

QUARTERLY

OF

APPLIED MATHEMATICS

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BOOK REVIEWS

Differential equations: geometric theory. By S. Lefschetz. Second edition. Interscience Publishers, New York, 1963. X + 390 pp. \$10.00.

The first edition appeared in 1957 (see this Quarterly, 16 (1958)448). The only major changes made in the present edition are an extension of the material on Liapunov's direct method and its converse and a revision of the treatment of stability in product spaces.

Introduction to the theory of finite-state machines. By Arthur Gill. McGraw-Hill Book Co., New York, San Francisco, Toronto, London, 1962. xi + 207 pp. \$9.95.

The book under review is a survey of the theory of synchronous, deterministic finite-state (sequential) machines. This theory is less than ten years old, and had until recently been widely scattered throughout various journals. Drawing heavily on papers by Huffman, Moore, Mealy and many others, the author has written an introductory text considering fundamental analytic aspects of this theory. The chapters concern the basic model (1) and its representations (2), machine minimization (3), experiments for identification of states (4) and machines (5), finite-memory machines (6), and input-restricted machines (7).

The author has written a very useful book, which is of interest to applied mathematicians and engineers concerned with computation, communication and control; it should also appeal to specialists in many other fields. The book is well organized and fairly carefully written. The definitions, theorems and algorithms occasionally become hard to follow, but the numerous examples help to circumvent the difficulty. The notation and terminology are generally well chosen. The problems at the end of each chapter (119 in all) will develop as well as test the reader's understanding of what he has read.

The book is essentially elementary in its approach. Various associated concepts (e.g., the order of information losslessness, decomposition, iterative networks, realizations and state assignments) are never mentioned. The basic literature sources are sketchily credited (Moore and Mealy models are used, but not identified as such), and references are limited strictly to what is discussed. The problems infrequently go beyond the concepts of the text. Although these conscious omissions do not necessarily detract from an introductory book, some of them could easily be rectified in a subsequent edition, at great benefit to many readers.

The author is not too concerned about practical considerations. For example, he gives a procedure for constructing experiments to identify faults in finite-state machines. However, this procedure is applicable only when each faulty version of a given strongly connected machine is itself strongly connected. This restriction along with other factors may drastically restrict the utility of the procedure. As another example, he makes no comparison of the relative computational effort required by the various minimization techniques, nor does he indicate how large a machine these techniques might handle.

This reviewer found only one section which was really hard to follow, namely the last section of Chapter 5, on information losslessness. The author's version of Huffman's losslessness test is exceedingly hard to read. In addition, although the author has attempted to correct a flaw in Huffman's version, he has unfortunately not succeeded: condition 1 on page 151 should read "The sum of the number of elements in the sets $S_1(\sigma_u), \dots, S_r(\sigma_u)$ (in the first subtable) is less than the size of the input alphabet." An intuitive statement of conditions 1 and 2 would have helped immensely. Incidentally, Huffman class I and class II machines are introduced (anonymously) only in the last problem of the chapter.

All things considered, the book is a good place to begin for someone interested in learning about the theory of finite-state machines. From then on, the reader will find it rewarding to hunt through the various journals for advanced and more recent developments.

PETER G. NEUMANN

(Continued on p. 46)

BOOK REVIEWS

(Continued from p. 34)

Numerical analysis. By N. Macon. John Wiley & Sons, Inc., New York, 1963. XIII + 161 pp. \$5.50.

This is a textbook for a one-semester introductory course in numerical analysis at the sophomore or junior level. The scope of the book is best indicated by the following list of the titles of its chapters with selected section headings: Basic concepts (Electronic digital computers, Stored programs, Developing a program, Flow charts, On errors); Approximations of functions by polynomials (Power series, Solution of differential equations); Iterative methods of solving equations (Newton's method, Iteration in the complex plane); Matrices and systems of linear equations (The determinant of a square matrix, Operations with matrices, Linear equations, The inverse and adjoint matrices, The solvability of sets of linear equations); Computational methods with matrices (The evaluation of determinants, Matrix inversion and simultaneous linear equations, Iterative methods, Errors, residuals, and "condition," Iteration combined with elimination); The characteristic values and characteristic vectors of a matrix (Linear independence of the characteristic vectors, The Iterative Process); Interpolation (The interpolating polynomial, Iterated linear interpolation, Interpolation with two independent variables, The remainder term for polynomial interpolation); Differentiation and integration (Numerical differentiation, Numerical integration, Gaussian quadrature); Remainder terms for the integration formulas; Ordinary differential equations (The Euler-Cauchy method, Improved methods of step-by-step approximation, Flow charts for the Euler-Cauchy, Heun, and midpoint methods); Systems of first-order equations; Difference equations (Stability of step-by-step methods). The material is concisely presented "in such a way that the reader must provide a great deal of the exposition himself, much as he would when reading a mathematical journal or treatise." Each chapter is followed by a set of exercises.

W. PRAGER

Selected numerical methods for linear equations, polynomial equations, partial differential equations, conformal mapping. Edited by C. Gram, J. Gjellerup, Copenhagen, 1962. XII + 308 pp. \$10.14.

The self-contained chapters of the volume report on studies sponsored by the Carlsberg Foundation at the Danish Institute of Computing Machinery (Regnecentralen) in Copenhagen. The contents of the book are best indicated by the following chapter and selected section headings: *Linear equations* by C. Andersen and T. Krarup (Direct methods, Iterative methods), *Partial differential Equations* by C. Gram, P. Naur, and E. T. Poulsen (Linear elliptic equations of the second order, the heat equation, Parabolic nonlinear equation, Hyperbolic equations, A single equation of order higher than two), *Conformal mapping* by C. Andersen, S. E. Christiansen, O. Møller, and H. Tornehave (Explicit mappings, Conformal mappings by means of orthogonal functions, Integral equation methods, The application of the Lichtenstein-Gershgorin integral equation, Conformal mapping of nearly circular regions onto the unit disc using expansion in powers of a small parameter), *Polynomial equations* by T. Busk and B. Svejgaard (Iterative methods, Bounds for the roots, Direct methods).

ALGOL procedures are presented for the methods recommended in Chapters 1 through 3, but not for the solution of polynomial equations.

Nearly half the book is taken up by the chapter on conformal mapping which constitutes the most complete treatment of the subject available.

W. PRAGER

(Continued on p. 56)

BOOK REVIEWS

(Continued from p. 46)

An elementary introduction to the theory of probability. By B. V. Gnedenko and A. Ya. Khinchin. Translated from the 5th Russian edition by L. F. Boron with the editorial collaboration of Sidney F. Mack. Dover Publications, Inc., New York, 1962. xii + 130 pp. \$1.45.

This is an authorized, unabridged translation of the 5th Russian edition published in 1961.

Theory of lubrication—with applications to liquid- and gas-film lubrication. By Nicolae Tipei. Stanford University Press, California, 1962. xvi + 566 pp. \$15.00.

This is a book from the pen of an engineer who combines an appreciation of the practical aspects of lubrication with an outstanding talent for translating them into precise analytic terms and coupled with an ability to solve the mathematical problems which present themselves as a result. The exposition is clear and the aim of the book "to present as complete a picture as possible of lubrication theory" is admirably fulfilled.

Except for the rather rare case of turbulent lubricating films, the theory of lubrication rests on well-established differential equations. Consequently, most of this theory is a matter of mathematical techniques, but the nature of the equations is such that challenging problems in classical, mathematical analysis are posed. A minor weakness of the book stems from the fact that digital computers have not yet become an every-day tool of the designer in the author's native Roumania. This somewhat inhibits the derivations, because of the self-imposed constraint to present formulae which are suitable for hand-computation.

Problems arising from the variability of the viscosity of liquid, as opposed to gaseous, lubricants, as well as compressibility effects in gaseous films are treated by the invocation of heuristic hypotheses rather than by the inclusion of the energy equation in the analysis. This faithfully reflects the present status of the theory, but it is felt that the inclusion of the energy equation (which is linear in the absence of compressibility and in the case of constant properties) may prove less formidable than appears at first sight.

The readers of this Quarterly will not be interested in the technical details of the book, sound though they are, but will appreciate the information that here is an excellent, mathematically-biased introduction to a subject which still abounds in many challenging as well as routine problems awaiting solution.

The translation reads refreshingly well; it was prepared by Scientific Literature Consultants under the editorship of William A. Gross whose deep understanding of the subject and pleasant style of writing have impressed themselves on every page of the book. The reviewer hopes that Dr. Gross will find it possible to include an Index before the book is reprinted, because its absence constitutes a severe handicap in a treatise which is not meant to be studied, and learned from cover to cover.

J. KESTIN

Contributions to order statistics. Edited by A. E. Sarhan and B. G. Greenberg. John Wiley & Sons, Inc., New York, London, 1962. xxv + 482 pp. \$11.25.

This book is a combined effort by sixteen authors to survey the theory and applications of order statistics developed mainly during the last fifteen years or so. Although various sections or chapters have been written by different authors, it is not a collection of unrelated papers. The book has two parts one on general theory and the other on applications. The orientation is toward the use of order statistics in parameter estimation, and the book is written mainly for applied statisticians.

G. F. NEWELL

(Continued on p. 73)

BOOK REVIEWS

(Continued from p. 56)

The mathematical theory of optimal processes. By L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelidze, E. F. Mishchenko. Authorized translation from the Russian by K. N. Triroff. Editor, L. W. Neustadt. Interscience Publishers (Div. of John Wiley & Sons, Inc.) New York, London, 1962. viii + 360 pp. \$11.95.

One of the major problems of the modern mathematical theory of control processes can be posed in the following terms: "Given a vector differential equation $dx/dt = g(x, y)$, $x(0) = c$, where x represents the state of a physical system at time t , the state vector, and $y(t)$ represents the control vector, determine $y(t)$ so as to minimize a given scalar functional $J = \int_0^T h(x, y, t) dt$, where x and y are subject to local constraints of the form $r_i(x, y) \leq 0$, $i = 1, 2, \dots, N$, global constraints of the form $\int_0^T h_i(x, y) dt \leq k_i$, and terminal conditions of the form $f_i(x(T), y(T), T) \leq 0$." In some cases of importance, T itself depends upon the history of the process, $T = T(x, y)$, and, indeed, may be the quantity we wish to minimize.

The book under review represents a fine and substantial contribution to a new mathematical domain. The major theme of the work is the "maximum principle," an analytic condition which provides important information concerning the structure of extremals, in the terminology of the calculus of variations, or of optimal policies, in the parlance of dynamic programming and control theory.

Since the book is an excellent one which will be widely read and used, it is worthwhile to analyze its objectives and results carefully within the framework of the classical theory of the calculus of variations, and with the desiderata of modern control theory in mind.

In the simplest version of classical variational theory, there are no local or global constraints. The first variation yields the Euler equation, generally a nonlinear differential equation, with two-point boundary conditions. For a variety of reasons, this direct approach is seldom effective computationally. If global constraints are present, Lagrange multipliers may be used to reduce the problem to one without constraints, at the expense of further computational difficulties.

If local constraints of the type indicated above are present, as they are in a large number of the most important classes of processes, the situation is even more complex. This is due to the fact that sometimes the Euler equation holds and sometimes the constraints determine the extremal, or policy. Hence, the analytic and computational difficulties that existed before as far as effective algorithms for the solution are concerned are now compounded.

Nevertheless, analogues and extensions of the classical results can be obtained. The pioneering work is

F. A. Valentine, "The Problem of Lagrange with Differential Inequalities as Added Side Conditions," *Contributions to the Calculus of Variations, 1933-1937*, University of Chicago Press, 1937, pp. 407-448.

Results of Valentine were used by Hestenes in some unpublished work on constrained trajectories in 1949, and in 1961, Berkovitz in his paper

L. D. Berkovitz, "Variational Methods in Problems of Control and Programming," *J. Math. Anal. and Appl.*, Vol. 3, 1961, pp. 145-169,

showed how the "maximum principle" and results of greater generality could be obtained from Valentine's work combined with the classical calculus of variations.

The principal point of all this discussion is that the "maximum principle" does not provide us with any analytic approaches which we did not already possess, and does not seem to aid us in the fundamental objective of providing numerical answers to numerical questions. Unfortunately, at the present time, we possess no straightforward approach to the effective analytic solution of constrained variational problems.

This does not diminish the value of the book. Its very elegant presentation of results pertaining to extensions of classical problems and its consideration of processes involving time delays and stochastic elements will have a very stimulating effect upon research in this new field. It will serve the very useful purpose of focusing attention upon new, fascinating, and significant areas of investigation.

Let us now discuss some of the contents of the volume, and present some detailed comments. The

authors present a general treatment of the control process formulated above, using the "maximum principle" (which is, as Berkovitz points out, a restatement of the Weierstrass condition as adapted by Valentine), and discuss some quite interesting examples in detail. In particular, they consider the "bang-bang" control process, where $dx/dt = Ax + y$, y is constrained by the conditions that its components can only assume the values ± 1 , and it is desired to reach the origin in minimum time. Following this, they discuss a control process involving retardation (work of Kharatishvili), pursuit processes (work of Kelenozheridze), some interesting applications to approximation theory, problems involving constraints on state variables, and finally some stochastic control processes. The discussion of pursuit and stochastic control seems far more difficult and involved than one based upon the functional equation approach of dynamic programming, and, is based upon "open loop" control, rather than feedback control.

The authors indicate the intimate relation between dynamic programming and the calculus of variations, and state (on p. 7): "... Thus, Bellman's considerations yield a good heuristic method, rather than a mathematical solution of the problem," and again (on p. 73): "Thus, even in the simplest examples, the assumptions which must be made in order to derive Bellman's equations do not hold."

These statements provoke some further remarks. In the first place, if one refers to Berkovitz's article, it will be seen that the equations derived from the functional equation approach can be made completely rigorous in a number of cases. In those cases where lines of discontinuity, or more generally, surfaces of discontinuity exist ("switching surfaces"), we have a situation similar to the existence of shocks in hydrodynamics. The classical equations exist on both sides of the shock and the problem is now that of continuation of the solution from one region to the other.

Perhaps even more important is the following consideration. At the moment, we intend to base computational algorithms on the use of a digital computer. Consequently, there is some merit in formulating control processes in discrete terms from the beginning. If we proceed in this fashion, all problems of existence of extremals vanish and we face directly the fundamental problems of numerical solution and determination of the structure of optimal policies. Dynamic programming can now be applied in a uniform fashion to the study of deterministic, stochastic and adaptive control processes. If we so desire, we can establish that various limits exist as the discrete process merges into a continuous one.

The digital computer can be used for mathematical experimentation with the hope of discerning the structure of optimal policies from the solution of particular problems.

Let us finally note that the authors make no mention of a number of other techniques available for the study of constrained variational problems: function space methods, see

J. P. LaSalle, "Time Optimal Control Systems," *Proc. Nat. Acad. Sci. USA*, Vol. 45, 1959, pp. 573-577,

gradient techniques of the type used by Bryson and Kelley, see

G. Leitmann (editor), *Optimization Techniques*, Academic Press Inc., New York, 1962, quasilinearization, see

R. Bellman, H. Kagiwada, and R. Kalaba, "A Computational Procedure for Optimal System Design and Utilization," *Proc. Nat. Acad. Sci. USA*, Vol. 48, 1962, pp. 1524-1528, and finally, techniques based on the Neyman-Pearson lemma, see

R. Bellman, I. Glicksberg, and O. Gross, *Some Aspects of the Mathematical Theory of Control Processes*, The RAND Corporation, R-313, 1958, where a number of different approaches are given.

Taking all that has been said into account, there is no question that this book is an important contribution, and one that must be read by everyone working in the theory of control processes. Its translation is a fitting tribute to a great mathematician and his distinguished colleagues.

RICHARD BELLMAN

A guide to IBM 1401 programming. By Daniel D. McCracken. John Wiley & Sons, Inc., New York, London, 1962. viii + 199 pp. \$5.75.

The IBM 1401 Data Processing System is a small electronic computer, designed to handle alphabetic information as readily as numeric, with extensive capabilities for high speed handling of data on magnetic tapes and punched cards. It is used as a complete data processing system by small businesses, and as an auxiliary data editor for larger commercial and scientific computers.

This book is an elementary, lucidly written, introduction to the use of the 1401 in business data processing. It is well-suited to the needs of aspiring 1401 programmers who have no previous experience with computers or punched cards.

B. A. CHARTRES

Proceedings of the First Symposium on Engineering Applications of Random Function Theory and Probability. Edited by John L. Bogdanoff and Frank Kozin. John Wiley and Sons, Inc., New York, London, 1963. $x + 421$ pp. \$8.75.

The purpose of this symposium was to acquaint engineers with some of the applications of the theory of stochastic processes. The book contains papers by S. S. Shu (introduction), M. Kac (on zeros of Gaussian processes), A. J. F. Siegert (random functions), R. B. Murphy (reliability), A. M. Freudenthal (reliability), E. T. Jaynes (information theory and statistics) I. Dyer (aerodynamic noise), E. W. Montroll (highway traffic), R. E. Kalman (filtering), and D. Slepian (communication engineering). Most papers are expository, and emphasize in varying degrees the mathematical techniques and the applications. In all cases the papers are directed toward the non-specialist and collectively they form an excellent introduction to the subject. This is one of the rare cases in which a group of papers by authors with diverse styles and interests makes a book in which this variety is an asset.

GORDON F. NEWELL

Nonlinear automatic control. By John E. Gibson. McGraw-Hill Book Co., Inc., New York, San Francisco, Toronto, London, 1963. xviii + 585 pp. \$16.50.

This is one of a number of recent books which deal with the vast field of nonlinear control systems. It is distinguished by being written as a text for engineering graduate students and it contains a large number of worked-out examples and exercises. The author does not hesitate to enter "the realm of Volkssatz" in plausibility arguments but does give some rigorous proofs and gives references for others. Quite a bit of space is devoted to discussion of the significance of results.

The work does not quite deserve the adjective monumental, but the following list of chapter topics will indicate the breadth of the coverage: Linear Systems, Sample Data Systems, Perturbation Methods, Phase Plane Methods, Second Method of Liapunov, Describing Functions, Optimum Switched Systems, Adaptive Systems. The author indicates that much of the material on time-variable and nonlinear systems can be covered in a single semester, but a thorough study of all the topics in the book could easily occupy a full year.

The book will probably appeal much more to engineers than to mathematicians because of its style which is at times almost conversational. The reviewer found the opinions, conjectures, and indications of the need for further research quite appropriate in this work which is addressed to students and deals with many newly explored topics. On the other hand, those already working in the field of control may well find the book a convenience because of its many references and an aid to organization of thought because of the many diverse topics presented in relatively homogeneous manner and notation.

DEAN C. KARNOPP

X-ray optics—the diffraction of X-ray by finite and imperfect crystals. By A. J. C. Wilson. Methuen & Co., Ltd., London, and John Wiley & Sons, Inc., New York, 1962. vii + 147 pp. \$4.50.

This Methuen Monograph is a revision of a small classic originally published in 1949. The subtitle "The Diffraction of X-Rays by Finite and Imperfect Crystals" states the chief emphasis of the book. The profile of each reflection from such a crystal is systematically broadened and distorted from that produced from a perfect, or ideally imperfect, crystal. The general level of sophistication is such that the advanced undergraduate can follow the discussion. It may be suspected that he could follow it more readily if more powerful and condensed notation were used. In the revision, about 15% new material has been added, details have been improved, and the references have been increased and brought up to date.

G. B. CARPENTER