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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\surd$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp [(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos (\pi x/2b)}{\cos (\pi a/2b)} \text{ is preferable to } \frac{\cos \frac{\pi x}{2b}}{\cos \frac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u \, du \text{ is preferable to } \int \frac{\sin u}{u} \, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t \, (a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n] \cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n) \cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354–372 (1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

CONTENTS

BOOK REVIEWS

Mathematics—its content, methods, and meaning. Three volumes. Edited by A. D. Aleksandrov, A. N. Kolmogorov, M. A. Lavrent'ev. Translated in part by S. H. Gould, T. Bartha and K. Hirsch. The Massachusetts Institute of Technology Press, Cambridge, 1964. xi + 359 pp. (Volume I), xi + 377 pp. (Volume II), xi + 356 pp. (Volume III). (10.00 per volume, or the set of three volumes $30.00)

This work (three volumes) was translated from a 1956 Russian edition by S. H. Gould, Kurt Hirsch and Tamas Bartha. The American Mathematical Society, supported by the National Science Foundation, originally translated and published the work.

The book is the result of an effort to present to people of other disciplines a picture of the content of various mathematical disciplines together with their development. The reader is presumed to have some mathematical training but not necessarily that of a mathematician. In fact, this is a book to show others what mathematicians do. It will nevertheless be of interest to Mathematicians as well as to Physicists, Chemists, Engineers, and to anyone concerned with mathematics.

On one hand, this work might be described as a semi-popular book about mathematics. On the other hand, it contains a large amount of serious mathematics. The authors seem to have accomplished extremely well a task that few mathematicians would undertake. The result seems to the reviewer to be a most valuable one.

The following list of topics may help to indicate to the reader the objectives of the book.

Volume I. Chapter I, A general View of Mathematics, A. D. Aleksandrov; Chapter II, Analysis, M. A. Lavrent'ev and S. M. Nikol'skii; Chapter III, Analytic Geometry, B. N. Delone; Chapter IV, Algebra; Theory of Algebraic Equations, B. N. Delone; Chapter V, Ordinary Differential Equations, I. G. Petrovskii.

Volume II. Chapter VI, Partial Differential Equations, S. L. Sobolev and O. A. Ladyzenskaja; Chapter VII, Curves and Surfaces, A. D. Aleksandrov; Chapter VIII, The Calculus of Variations, V. I. Krylov; Chapter IX, Functions of a Complex Variable, M. V. Keldyš; Chapter X, Prime Numbers, K. K. Mardzanisvili and A. B. Postnikov; Chapter XI, The Theory of Probability, A. N. Kolmogorov; Chapter XII, Approximations of Functions, S. M. Nikol'skii; Chapter XIII, Approximation Methods and Techniques, V. I. Krylov; Chapter XIV, Electronic Computing Machines, S. A. Lebedev and L. V. Kantorovič.

Volume III. Chapter XV, Theory of Functions of a Real Variable, S. B. Stečkin; Chapter XVI, Linear Algebra, D. K. Faddeev; Chapter XVII, Non-Euclidean Geometry, D. A. Aleksandrov; Chapter XVIII, Topology, P. S. Aleksandrov; Chapter XIX, Functional Analysis, I. M. Gel'fand; Chapter XX, Groups and Other Algebraic Systems, A. I. Mal'cev.

ROHN TRUELL

Introduction to mathematical statistics. By Robert V. Hogg and Allen T. Craig. Second Edition. The Macmillan Co., New York and Collier-Macmillan Ltd., London, 1965. x + 383 pp. $8.50.

This is the second edition of a well-known text, written for students with a knowledge of advanced calculus. Amongst additions in this edition are a treatment of conditional probability, sequential analysis and expanded ones of hypothesis testing and analysis of variance. There is, now, a discussion of decision functions and Bayersian statistics.

Features of the book are its clarity of presentation and the wealth of useful exercises and examples, both worked and unworked. To single out an instance, the chapter on sufficient statistics is thoroughly modern in spirit, including the Rao-Blackwell theorem and a discussion of completeness. There is, however, no introduction to the concept of statistical invariance which is, perhaps, a pity. But the book fulfills its purpose very well indeed.

W. FREIBERGER

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BOOK REVIEWS

(Continued from p. 304)

Non-classical shell problems. Proceedings of International Association for Shell Structures Symposium in Warsaw, 1963. Edited by W. Olszak and A. Sawczuk. North-Holland Publishing Co., Amsterdam, 1964. xx + 1185 pp. $28.00.

This volume of nearly twelve hundred excellently produced pages contains sixty-seven papers concerned with the introduction of thermal, viscous, creep and plastic effects into the theory of thin shells. Many of the papers deal with shallow shells and with shells of revolution, just as in the ltierature on elastic shells. The individual papers, most of them mathematical in character, together with the numerous references contained in them represent a useful summary of the state of the art in relation to the better known and more fully developed theory of the "classical" elastic shell.

E. Reissner

Topological vector spaces. By A. P. Robertson and W. J. Robertson. Cambridge University Press, American Branch, New York, 1964. viii + 158 pp. $4.95.

This readable book is almost completely self-contained. The table of contents is as follows: I. Definitions and elementary properties; II. Duality and the Hahn-Banach theorem; III. Topologies on dual spaces and the Mackey-Arens Theorem; IV. Barrelled spaces and the Banach-Steinhaus theorem; V. Inductive and projective limits; VI. Completeness and the closed graph theorem; VII. Some further topics; VIII. Compact linear mappings. The last two chapters include discussion of tensor products, the Krein-Milman theorem, and the Riesz theory of linear mappings (which includes much of the theory of Fredholm integral equations). The authors pay due attention throughout to motivation and applications, and point out alternative lines of development in supplements at the end of most chapters. The book would be somewhat easier to use, however, if it contained an index of symbols.

Thomas A. Brown

Integration of equations of parabolic type by the method of nets. By V. K. Saul'yev. The Macmillan Co., New York, 1964. xvii + 343 pp. $12.00.

The Russian edition appeared in Moscow in 1960. The present translation has been prepared by G. J. Tee and edited by K. L. Stewart. (The "method of nets" is usually called "method of finite differences".)

Tables of the function $w(z) = e^{-z^2} \int_0^z e^{x^2}\, dx$ in the complex domain. By K. A. Karpov. The Macmillan Co., New York, 1965. xxi + 519 pp. $19.75.

The Russian edition appeared in Moscow in 1954. For the present edition, the introductory material has been translated by D. E. Brown.

Tables of the normal probability integral, the normal density, and its normalized derivatives. Edited by N. V. Smirnov. The Macmillan Co., New York, 1965. xvi + 125 pp. $7.50.

The Russian edition was published in Moscow in 1960. For the present edition, the introductory material has been translated by D. E. Brown.

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BOOK REVIEWS

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Extraction of signals from noise. By L. A. Wainstein and V. D. Zubakov. Translated from the Russian by Richard A. Silverman. Prentice-Hall, Inc., New Jersey, 1962. xii + 382 pp. $12.00.

The book consists of three parts. The first part is devoted to optimum linear filters, together with some background material on stationary random processes and linear filters. Two types of optimum filters are discussed: the Wiener filter which minimizes the mean-square error and the matched filter (generalized for non-white noise) which maximizes the signal-to-noise ratio.

The second and major part of the book, gives an extensive treatment of optimum detection of radar signals in noise where the optimality criterion this time is maximization of a posteriori probability. After some discussion of a priori and a posteriori probabilities, likelihood ratios and decision rules, much space is devoted to the derivation of test statistics for detecting various types of radar echo signals in Gaussian noise. The radar signals considered are completely known waveforms, those containing random parameters (amplitude and phase), coherent and incoherent trains of pulse-modulated sinusoids with and without random parameters. This part also includes optimum measurement (estimation) of signal parameters, arrival time in particular, using the same optimality criterion. Unlike the treatment in the first part, here the signals and noises are represented by finite sequences of random variables, requiring no more than a background in elementary probability theory. Many examples are given where the false-alarm and detection probabilities are explicitly calculated and sufficient physical motivation is given.

The third part explores a variety of auxiliary subjects, including Gaussian variables, modulation of sinusoids by Gaussian processes, clutter echoes, etc.

While the treatment of stochastic processes as well as that of the fundamentals of statistical inferences is here quite limited, the book may be recommended as supplementary reading for its extensive treatment of concrete and physically motivated examples in optimum detection of radar signals.

T. T. Kadota

Introduction to basic Fortran programming and numerical methods. By William Prager. Blaisdell Publishing Co., New York, Toronto, London, 1965. ix + 203 pp. $6.00.

This is a carefully-written introduction to automatic computation. It is available as a paperback textbook and is designed for a one-semester course in applied mathematics following the elementary calculus. Such a course allows the early use of numerical methods which do not suffer from the limitations or tedium associated with hand computations. The text is directed toward the importance of careful analysis before computation. Error analysis and control are stressed at an early stage. There are many well-chosen examples and each chapter is concluded with an adequate set of illustrative examples. The presentation is lucid and also has a high level of mathematical rigor and an accurate introduction to Fortran.

Approximately half of the text is devoted to programming and to algorithms using the basic Fortran language for the IBM 7070. This Fortran includes all of the elementary statements with the exception of the subroutine, function and common. If desired, however, these statements could easily be taught in place of the section on sorting. The use of magnetic tapes is described. A check list of common programming mistakes and suggestions for reducing programming mistakes are included.

The principal areas of numerical methods that are covered include modern treatments of: computations with polynomials; polynomial and trigonometric interpolation; quadrature and quadrature with repeated subdivision of interval; iterative methods for solution of a nonlinear equation or a system of two nonlinear equations; linear algebraic equations; one-step and multi-step methods for numerical integration of ordinary differential equations. The reader is guided to additional study by many references to original articles and also to supplementary texts.

Clay L. Perry