QUARTERLY

OF

APPLIED MATHEMATICS

VOLUME XXIV JANUARY • 1967 NUMBER 4

QUARTERLY OF APPLIED MATHEMATICS

This periodical is published quarterly by Brown University, Providence, R. I. 02912. For its support, an operational fund is being set up to which industrial organizations may contribute. To date, contributions of the following companies are gratefully acknowledged:

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Second-class postage paid at Providence, Rhode Island, and at Richmond, Virginia

WILLIAM BYRD PRESS, INC., RICHMOND, VIRGINIA

QUARTERLY

OF

APPLIED MATHEMATICS

VOLUME XXIV 1966

Printed by the
William Byrd Press, Inc.
Richmond, Virginia

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BOOK REVIEWS

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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\surd$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\,[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos\,(\pi x/2b)}{\cos\,(\pi a/2b)} \text{ is preferable to } \frac{\cos\dfrac{\pi x}{2b}}{\cos\dfrac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u\, du \text{ is preferable to } \int \frac{\sin u}{u}\, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t\, (a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n] \cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n) \cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354–372 (1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten.*

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

—BOOK REVIEW SECTION—

Mathematical explorations in behavioral science. Edited by Fred Massarik and Philburn Ratoosh. Richard D. Irwin, Inc. and The Dorsey Press, Homewood, Illinois, 1965. viii + 387 pp. $7.95.

"This volume," say the editors, "was conceived with the view that much needs to be done *now* to close the gap between the abstract, apparently self-sufficient mathematical model and the empirical data needed to breathe life into the model." What they did, in November, 1961, was to "invite a group of scholars from fields such as psychology, sociology, business administration, political science, and mathematics" to meet for four days at Cambria Pines, California. The volume, four years later, assembles some of the papers given at the meeting. At least six of the 22 chapters have meanwhile been published elsewhere, these accounting for one-third of the total text.

How successful was the meeting in closing the gap between mathematical theories and empirical data? In subject matter, the papers cover the waterfront. Five of the chapters are programmatic, or develop methodological points without applying them to data; six present mathematical theories without empirical data; two provide data without mathematical theories; two discuss methodology and apply it to sample data; and six (by the most generous classification) test mathematical theories with empirical data. Thus, the batting average is just under 300, and we must conclude that the theory-data gap may as well have been widened as narrowed.

Applied mathematicians may be interested in what kinds of mathematics these behavioral scientists find useful. Some elementary algebra, Bayesian decision theory, game theory, simple stochastic processes, and a little calculus cover most of the ground. The two chapters making the most "serious" use of mathematics are Coleman's "Diffusion in Incomplete Social Structures," and Arrow's "The Economic Implications of Learning by Doing." Both have previously been published elsewhere.

Papers published in miscellanies like this rapidly become lost from view. It is to be hoped that the chapters that test theory with new data—in particular those of Attneave on detection of pattern in sequences, Bartos on bargaining, Solomon on effects of group size on performance, and Catton on "social mass" will somehow be saved from that fate. Since the interesting chapters by Lieberman (game theory), Meier (description of a library as an information processing system), and Coleman and Arros (mentioned previously) are also available in other publications they are presumably in less danger of oblivion.

Herbert A. Simon

Six lectures on modern natural philosophy. By C. Truesdell. Springer-Verlag New York, 1966. 117 pp. $4.50.

As Professor Truesdell remarks in the final lecture of this series, "It is tasteless to recommend one's own taste, but scarcely honest to recommend any other." The book as a whole is an extended explanation of Truesdell's taste. The first five essays are expositions of recent work that the author particularly admires, and the last, on *Method and Taste in Natural Philosophy,* is a trenchant general apology for the area of research that he has done so much to encourage.

Although Professor Truesdell's paragons are unexceptionable, his anathema falls on everything that is not precise mathematics and almost everything that is. The book should be kept away from hypertensives. After much head-holding and hand-wringing, I have concluded that I must recommend it to all others.

A. C. Pipkin

Theoretical hydromechanics. By N. E. Kochin, I. A. Kibel and N. V. Roze. Edited by J. R. M. Radok. Translated from the fifth Russian edition by D. Boyanovitch. Interscience Publishers, New York, London, Sydney, 1964. v + 577 pp. $20.00.

This translation of the fifth Russian edition of this basic work represents considerable modification and some modernization of the material presented in the previous edition. The chapter headings are: I. *The Kinematics of a Fluid Medium.* II. *The Fundamental Hydrodynamic Equations for an Ideal Fluid.* III. *Hydrostatics.* IV. *The Simplest Cases of Motion of an Ideal Fluid.* V. *Vortex Motion of an Ideal Fluid.* VI. *The Plane Problem of Motion of a Body in an Ideal Fluid.* VII. *The Three-Dimensional Problem of the Motion of a Body in an Ideal Fluid.* VIII. *The Wave Motion of an Ideal Fluid.*

The classical theory of Hydrodynamics is set forth clearly. The mathematical development is based upon vector analysis—tensors are not used. Throughout the text one finds a wealth of completely worked examples, others partially developed and many excellent illustrative exercises.

Chapters I, II and III contain derivations of the momentum, continuity and energy equations in vector form. Both Eulerian and Lagrangian developments are applied and their respective advantages detailed. Various special considerations, pertinent to the later chapters, are included here. The concepts of vorticity, streamlines, barotropic and baroclinic media and pressure considerations receive attention.

In Chapter IV both steady state and irrotational motion are the simple cases referred to in the subtitle. Bernoulli's, Cauchy's and the Bernoulli-Euler integrals are developed. Thomsen's theorem is proved and its implications examined. In the discussion of irrotational motion we find the introduction of the stream function and velocity potential—thereby inducing a natural interaction with complex function theory.

The fundamental equations of vortex motion, Helmholtz's vortex theorems and vortex formation constitute the first third of Chapter V. In the second third the determination of the velocity field from a given vortex field is carried out. Examples include single vortex filaments, two rectilinear vortex filaments and vortex layers. New to this volume, the last third of Chapter V applies Liapunov's Direct Method to investigate the stability of vortex streets.

The motion of bodies in ideal fluids is the concern of Chapters VI and VII (170 pages). Here we first meet a warning that many of these results disagree with reality. Many familiar examples appear. Included are motion of a circular cylinder, Kutta-Joukowski and Blasius-Chaplygin formulae and flow around various profiles. The two dimensional problem is herein augmented by exposition of the cascade and cavitation theories. Discontinuous flow is analyzed by Kirchhoff's method and orifice flow by the Joukowski-Michell procedure. In three dimensions, considerations are more limited being restricted to motion of a sphere and ellipsoid. Rankine's source and sink method is described and applied.

Chapter VIII on waves has been augmented, in this edition, to include the application of hydrodynamics to meteorology and the flow of a baroclinic fluid about obstacles. In addition to the development of the fundamental equations, excellent discussions on standing, progressive and trochoidal waves are included.

The presentation throughout the book is thorough and the translation has been very capably done. While the subject index is adequate the author index is almost nonexistent. There has been little attempt to include recent developments. One cannot discover the status of current research from this volume. There are less than a dozen references to the post 1950 period.

A strong feature of the book is the excellent discussion of the implications of the mathematical arguments. When this is added to the thorough treatment and wealth of examples, one has as a result an excellent textbook for a first graduate course.

W. F. Ames

Asymptotic expansions. By E. T. Copson. Cambridge University Press, New York, 1965. 120 pp. $6.00.

This book is a valuable addition to the small but slowly growing list of works dealing with various aspects of asymptotic analysis. It is a revised and enlarged version of the author's well-known monograph on *The Asymptotic Expansion of a Function Defined by a Definite Integral or Contour Integral.*

That monograph, which first appeared in 1943 and in a revised edition in 1946, has been out of print for many years and the appearance of the present book is therefore particularly welcome.

The first part of the book deals with general properties of asymptotic sequences and series, integration by parts, the method of stationary phase, and the method of Laplace. Watson's Lemma is then introduced and used to discuss Debye's method of steepest descents and Riemann's saddle-point method.

In the application of the method of steepest descents to Airy functions it is shown that a single application of the method together with the use of connection formulas suffices to determine all of the required expansions for all values of arg z. The expansions so obtained have overlapping sectors of validity in which the two expansions differ by a subdominant series. This situation is not inconsistent, according to the usual interpretation, but is merely an example of the Stokes phenomenon. In some situations, however, it would appear desirable to have a criterion by which the expansions can be restricted to non-overlapping sectors. Such a criterion has recently been suggested by Olver based on his theory of error bounds for asymptotic expansions of functions defined by ordinary linear differential equations but a corresponding theory of error bounds for functions defined by contour integrals does not appear to exist at the present time.

Finally, the author discusses the important theory of Chester, Friedman, and Ursell for obtaining uniform expansions according to the method of steepest descents when the integrand has two nearly coincident saddle-points.

W. H. Reid

Ergodic theory in statistical mechanics. By I. E. Farquhar. Interscience Publishers, London, New York, Sydney, 1964. viii + 235 pp. $12.00.

A great many works on statistical mechanics have been published recently, and it is certainly a welcome surprise to find one that has something both new and correct to say. Dr. Farquhar sets himself the task of explaining mathematical rigorous ergodic theorems in the context which gave rise to ergodic theory originally, namely, statistical mechanics. After an introductory chapter designed to clear away some of the mists of confusion and error generated by "physical" writers on statistical mechanics, followed by a general discussion of the methods of averaging, he gives brief statements and interpretations of the theorems of Birkhoff, Lewis, v. Neumann, and Hopf. The next chapter concerns the application of these theorems to statistical mechanics. Next is a presentation of the ideas of Khinchin, with the refinement by Mazur and Van der Linden to account for weak interaction, followed by Uhlhorn's theory based on definition of the phase space of an experiment and by a sketch of the generalization of Hopf's theorem by Albertoni, Bocchieri, and Loinger. Nowhere else can an integrated, definite, and correct presentation of all this material be found.

The last third of the book concerns ergodic theory in quantum mechanics. Here the results of Bocchieri and Loinger are emphasized, according to which the ergodic property is shown to follow from v. Neumann's probability assumption, independently of the time evolution of the system.

Dr. Farquhar concludes that despite a number of concrete theorems, "the current verdict on ergodic theory in its relation to statistical mechanics . . . [is] *not proven.*"

C. Truesdell

Statics of granular media. By V. V. Sokolovskii. Translated by J. K. Lusher. English translation edited by A. W. T. Dan el. Pergamon Press, Oxford, London, Edinburgh, New York, Paris, Frankfurt, 1965. xiv + 270 pp. $12.00.

Despite the muddled statement on the second title page, this is an English translation of the third Russian edition, published in 1960, of the book *Statika Sypuchei Sredy.* This book was first published in 1942 and it is a standard text on two-dimensional stress fields in soils with application to the stability of foundations, retaining walls and slopes. The second Russian edition (1954) was issued in translation in 1956 (Butterworth, London). The third Russian edition is a condensed version of the second (243 pp. + 160 fig. versus 275 pp. + 200 fig.), but it also contains some new tables and solutions to a number of new problems, involving discontinuous stress fields or curvilinear boundaries. The translation is well done.

R. T. SHIELD

Mathematical analysis, functions, limits, series, continued fractions. Edited by L. A. Lyusternik and A. R. Yanpol'skii. Translated by D. E. Brown and translation edited by E. Spence. Pergamon Press, Oxford, London, Edinburgh, New York, Paris, Frankfurt, 1965. xi + 404 pp. $10.00.

This book is essentially an outline of selected topics of interest to applied mathematicians. It quotes key results but has few derivations or proofs. The emphasis is on those topics in mathematical analysis that form the foundation for modern methods of numerical analysis. Chapter I (35 p.) reviews classic notions in real variable. Chapter II (45 p.) discusses functions of several variables including contraction mappings and convex functions. Chapter III (78 p.) gives a more thorough review of infinite series. Chapter IV (67 p.) describes orthogonal series with emphasis on orthogonal polynomials. Chapter V (61 p.) reviews the theory of continued fractions and Chapter VI (72 p.) is a summary of "some special constants and functions". The reviewer noticed quite a few errors. This could be a serious flaw for people who may wish to use the book as a "handbook". The book does, however, give an excellent survey of many topics seldom found in a general text. It also gives many special examples.

G. F. NEWELL

Distribution theory and transform analysis—an introduction to generalized functions, with applications. By A. H. Zemanian. McGraw-Hill Book Co., New York, St. Louis, San Franciso, Toronto, London, Sydney, 1965. xiii + 371 pp. $13.75.

The book develops the theory of distribution in a fairly elementary way. Thus instead of introducing a topology in the space D of test functions, only the concept of sequential convergence is introduced. Most of the main theorems of the Schwartz theory are obtained by this simplified approach, including those of convolutions, Fourier transforms and Laplace transforms. Some applications are given to differential and difference equations, and to some physical systems. Finally, there are numerous examples and problems.

The book should be very useful for graduate students of engineering and science. It should appeal also to mathematics majors at their senior level.

AVNER FRIEDMAN

Topics in advanced mathematics for electronics technology. By Stephen Paull. John Wiley & Sons, Inc., New York, London, Sydney 1966. xi + 420 pp. $4.95.

An introduction to topics in mathematics, this book is aimed at problems in electronics of the electric circuit and network variety. The book is not concerned with physical electronics. The contents of the eight chapters are infinite series, Fourier series, Fourier integrals, ordinary differential equations, gamma and Bessel functions, vector analysis and determinants and matrix algebra. The mathematical treatment is elementary to intermediate and the network analysis is intended for engineers at an undergraduate level. The book seems to be clear and well written.

ROHN TRUELL

Spectral analysis of economic time series. By C. W. J. Granger in association with M. Hatanaka. Princeton University Press, New Jersey, 1964. xviii + 299 pp. $8.50.

The recent development of Spectral Analysis by J. W. Tukey and others has found important applications in many fields of Science. The recently popularized Fast Fourier Series methods (not discussed in this book) should do much to reduce computer costs of both research and routine data reduction, and thus stimulate further use of Spectral Analysis.

It is inevitable that the methods should be tried on Economic data, which so often occur in the form of time series, and it is also inevitable that this should be difficult and require special techniques and abilities.

The challenge to study the stock market (and by implication make oneself a fortune) occurs to most people when they first meet Spectral Analysis. Students of these matters have long held that there were cycles of business activity, as well as cycles of the stock market, but the failure of classical Fourier Analysis (with the requirement of perfect periodicity) discouraged further work. Spectra methods, with their recognition of "noise" and approximate periods has stimulated further research on the matter. If only the phenomena were guided by linear mechanisms, then the simple decomposition into sines and cosines would do much to elucidate matters, but when the governing mechanisms are probably both nonlinear and nonstationary, then there are many difficulties to be overcome before one can expect success (and the fortune). However, the importance of Economic control is so great that anything that can be found of a fundamental nature is well worth the effort.

This book represents a serious attempt to apply Spectral theory to Economic time series. It is written for economists who are not skilled mathematicians, and often states the mathematical results without proving them. After a careful introduction to spectral analysis the book takes up cross-spectral analysis, processes involving feedback, nonstationary time series, and demodulation. Some interesting applications to business cycle indicators and inventory cycle theory are given, and serve to reveal the immense difficulty of the field.

The authors do not discuss the many results given in "The Random Character of Stock Market Prices", edited by Cootner (MIT Press) which also use Spectral Methods to good effect.

It seems to this reviewer that the importance of Spectral Methods to economic theory is such that anyone trying to do serious research in the field will have to master the many results already known in the field, and be prepared to occasionally develop new results on his own. Unfortunately we still do not have a decent introductory test to the field for the nonmathematically inclined and this lack will continue to retard its development.

R. W. Hamming

Calculus on manifolds—a modern approach to classical theorems of advanced calculus. By Michael Spivak. W. A. Benjamin, Inc., New York and Amsterdam, 1965. xii + 144 pp. $7.00.

In this small book the author has given us a beautiful account of the classical multidimensional calculus from the modern geometric approach. The beauty of the book resides in the fact that although we are always given the "right" definition and the "right" proof the book is warm and easily approached.

Chapter 1 treats functions on euclidean space and those theorems about compact subsets of euclidean space as are necessary for integration theory.

Chapter 2 centers about the theory of differentiable functions of R^n to R^m. The author says that $f : R^n \to R^m$ is differentiable at 0 if there exists a linear transformation $\lambda : R^n \to R^m$ such that

$$\lim_{n \to 0} \frac{|f(a + h) - f(a) - \lambda(h)|}{|h|} = 0$$

The culminating theorem in this development is the Implicite Function Theorem.

Chapter 3 treats Riemann integration on euclidean spaces and includes such topics as measure and content zero of a set and the easy part of Sard's theorem.

Chapter 4 begins with a brief account of tensor product, and exterior algebras. This is followed by a discussion of vector fields and forms which includes the exterior derivatives of forms and the Poincaré lemma.

The chapter closes with a treatment of singular chains and Stokes' theorem.

Chapter 5 is devoted to a treatment of integration theory for submanifolds with boundary of euclidean space. The book closes with the classical statements of Green's and Stokes' theorems.

LOUIS AUSLANDER

Ergodic theory and information. By Patrick Billingsley. John Wiley & Sons, Inc., New York, London, Sydney, 1965. xiii + 193 pp. $8.50.

The two central results of this book are the Kolmogorov–Sinai proof of the non-isomorphism of the 2-shift and 3-shift (in ergodic theory), and the coding theorem for a discrete memoryless channel in Shannon's coding theory. The description of the former largely follows Halmos, "Entropy in Ergodic Theory", University of Chicago notes, 1959, the proof of the latter is that given by the reviewer (Illinois Journal of Math. 1, No. 4, (1957), 591–606). There are also chapters on the ergodic theorem and convergence of entropy, and what is intended to an introduction to coding theory which does not "go deep", according to the author's preface.

The author's intention has been to write an "interesting book", in which he has succeeded, making use of a conversational style and a goodish number of examples, and "to link coding theory as closely as possible to ergodic theory . . .". We will shortly see that what link he has shown to exist is trivial indeed, but no closer link has yet been demonstrated by anyone else. The first four chapters on ergodic theory are excellent. The fifth and last, which is intended to be an exposition of the basic ideas of coding theory, leaves much to be desired. We will now discuss the two criticisms in detail.

It is always exciting to demonstrate a link between two branches of mathematics, between which no connection had been thought to exist. In the case of ergodic theory and information (coding) theory the existence of a connection has often been vaguely asserted. What would constitute a meaningful link between the two theories? It seems to the reviewer that to demonstrate such a link one has to show either that important theorems in one theory already follow from theorems in the other theory, or that there are general theorems which subsume important results of both theories, or that, using the methods of one theory, one can obtain important new results in the other. The author has not, and indeed no one else has, shown any of these. (This, of course, in no way precludes the possibility that a meaningful connection will be demonstrated in the future). The connection claimed in the book is supposed to come from the use of entropy. Actually, even this is not the same in both theories. In ergodic theory the crucial role is played by the entropy of a transformation, which plays no role whatever in coding theory. In ergodic theory the entropy of a measure preserving transformation is an invariant under isomorphism. It is the clever use of this device which enables one to prove that the Bernoulli 2-shift and 3-shift are not isomorphic. In coding theory entropy enters in an *inescapable* way in the very statements of the theorems themselves. The capacity of a discrete memoryless channel is the entropy of a probability vector minus a linear combination of entropies of probability vectors.

As befits his interesting style, the author tries to motivate the introduction of entropy in an intuitive way. He regards entropy "as a measure of the amount of randomness in a single roll of the die" or other chance experiment. In this he follows the now standard explanation which has its origin in the axioms given by Shannon[1] in his justly celebrated paper in Bell System Tech. Journal, 27 (1948),

[1]These axioms determine entropy to within a multiplicative constant and have been the object of much investigation. However, Shannon (ibid.) remarked that he would not make use of these properties, and that the reason for the introduction of the entropy function is its role in coding theory. (The suggestive terms "entropy" and "information theory" for coding theory seem to be due to Shannon). Entropy arises in coding theory in a natural and essential way, but no non-trivial use of the axioms referred to above has yet been made in coding theory. Some authors have tried to found a theory of statistical inference on the notion of entropy as a measure of randomness. The achievement of a truly meaningful link between statistical inference and information theory would be an exciting scientific step yet to be achieved.

379–423, 623–656. Of course, what is an intuitive explanation is a subjective matter, but the reviewer questions whether this explanation would really motivate a competent mathematician to rediscover, or even to understand better, the proofs of the book's main theorems, or to discover new theorems. In the simple Lemmas 3 and 4 of page 172 of the proof of the coding theorem and in their proofs, the author has a simple, intuitive, combinatorial explanation of the essential role of entropy in coding theory. Using these results an intuitive explanation of the coding theorem has been given. The explanation given by the author at the top of page 170 explains only why one can send *two* different messages, which is trivial. The theorem actually asserts that one can send essentially 2^{nC} different messages, an enormous number largely independent of the maximum probability of error! This is a truly remarkable theorem.

In the author's description of coding theory there is a source which generates messages stochastically with a given distribution μ. The messages are coded, sent over the channel, and then decoded. The results depend heavily upon μ. Shannon, in all his papers but one, does not really use this model. This model is used by him only in his contribution to R. E. Machol (editor) "Information and decision processes", New York, 1960, McGraw-Hill Book Co. (a reference not given by the author). It is the model used by Dobrushin in Uspyekhi Mat. Nauk, 14(90), 1959, 3–104, and in several other papers, though by no means in all. The model used by Shannon in all his other papers and also used in many Russian papers is really independent of any μ. But then, in order to study noisy channels (the interesting problem), these authors use what Shannon calls a "fidelity criterion", missing in the author's description. The principal coding theorem proved by the author of the book under review does not depend on μ! Surely the reader may be forgiven a certain amount of confusion.

The choice of references in coding theory is not entirely felicitous; essential references are omitted and irrelevant ones given. What suggestion is made of the history of the subject is not really accurate.

As has been suggested earlier, the first four chapters of this book constitute a lively, charming and interesting account of an interesting branch of mathematics. They are recommended with enthusiasm for both professional and amateur mathematicians. It is only the fifth and last chapter which does not measure up to the standard of the first four. Perhaps this deficiency can be remedied in a future edition.

J. Wolfowitz

Time-lag control systems. By M. Namik Oguztoreli. Academic Press, New York, London, 1966. xii + 323 pp. $13.50.

This book is divided into two parts: I. Ordinary delay-differential equations (164 pages). II. Optimal processes with time delay (110 pages). Except for a few isolated topics, the first part lays the foundation for the study of optimal processes. This consists first of a discussion of existence, uniqueness, continuous dependence on parameters, etc. for general delay equations even with the right hand sides measurable in the independent variable. The presentation on this topic is satisfactory. The next basic topic is concerned with linear homogeneous and nonhomogeneous systems. Having nicely derived the integral representation of the solutions of the linear system, the author turns to the important question of the derivation of the differential equations that the kernel functions must satisfy. This "derivation" is beyond description and the interested reader will be forced to go to other sources for information. These basic topics comprise approximately two thirds of the first part. The remaining miscellaneous topics are a few noninformative pages of stability; a chapter on nonlinear equations with functional power series for right hand sides that seems to be of little general interest; and a chapter on piecewise linear systems.

In the second part, the author formulates optimal processes in such a way that most of the extensive literature on these questions for ordinary differential equations can be carried over almost verbatim. In fact, this seems to be what the author has done, even to the extent of some mistakes from papers in ordinary differential equations. At other times, a full awareness by the author of the implications of some of the theorems that he states would have led to a clearer and more concise presentation. If part II had been better organized, this could have been a valuable contribution.

It is difficult to determine the audience for such a book. It is extremely theoretical with no examples and lacks the preciseness that is usually associated with mathematics.

Jack K. Hale

The Fourier transform and its applications. By Ron Bracewell. McGraw-Hill Book Co., New York, San Francisco, Toronto, London, Sydney, 1965. viii + 381 pp. $11.75.

Many engineering educators would doubtless agree with the author of this very readable book that while the (rigorous) development of Fourier transform theory is normally covered only in advanced mathematics courses, the usefulness of this analytical tool for discussing physical phenomena or the operation of physical devices makes it desirable to introduce it at earlier levels of a student's development.

The book emerges, then, as an attempt (successful, in the reviewer's opinion) to provide the student with an appreciation of the breadth of applicability of the Fourier transform to diverse problems of physical origin, and to develop in him some measure of insight into time-domain vs. frequency-domain relationships, as well as manipulative skill in using the transform. This is accomplished through the use of physical interpretation of mathematical expressions (wherever possible), a profusion of illustrations which aid the exposition measurably, and a collection of thought-provoking problems at the end of each of 16 of the book's total of 19 chapters.

The text can be divided into two approximately equal parts; one part contains basic material on Fourier transforms and related topics, e.g., definitions, conditions for existence, basic properties, convolution, generalized functions, relationship of frequency-domain behavior to time-domain behavior of functions and systems, etc.; the second part deals with special extensions and applications of the theory, and branches out into a discussion of other transforms and their applications. Some of the topics covered in this part are filtering, sampling, multidimensional Fourier transforms, antennas, television, convolution in statistics, noise, beat conduction and diffusion, and the Laplace, Hankel, Z, Mellin, Abel and Hilbert transforms.

Since the text is directed at students with simple mathematical backgrounds, there are the inevitable rough spots in the exposition due to the resolution of mathematical questions by formal manipulation or plausibility arguments (this is particularly true of the section on Fourier transforms of generalized functions). There is also a notable omission from the discussion on convolution, namely, frequency-domain convolution and its applications.

Much more surprising is the existence of an engineering textbook anachronism in the chapter on Laplace transforms, where it is stated that transform methods simplify the solution of linear differential equations. This distortion is compounded by lack of mention of the transform method's value as a vehicle for constructing design and synthesis techniques via the use of transfer functions.

The above criticisms do not vitiate the book's good qualities, and the book is therefore recommended as a reference or text at the undergraduate level.

LEONARD WEISS

Radiative transfer on discrete spaces. By Rudolph W. Preisendorfer. Pergamon Press, Oxford, London, Edinburgh, New York, Paris, Frankfurt, 1965. xi + 462 pp. $20.00.

The subject of radiative transfer is of continually increasing interest and importance to astrophysicists, meteorologists, and oceanographers; as well as neutron physicists and reactor engineers, who refer to the discipline as *transport theory*. This book is written primarily from the viewpoint of the person interested in radiative transfer in the hydrosphere, but much of the material will be of value to other practitioners.

The subject matter is divided into four parts. The first is entitled "Fundamentals" and consists of a careful discussion of many classical notions together with such new concepts as the principle of invariance and the general idea of invariant imbedding. Special attention is paid to operational definitions of various terms with discussions of actual instruments used in measurements when that is appropriate. The author is led to quite precise mathematical derivations of fundamental equations and to the careful statement of various mathematical-physical principles.

The second and third parts form the heart of the work and are devoted to the discrete space theory together with its applications. By a discrete space is meant, roughly speaking, one in which spacial and angular dependences have been discretized. While this is done for both one and three spacial dimensions, especial attention is given to the latter case. The region is divided into a cubical lattice and the angular dependence is discretized to twenty-six specific directions corresponding to the twenty-six cubes adjacent to any given cube. Principles are restated for the discretized situation and equations are worked out in great detail, especially those arising from the invariant imbedding principle. An actual numerical study of one set of experimental data resulting from an investigation of radiation in a lake is described and comparison of the results with the data is made.

The final portion of the book, entitled "Advanced Topics", takes up a variety of subjects, none of them in detail characterizing the earlier chapters. It is shown how light polarization can be included in the development. A connection is made with the theory of Markov chains. The relationship of radiative transfer with classical electromagnetic theory is explored and the radiance function is associated with the Poynting vector. The final chapters are devoted to a discussion of an axiomatic formulation of radiative transfer theory and to a brief statement of some outstanding mathematical problems.

This book contains valuable information for both the theoretical physicist and the mathematician. Unfortunately, the material is presented in such a way as to make the going difficult for both. The physical scientist and engineer will likely discover that the mathematical symbolism is excessive and frequently does not seem to provide any genuine enlightment. The mathematician may find a tendency to obscure the relationship between *principles* and *theorems*. (For example, the connection between the principle of interaction (page 114) and the general notion of a Green's function is not mentioned until page 331). The reviewer feels that many readers will find the detail excessive and the frequent philosophical excursions unnecessary—and as a consequence they may find the book too long and the price too high.

But there is much new material, much material that is presented in an unusual manner, many unique viewpoints and ideas. The reader already fairly familiar with the field and not about to be thwarted by the shortcomings mentioned in the previous paragraph will find the book of considerable interest and importance.

G. Milton Wing

Theoretical and mathematical biology. Edited by Talbot H. Waterman and Harold J. Morowitz. Blaisdell Publishing Co., New York, Toronto, London, 1965. xvii + 426 pp. $12.50.

Biology, as a significant component of man's intellectual and technical environment, is on the threshold of a new era. The excitement of this impending revolution comes across in the pages of this book. In fact, the revolution would no doubt be much further along if recent generations of biology graduate students had been able to read books like this one. Now that it is available, it can be confidently expected to "provide a significant stimulus for the revolution in biology" as the dust jacket proclaims. For the non-biologist mathematician or physicist, it provides a remarkably broadly-based sketch of what it is that some of the "avant garde" in biology are up to. There has long been a need for a summary book of this sort with just this title.

Like so many books these days, the contents were not in fact written as a book, but as separate lectures, essays or review articles, each one aimed at a different audience. The editors have made an effort to stitch these components together by sectional arrangement, introductory paragraphs, and their own chapters,—but one still has the feeling that most of what is here one has seen before, some of it in several different places.

The Seventeen chapters are somewhat arbitrarily arranged into five sections, summarized here to indicate the scope of the book:

I. Introduction: What theoretical biology has been, is, and is becoming. (3 Chapters); II. Physical and Chemical Analysis: Thermodynamics, rate theory, origin of life and molecular biology, nerve membranes, mechanism of hearing. (4 Chapters); III. Statistical Analysis: Multivariate Statistics, Morphometrics. (2 Chapters); IV. Computer Applications: Analog and Digital. (2 Chapters); V. Systems Analysis: General Principles, Metabolic Analogies, Information processing, Genetics, ecology. (5 Chapters); —)-: A review and critique of the book by one of the editors. (1 Chapter)

The book also reflects, in a fascinating and probably unavoidable way, one of the pedagogical dilemmas in theoretical biology. Throughout the book, one constantly wonders what audience the book is appropriate for. For mathematicians, in the hope of fascinating them into furthering the biological revolution? A worthy cause, and one which the book may well serve—but most of it certainly wasn't written with this in mind. For the practicing theoretical biologists? No, he's already read most of this stuff! For the biologist who doesn't know what theoretical and mathematical biology is, and would like to know? Perhaps—but unfortunately the mathematics used in many places here will be very likely to render the book unreadable for him. We are left, then, with the biology student, graduate or undergraduate, who has been puzzled by the fact that his science has so little explicit theory and so few theorists, in spite of its reliance on physics and chemistry, in which theory occupies an honored, well-established and essential place. If this book succeeds in persuading such students that they had better learn their mathematics and learn it well if they hope to become useful biologists during the next couple of decades, it will indeed have fulfilled its avowed purpose.

In any event, this reviewer recommends this book enthusiastically to mathematicians of all sorts who would like to take a peek at what is going on in a part of science which should interest them now, and about which they'll be hearing much, much more in the near future.

PETER A. STEWART

Automatic numerical integration. By J. A. Zonneveld. Mathematish Centrum, Amsterdam, 1964. 110 pp. $3.00.

The author derives and tabulates Runge–Kutta methods of various orders for (1) first order systems of equations (2) second order systems with first derivatives (3) second order systems without first derivatives. Criteria for automatic determination of an acceptable step length are presented and discussed. The monograph also contains six ALGOL 60 procedures and a number of numerical examples. It should prove a useful compendium for a computer lab reference library.

P. J. DAVIS

Calculus made easy. By Silvanus P. Thompson. Macmillan & Co., New York, 1965. vi + 250 pp. $1.95.

This is a paperback edition of an old landmark. I first met the book as an undergraduate; a roommate threw it to me and said, "It's worth a few laughs." I must admit that I didn't much like it then as a place to learn calculus, and I still don't. But to be fair, I've met fine men and women who have.

When I read some of the things in this book, I find myself wincing; partly, I think, because I grew up in the limit school of mathematics, but partly because I don't really believe that what Silvanus says helps anyone. There are innumerable places where, instead of lifting the veil of obscurity, he draws it down more heavily over the pretty features of the subject. There are plenty of books where calculus is made just as easy. And as far as the cookbook (or as we now say, the algorithmic) aspects of calculus, you can get it just as well from the plastic cards sold in school book shops.

The printing record of the book has been remarkable. It has had three editions since 1910, and has been reprinted every three years since then, on the average. To cop a phase from TV, Silvanus must have done something right. I recall reading in a biography of Harold Macmillan, that the Macmillan fortune was based upon the vast popularity of Hall and Knight's "Algebra". A bust of Hall and/or Knight was subsequently set up in the Macmillan Board Room. Thompson surely merits a small bas-relief.

With his famous phrase "What one fool can do, another can," the author promises to reveal to us at small cost the mysteries of the mathematical cult. Is this the secret of its longevity? After all, who can resist a delicious shudder at seeing the next man reduced to water and ashes? Or is it the title of the book which is in the great do-it-yourself tradition? I don't know. Perhaps, after all, he has made calculus easy.

One thing is clear though; deltas may come and epsilons may go, but Silvanus P. Thompson, F.R.S., 1851–1916, apparently goes on forever.

PHILIP J. DAVIS

Tables of Laguerre polynomials and functions. By V. S. Aizenshtadt, V. I. Krylov and A. S. Metal'skii. Translated by Prasenjit Basu. Pergamon Press, Oxford, London, Edinburgh, New, York, Toronto, Paris, Frankfurt, 1966. xv + 150 pp. $8.00.

Laguerre polynomials and functions are widely used in many problems of mathematical physics and quantum mechanics, for example, in the integration of Helmholtz's equation in paraboloidal coordinates, in the theory of the propagation of electromagnetic oscillations along long lines, in the solution of Schrödinger's equation for hydrogen-like atoms, etc., as well as in the expansion in series of an arbitrary function in the interval $(0, \infty)$.

The tables contain the values of Laguerre polynomials and Laguerre functions for $n = 2, 3, \cdots, 7$; $s = 0(0.1)1$; $x = 0(0.1)10(0.2)30$ and the zeros and coefficients of the polynomials for $n = 2(1)10$ and $s = 0(0.05)1$.

Methods of matrix algebra. By Marshall C. Pease, III. Academic Press, New York and London, 1965. xviii + 406 pp. $13.75.

In his Foreword, the author states that "This book is primarily directed toward the student at or near the graduate level in physics or some related field . . .".

In consequence, much of the subject matter is introduced, interpreted, or illustrated with references to physical applications. The book covers a wide variety of both standard and nonstandard matrix topics (including a chapter on non-positive-definite quadratic forms—referred to as "improper inner products"), in varying degrees of depth without exceeding normal expectations of the mathematical sophistication of the audience to which the book is addressed. The concluding five chapters deal exclusively with applications, and there are a number of problems given at the end of each chapter of the book. Computational problems involving matrices are *not* discussed.

The style of writing is relatively informal, and explanations are generally easy to follow.

The book is recommended as a reference for students whose prior exposure to matrices and linear algebra has not been deep and who wish to be apprised of physical applications as they learn the theory.

Chapter headings are as follows: I) Vectors and Matrices; II) The Inner Product; III) Eigenvalues and Eigenvectors; IV) Hermitian, Unitary, and Normal Matrices; V) Change of Basis, Diagonalization, and the Jordan Canonical Form; VI) Functions of a Matrix; VII) The Matricant; VIII) Decomposition Theorems and the Jordan Canonical Form; IX) The Improper Inner Product; X) Singular and Rectangular Operator; XIII) The Commutator Operator; XIV) The Direct Product and the Kronecker Sum; XV) Periodic Systems; XVI Application to Electromagnetic Theory; XVII) Sturm–Liouville Systems; XVII) Markov Matrices and Probability Theory; XIX) Stability.

LEONARD WEISS

Elements of the theory of nonlinear oscillations. By N. V. Butenin. Translated from the Russian by Scripta Technica. Blaisdell Publishing Co., A Division of Ginn and Co., New York, Toronto, London, 1965. viii + 226 pp. $7.50.

This book illustrates a number of methods, some of which are only approximate, for studying nonlinear phenomena. This is done by applying the methods to a series of examples of one degree of freedom and to systems with gyroscopic forces (two degrees of freedom).

No attempt is made to justify the methods and this is not the purpose of the book. The author's aim is to provide an introduction to nonlinear phenomena and to some of the methods available for their study. This is a prerequisite for a more extensive study of the theory and it is through such examples that one achieves an appreciation of the theory and learns to apply it. Although there are other books of this type, this one is concise and is suitable for a one-semester course for engineering students following an introductory course in ordinary differential equations. It is well written and the translation is good. A serious drawback to the book as a text is that the instructor will have to provide his own exercises.

J. P. LASALLE

Real and abstract analysis—a modern treatment of the theory of functions of a real variable. By Edwin Hewitt and Karl Stromberg. Springer-Verlag, New York, 1965. viii + 476 pp. $9.50.

In this book the authors have given an account of the essential topics for a rigorous course in advanced analysis. The book also contains some more advanced material and many applications of the theory. Chapter One deals with set theory, including the axiom of choice, cardinal and ordinal numbers, and a construction of the real and complex number fields. There follows a chapter on topology and a detailed treatment of spaces of continuous functions. Integration theory is the subject of Chapter Three. The account given starts with the Riemann–Stieltjes integral and culminates, through the Daniell extension procedure, with the full generality of integration on arbitrary measure spaces. Chapter Four is an introduction to functional analysis giving the basic theorems on Banach and Hilbert spaces, together with applications to Fourier transforms and special functions. In Chapter Five there is an account of differentiation including the Lebesgue–Radon–Nikodyn theorem and several applications. Finally there is a chapter devoted to integration on product spaces.

The authors are to be congratulated on their choice of material for this book. The presentation is clear and complete and the more peripheral sections are all interesting and often contain illuminating comments. While the book as a whole would be too much for all but the best students, with some omissions, an excellent course could be given from it. Suggestions as to what might be left out are given in the preface. The presence of many applications, both in the text and in exercises, should make this a most suitable book for prospective applied mathematicians.

The index of some fourteen pages should help in the use of this work as a reference. However for this it is necessary to be familiar with the notation, because special symbols, once introduced, are used without further reminders. The production leaves something to be desired. Too often the pages have a rather cramped appearance, frequently the same symbols are printed on the same line in different sizes and fractional exponents are in disproportionately large type. However these are minor blemishes in an excellent book.

J. A. Erdos

Fundamental analogue techniques. By R. J. A. Paul. The Macmillan Co., New York, 1965. x + 216 pp. $6.50.

This brief but most interesting volume is intended to serve as an introduction to the fundamental concepts and techniques underlying the use of electronic analog computers. In order to avoid superficiality, the author has chosen to treat in considerable detail topics relevant to the simulation of dynamic systems while omitting, except in brief references, application areas such as partial differential equations, random noise, statistics, optimization, and hybrid computations. Within these constraints, the author has fulfilled his purpose remarkably well and has produced a most useful text.

The book opens with an introductory chapter placing analog computing techniques in a general perspective. The basic principles of the electronic differential analyzer and the operation of the major linear components are then briefly reviewed. This is followed by a chapter on programming, scaling and checking procedures including a detailed discussion of amplitude scale-factoring.

Chapter IV is devoted to "dynamic analogies and network element transforms" and describes a number of techniques which are invaluable in the development of the mathematical models of mechanical and electro-mechanical systems which are then programmed on the computer. A separate chapter is devoted to iterative operation of the analog computer and draws heavily upon results obtained with the DYSTAC computer as well as upon the authors own experience at the College of Aeronautics at Cranfield, Great Britian. The final chapter discusses the simulation of rational transfer functions in considerable detail. Together with three appendices devoted respectively to unilateral s-multiplied Laplace transforms, generalized two-port networks, and symmetrical lattice networks, this last chapter constitutes actually an interesting treatment of active network synthesis.

This text is recommended as a relatively compact introduction to analog computing and should also appeal to analog computer users specializing in dynamic simulations.

Walter J. Karplus

Nonlinear partial differential equations in engineering. By W. F. Ames. Academic Press, New York and London, 1965. xii + 511 pp. $16.00.

In no area of applied mathematics is there a greater gap between the knowledge the mathematician can supply and the information needed in the applications than in the large and difficult field of nonlinear partial differential equations. The applied scientists and the mathematicians approach this situation in different ways, each justified by the nature of his science: The mathematician formulates problems that appeal to his esthetic sense and which he can analyze with the degree of rigor required by the professional code of his craft (a code that is being revised every generation, to be sure). If the problem has still some resemblence with the mathematical model for a physically relevant system, so much the better. The applied scientist, no matter how much he may appreciate pure mathematics intellectually, has to tackle problems the mathematicians cannot solve yet. He therefore adds to his arsenal of legitimate modes of justification all sorts of plausibility arguments, as long as they convince him and don't contradict physical evidence.

The author of this book is well acquainted with the mathematician's viewpoints, but he is an engineer writing for engineers. In this book he has collected descriptions of a large number of methods that have been used to solve nonlinear mathematical problems in applied mathematics and engineering. The amount of material is enormous and the methods and viewpoints vary with the source from which it was taken from being purely mathematical to being based mostly on a hope and a prayer. By necessity, the presentation is therefore brief and not always self-contained. To an engineer who is confronted with problems involving nonlinear partial differential equations this collection will be a very valuable help. He will find, arranged according to methods, concrete discussions, mostly in the form of examples, of a great wealth of techniques supported by extensive references to the recent literature. About four hundred different articles and books are listed.

After a brief introductory section in which the physical origin of some nonlinear partial differential equations is discussed, there is a chapter describing numerous transformations of the dependent and independent variables, by means of which certain problems can be substantially simplified. Next, the author lists, in two long chapters, methods for finding special classes of solutions to special differential equations. It is his opinion that too much emphasis on the exact fulfillment of the boundary conditions of a given problem has led to an unjustified neglect of such explicit special solutions. Approximate solutions such as regular and singular perturbations and various other series expansions make up the next two chapters. Finally, there is a very long chapter on numerical methods, primarily of the discrete variable type.

As a mathematician, this reviewer is probably biased in favor of the standard mathematical terminology and the mathematician's mode of expressing ideas. But I cannot help feeling that the regrettable difficulties of communication between mathematicians and engineers could be eased if on the engineering side more effort were made towards clarity of expression. This book, while clearer than many others, still leaves too much to the reader's ability to read between the lines: He has to find out that "equivalent" is meant when two equations are called "compatible" (p. 54), that, when a second order differential equation is said to have been "factored" it has only been replaced by a system of two simultaneous first order differential equations (p. 29), etc. Here is an example of a different type of vagueness in the terminology: After defining "singular perturbations" in the usual way on p. 211 the author illustrates the concept by an example (Proudman and Pearson's analysis of viscous flow past a sphere at low Reynolds number) which, on the face of it, does not at all conform to that definition. It is true that this problem can probably be reformulated so as to involve a singular perturbation, but this would require some far from obvious transformations.

Wolfgang Wasow

Integral transforms and operational calculus. By V. A. Ditkin and A. P. Prudnikov. Translated by D. E. Brown. English translation edited by I. N. Sneddon. Pergamon Press, Oxford, London, Edinburgh, New York, Paris, Frankfurt, 1965. xi + 529 pp. $15.50.

This is a translation, by D. E. Brown, of the original Russian edition published in 1961. It is a handbook and tables of integral transforms of functions over unbounded intervals. The first four chapters cover properties of Fourier, Laplace, Bessel and, briefly, other transforms including Hilbert and Laguerre transforms. Conditions of validity are usually stated in their more efficient forms, using concepts from Lebesgue theory of integration. Most properties are stated without proofs, which are to be found in references listed in the Bibliography at the end of the book. For some of the transformations basic operational properties on differential or integral forms are not emphasized or not stated; they are sometimes indicated by an example. Chapter 5 gives a brief presentation of a modified form of Mikusinski's operational calculus based on operators.

Chapters 6 to 12 consist of about 350 pages, printed lengthwise on the pages, of tables of transforms and their properties. The tables include about 400 each of Fourier cosine and sine transforms and Laplace transforms, some 60 Mellin transforms and 300 Bessel or Hankel transforms.

R. V. Churchill

Applied complex variables. By John W. Dettman. The Macmillan Co., New York and Collier-Macmillan Ltd., London, 1965. ix + 481 pp. $9.95.

The book is divided into two parts. Part I, entitled "Analytic Function Theory", gives a careful introduction to the classical theory of functions of a complex variable. It is stated in the Preface that no advanced calculus background is assumed, but this should not be interpreted to mean that the book is suitable for engineering students whose exposure to mathematics is limited to a manipulatory calculus course. A certain amount of mathematical sophistication—of the kind usually acquired in an advanced calculus course—would appear to be prerequisite for a successful use of the book.

Part II, called "Applications of Analytic Function Theory", is divided into the following five chapters: Potential theory; Ordinary differential equations; Fourier transforms; Laplace transforms; Asymptotic expansions. While all the topics treated are of interest to applied mathematicians, there are some whose connection with complex function theory would appear to be rather tenuous. There are hardly any branches of mathematical analysis which are completely deviod of the occasional use of complex algebra, but it is arguable whether this justifies their being regarded as applications of analytic function theory.

Throughout the book, the presentation is both clear and rigorous, and the number of exercises is adequate.

Zeev Nehari

Differential equations of applied mathematics. By G. F. D. Duff and D. Naylor. John Wiley & Sons, Inc., New York, London, Sydney, 1966. xi + 423 pp. $11.95.

This book is written as a text for senior undergraduates in applied mathematics and engineering. It presents, in a unified manner, the application of Fourier series and integral transforms to the study and solution of the standard partial differential equations of mathematical physics.

Chapter headings: I. Finite Systems; II. Distributions and Waves; III. Parabolic Equations and Fourier Integrals; IV. Laplace's Equation and Complex Variables; V. Equations of Motion; VI. General Theory of Eigenvalues and Eigenfunctions; VII. Green's Functions; VIII. Cylindrical Eigenfunctions; IX. Spherical Eigenfunctions; X. Wave Propagation in Space. Short tables of Fourier, Fourier Sine, Laplace and Hankel transforms are included, as well as a Bibliography. Each section includes many exercises, some of which are motivated by physical problems.

The authors have successfully maintained a close relationship between the mathematical methods and their applications. Throughout the text they have developed the background of mechanics necessary for the development of the equations of motion of continuous systems. Chapter V, exclusively devoted to this purpose, is particularly well written and will certainly motivate its readers to further study. The mathematical treatment is not very rigorous, and many theorems are stated with much stronger conditions than are necessary, but from a certain viewpoint this is desirable.

This book is a welcome addition to the texts available for the standard undergraduate course in Boundary Value Problems for engineers and physicists, with its emphasis on physical motivation and application to significant physical problems. A good advanced calculus course is necessary for a thorough understanding of the presentation.

E. F. INFANTE

Handbook of series for scientists and engineers. By V. Mangulis. AcademicPress, New York and London, 1965. viii + 134 pp. $6.95.

This is a useful reference work. Systematic tabulations are given both for expanding a given function into a series, and for finding the function represented by a given series.

Bernoulli (1713), Bayes (1763), Laplace (1813). Edited by Jerzy Neyman and Lucien M. LeCam. Proceedings of an International Research Seminar, Statistical Laboratory, University of California, Berkeley, 1963. Springer-Verlag New York Inc., 1965. ix + 262 pp. $9.00.

This volume represents the Proceedings of an International Research Seminar organized in 1963 by the Statistical Laboratory, University of California, Berkeley, on the occasion of the 250th anniversary of Thomas Bayes' "Essay towards solving a problem in doctrine of chance" and the 150th anniversary of Pierre-Simon de Laplace's "Essai philosophique sur les probabilites".

There are fifteen excellent articles by very distinguished speakers on a wide range of topics in probability theory (mainly stochastic processes) and mathematical statistics.

Coding theorems of information theory. By J. Wolfowitz. Second Edition. Springer-Verlag New York Inc., 1964. x + 154 pp. $6.75.

The second edition of this well-known book has been improved to the extent of the correction of a few misprints and the addition of a few remarks here and there. Moreover, some of the theorems have been replaced by stronger versions.

Handbook for computing elementary functions. By L. A. Lyusternik, O. A. Chervonenkis and A. R. Yanpol'skii. Translated from the Russian by G. J. Tee and edited by K. L. Stewart. Pergamon Press, Oxford, London, Edinburgh, New York, Paris, Frankfurt, 1965. xiii + 251 pp. $10.00.

This is an excellent handbook, addressed, among others, to digital computer programmers. It is concise, yet readable and theoretical, yet useful. It treats of binomial expansions; exponential, logarithmic, trigonometric, hyperbolic (and their inverse) functions; the standard orthogonal polynomials: Legendre, Chebyshev, Hermite, Laguerre; Gudermanians and hypergeometric functions.

WALTER FREIBERGER

Detection theory. By Ivan Selin. Princeton University Press, New Jersey, 1965. xiii + 128 pp. $5.00.

Signal detection, looked upon as a problem of the testing of statistical hypotheses on stochastic processes, receives in this book a treatment which is relatively elementary (assuming only "engineering probability theory") but quite comprehensive and certainly most readable. The contributions of recent years, by Grenander, Middleton, Zadeh and others, are well summarized and coherently presented.

WALTER FREIBERGER

Applied mathematics for engineers and scientists. By S. A. Schelkunoff. Second Edition. D. Van Nostrand Co., Inc., Princeton, N. J., Toronto, New York, London, 1965. xii + 496 pp. $9.50.

The first edition of this well-known textbook has been supplemented by three chapters: Solving by Series, Partial Fractions and Summation.

Queues and inventories—a study of their basic stochastic processes. By N. U. Prabhu. John Wiley & Sons, Inc., New York, London, Sydney, 1965. xi + 275 pp. $12.95.

Professor Prabhu's study of the basic stochastic processes of queues and inventories treats mainly single-server queueing systems and related inventory models. He concentrates on demonstrating the mathematical methods of analysing the various models, and covers those techniques which lead to explicit results.

Chapter 1 (Some Queueing Systems) is both an introduction and a broad survey of what is now "classical" (pre 1956) queueing theory. There is a discussion, mainly on stationary distributions, of $M/M/1$, $M/M/s$, and $M/M/\infty$; Erlang distributions as service in phases ($M/E_k/1$) and arrival in stages ($E_k/M/1$); $M/D/s$; Kendall's work on imbedded Markov chains; Takacs's integro-differential equation; and Lindley's work on waiting times. Chapter 2 (Transient Behaviour of the Systems $M/G/1$ and $GI/M/1$) uses combinatorial methods and supplementary variables in systems where the algebra remains tractable by virtue of the special properties of the negative exponential distribution and Poisson process. Both Chapter 2 and Chapter 3 (Some Imbedded Markov Chains) lean heavily on work of the author and U. N. Bhat to give neater proofs of known results and thereby develop further expressions for the two basic simple queues. Chapter 4 (The Queueing System $GI/G/1$; Bulk Queues) rounds off the first part of the book (pp. 1–173) with an elegant treatment of the applications of fluctuation theory to waiting times and busy periods in $GI/G/1$, and a brief discussion of bulk queues.

Inventories are discussed in the remainder of the book: "...we describe a few inventory models. Although we shall solve the optimization problem for a few of these, our main topic of study is the related stochastic processes" (p. 175). Chapter 5 (Some Inventory Models) is broader in scope than Chapter 6 (Moran's Model for the Dam) and Chapter 7 (Continuous Time Storage Processes) which explore in some depth the various possibilities leading to tractable algebra in Morands Dam model and its continuous time analogue. One feels that this second part of the book is not as comprehensive as the earlier section on queueing theory.

In the "Complements and Problems" at the end of each chapter the author summarizes some of the related literature, including special cases of the text preceding it, and indicates the relevance and interrelationships of some of the formulae derived. These notes are valuable in significantly increasing the scope of the book without adding unduly to its length.

Two minor points may be mentioned. In the first chapter the existence of equilibrium distributions is frequently assumed without proof: however, by omitting arguments relating to existence proofs the author can speed his way to exhibiting methods which lead to explicit solutions. The other point concerns notation: the reviewer was not always happy with the abundantly free use of the Stieltjes differential.

In the reviewer's experience the book is well adapted for use as a reference on standard models. Newcomers to the subject studying the book in detail should gain a fairly comprehensive understanding of the mathematical methods now used in the proliferating (and proliferated) literature.

D. J. DALEY

Mathematical theory of creep and creep rupture. By Folke K. G. Odqvist. Oxford University Press, New York, 1966. ix + 168 pp. $7.70.

This very informative and interesting monograph is addressed to mathematically oriented engineers who need to solve multiaxial stress problems of creep deformation and must design against creep rupture. The text is self-contained and is supplementary to "Kriechfestigkeit metallischer Werkstoffe" by F. K. G. Odqvist and J. Hult, Springer, Berlin, 1962.

Metals are idealized as rigid-nonlinear-viscoplastic-deteriorating solids obeying power term forms of stress-strain relations. Numerical values are tabulated for a number of steels and non-ferrous metals on this basis. A commutative law of creep (an equation of state) and a deformation law for plastic flow are assumed for simplicity along with a modification of Kachanov's rupture condition. Professor Odqvist points out the physical and mathematical objections to certain aspects of these simplifications but emphasizes their value in obtaining usable answers to problems of practical importance. Idealizations and experimental data are compared. He cautions that "The theories to be presented are mostly less than five years old and the observational facts to support them are scarce."

Boundary value and stability problems are discussed in considerable generality. Explicit solutions are given for tubes, beams, membranes, plates, and shells.

D. C. Drucker

Topics from the theory of numbers. By Emil Grosswald. The Macmillan Co., New York, and Collier-Macmillan Ltd., London, 1966. xiv + 299 pp. $8.50.

A certain evolutionary phase is long overdue in the teaching of the theory of numbers, namely the realization that algebra has been so strongly influenced by number theory that the first semester of number theory can be largely subsumed in the algebra courses. It is painful to see "primitive roots" taught to students who have learned "cyclic groups" elsewhere, for example. In accordance with this evolutionary process, there is a definite need for more mature books on number theory which dwell on problems of significance to the frontiers of mathematics and not on the combinatorial novelties that belonged to the bygone age when mathematics was a leisurely diversion for the training of the mind.

This present book by Grosswald serves admirably to make the student aware of how modern analysis and algebra were shaped by number theory. The author does this largely by using the prime number theorem, the partition function, and the "regular case" of Fermat's Last Theorem. The book is very well written and well organized. It does require somewhat advanced knowledge of analysis but elementary knowledge of algebra.

Any mathematician interested in teaching number theory will find it irresistible to try this book.

Harvey Cohn

Hill's equation. By Wilhelm Magnus and Stanley Winkler. Interscience Publishers, A Division of John Wiley & Sons, New York, London, Sydney, 1966. viii + 127 pp. $8.50.

This monograph is a beautiful introduction to the fascinating world of linear homogeneous second order differential equations with periodic coefficients. The discussion is centered around the stability of the solutions and not on specific series representations for the solutions. After a proof of the general theorem of Liapunov and Haust on the characterization of the intervals of stability, the authors turn to a treatment of some methods for determining more precise information in special cases. The coexistence problem is also discussed in detail.

Jack K. Hale