entirely. The use of the frequency stability criterion is emphasized, being presented as the Mikhailov method, but recognized by readers of English as quite similar to the graphical method built around Nyquist's proof of the stability of such systems.

In Part III, linear control systems which include time delay (transportation lag, as it is sometimes called in the English literature), distributed parameters, or sampling are discussed. The models are described by differential-difference equation in the first case, partial differential equations in the second, and difference equations in the third. In each case, specific examples are worked in detail, and applications made of the frequency stability criteria discussed in Part II.

In Part IV, piecewise-linear analysis and design of second-order systems in the phase plane, stability analysis by the direct method of Lyapunov, and analysis of non-linear oscillations are presented. The discussion of each topic includes detailed application to specific examples, and is based on a minimum of mathematics. The applications of Lyapunov's direct method made by Luré are treated, as are the approximate conditions for oscillation developed by Krylov and Bogoliubov.

In Part V, numericial methods for the solution of differential equations are presented. Both direct plotting of trajectories and the indirect method of approximation in the frequency domain developed by Solodovnikov are treated. The latter method is generally known in the United States as Floyd's method. The section does not acknowledge the existence of either analog or digital computers. Since most control engineers in the United States have access to both types of machines, the chapter is of doubtful value.

Generally speaking, Popov's book serves its stated purpose of being a guide to the self study of automatic control very well. Each method is carefully explained and immediately applied to one of the five typical problems selected for the study. The mathematical level is maintained at the minimum level necessary in each case. In the opinion of the reviewer, the selection of topics is somewhat limited, since the root locus method is not treated, random signals are omitted and, most seriously of all, the entire field of optimal control is ignored. One must admit that only a finite number of topics can be treated in an introduction, but with only Popov as a guide, the engineer will see many trees, but not the outline of the forest.

G. F. Franklin