

—BOOK REVIEW SECTION—

Introduction to cybernetics. By Viktor M. Glushkov. Translated by Scripta Technica, Inc. Academic Press, New York and London, 1966. x + 322 pp. \$11.75.

About twenty years have passed since Norbert Wiener coined the expression *cybernetics* to describe the interdisciplinary study of communication and control in man and machine. Since then there has emerged a vast literature devoted to this subject, most of it in the form of specialized papers published in journals but also some of it published in book form. It is a topic that lends itself to oversimplification and popularization with grandiose and rather nebulous outlooks. Enthusiasts are promising us wonderful (or sometimes frightening) results in the future. No doubt this sort of publicity has made cybernetics appear less than respectable to many scientists whose interests otherwise might have made them sympathetic to this general approach.

The present book is different: it is a professional job, written by a well-known authority in the field. Although intended as an introduction, it does not avoid technical difficulties. Of course, the author does not give complete derivations and discussions of the many and varied topics that he describes, but this would be impossible in a book of this size. He is concise in the exposition, sometimes to such an extent that a reader who is not already familiar with the material would find it hard to digest the content. This is true particularly in the treatment of finite automata in Chapter III.

It is interesting to see what Professor Glushkov chooses to include in the book, obviously considering it as belonging to the core of cybernetics, and what he leaves out. A central notion is that of an algorithm and its implementation via automata of different construction. This theme dominates the book and for an obvious reason. If one had to explain the massive research effort that has gone into cybernetics in recent years, most people would point to the advent of the digital computer. It is true that even before this event some work was done on computability, Turing machines, self-organizing systems etc., but once the technological capability of the computer became obvious it led to an explosive outgrowth of research activities in cybernetics. Therefore the emphasis on algorithms. The author uses the first chapter to describe some of the usual algorithmic systems. This is followed up in Chapters III and V by a discussion of finite automata as well as of computers and their programming; the latter is illustrated by a sketch of ALGOL-60.

Chapter IV deals with self-organizing systems and in particular with pattern recognition and learning in perceptions. This is perhaps the most practical part of the book, but it is disappointing that even here the exposition is completely abstract and general, when it would have been possible to point to concrete and exciting instances of practical implementation.

Chapters II and VI finally give an outline of propositional and predicate calculus and discuss construction of proofs.

Among things usually classified as cybernetics but only mentioned in passing in this book is mathematical model building in neurophysiology and psychology. But this would perhaps not have fitted in very well with the material of the present book. More surprising is the fact that the mathematical theory of languages, formal and natural, has not received more attention. The elements of algebraic linguistics are quite close to some topics of the book.

We should be thankful to Professor Glushkov for writing this volume and completing so well this difficult task. It may not be the ideal book for the beginner, parts of it being quite difficult, but a reader with a modest background in cybernetics will find a lot of valuable information in it.

The term "cybernetics" seems to have been accepted by the scientific community in the U.S.S.R. as well as in Central Europe. A number of periodicals appear with this term occurring in their names and institutes of cybernetics have been founded where respectable research work is going on. In the U.S., however, people seem to shy away from using the word, perhaps because of its negative publicity. It is a good term, though, and describes fairly well an area in which many research workers have a legitimate interest. Is it not time that we also could talk about cybernetics without feeling like charlatans?

ULF GRENANDER (*Providence, R. I.*)

Mathematics of automatic control. By Toshie Takahashi. Translated by Scripta Technica, Inc. Holt, Rinehart and Winston, Inc., New York, Chicago, San Francisco, Toronto. London, 1966. xiii + 434 pp. \$12.50.

The book, a translation from the 1961 Japanese edition, discusses a variety of topics in the theory of complex variables that are of use in the study of classical automatic control theory. The emphasis is on the mathematical topics and, although there is substantial physical motivation, actual design techniques for automatic controls are not discussed.

The treatment of complex functions is clear and reasonably complete (for the intended purpose) but is relatively standard, in that the material is available in other English-language texts which are concerned with 'transfer function' analysis for linear systems (see references below). The 'transfer function' is discussed in detail; the basic concepts are dwelt upon at some length, and numerous physical examples (electrical, mechanical, hydraulic) are given. In fact, the author seems to have taken pains to identify many of the mathematical concepts with their physical counterparts. The mathematics would be accessible to an engineering senior, or first-year graduate student. The book contains many worked out examples.

The section on Laplace transforms gives a nice summary, with proofs, of many of their properties. All the usual transfer function stability criteria are discussed and proved. Topics in filters, such as the Paley-Wiener theorem, positive real functions, and ideal filters, are discussed.

The book reads very well, is carefully organized, and would provide the reader with an adequate background, so that the specialized works on transfer functions, transfer function stability, etc., will be accessible. However, there is no mention of sampled data systems, very little on statistical questions, and nothing concerning the 'state variable' approach, or multi-input multi-output linear systems. The book by Kaplan provides a broader coverage of linear systems, and the book by Papoulis overlaps it in most crucial topics, and discusses others (e.g. Fourier series).

Kaplan, *Operation Methods for Linear Systems*, Addison-Wesley

Papoulis, *The Fourier Integral and its Applications*, McGraw-Hill

H. J. KUSHNER (*Providence, R. I.*)

The selected papers of E. S. Pearson. Issued by the Biometrika Trustees to celebrate his 30 years as Editor. University of California Press, Berkeley and Los Angeles, 1966. 327 pp. \$6.75.

This volume of 21 papers by E. S. Pearson was published on behalf of the Trustees of *Biometrika* to mark his retirement after 30 years of service as Managing Editor of that journal. The papers were originally published from 1928 to 1963, mostly in *Biometrika*. The selection includes such important contributions as *Methods of statistical analysis appropriate for k samples of two variables* (with S. S. Wilks), *The use of confidence or fiducial limits illustrated in the case of the binomial* (with C. J. Clopper), and *The choice of statistical tests illustrated on the interpretation of data classed in a 2×2 table*. A companion volume, entitled *Joint Statistical Papers*, will reprint the 10 papers written in collaboration with J. Neyman, which formed the basis of what is now known as the "Neyman-Pearson theory." This theory of testing hypotheses and confidence intervals has been fundamental in the modern mathematical development of statistical inference. Most of the papers in the present volume under review are concerned with the meaning and usefulness of this theory in clarifying existing procedures and devising new techniques.

T. W. ANDERSON (*New York, N. Y.*)

Applied regression analysis. By N. R. Draper and H. Smith. John Wiley & Sons, Inc., New York, London, Sydney, 1966. ix + 407 pp. \$11.75.

The authors of this book have had a considerable amount of experience as consulting statisticians for industry and have written with this experience as a background and with such applications in view. The result is a book that is both strong on the theoretical side (though never unnecessarily sophisticated mathematically) and well written from the applied point of view.

The reader is expected to have some knowledge of elementary statistical techniques, but their development of their topic is so full and carefully set out that anyone who has some rudimentary knowledge of statistics and matrix theory (together with a feeling for the applications) will have little difficulty in following it.

Of the 407 pages of the book, 316, or about three-quarters, are devoted to a standard though original treatment of the subject including two extensive bibliographies (one general, the other "nonlinear"); 30 pages to Answers to Exercises (which include a good deal of the numerical workings, together with comments where needed); and about 50 pages to computer printouts. The chapters are: 1. Fitting a Straight Line by Least Squares; 2. The Matrix Approach to Linear Regression; 3. The Examination of Residuals; 4. Two Independent Variables; 5. More Complicated Models; 6. Selecting the "Best" Regression Equation; 7. A Specific Problem; 8. Multiple Regression and Mathematical Model Building; 9. Multiple Regression Applied to Analysis of Variance Problems; and 10. An Introduction to Nonlinear Estimation.

Chapter 1 gives a brisk treatment of simple linear regression, followed through for a numerical example. Derivation of the usual algebraic identities is terse but effective, a more general and detailed treatment being reserved for later chapters.

Chapter 2 would be difficult to any reader who had not some previous familiarity with matrices; the development here is extremely rapid. Many important results, and such concepts as unbiasedness and minimum variance, are given "on the run" in the course of discussing an example.

Chapter 3 is devoted to the examination of residuals; this is an important practical tool that modern computer techniques have made readily available.

Chapter 4, dealing with only two regression (here called "independent") variables, seems to be retrogressive after the general treatment given in Chapter 2. However, it is a helpful discussion of the issues that arise in the practice of fitting regression relations; it is made more telling by the inclusion of diagrams showing the geometry of least squares. The discussion of the various criteria for the effectiveness of an additional variable would have been more useful if some opinions had been expressed about their relative merits.

Amongst other devices, dummy variates are introduced in Chapter 5; they are exploited fully in Chapter 9, where the fact that analysis-of-variance models are simply regression models, and that the choice of the correct model is just as important as in regression problems, is emphasized. Here too it is made clear that the examination of residuals plays as important a role in the analysis of variance models as in the familiar regression models.

The final chapter gives a lively discussion of nonlinear models, greatly assisted by diagrams showing the geometry of nonlinear estimation. The difficulties of calculation, let alone of testing significance or providing standard errors, are clearly shown, and some methods of improving convergence in particular cases are discussed.

The book has a refreshingly "practical"—one might say "computerized"—approach. A novel feature of the book includes the detailed working of large numerical examples, including computer printouts. The latter give on first inspection a somewhat unfinished appearance to the presentation and certainly do not make good reading, but actually are of immense value, since features of the printout such as any investigator might be presented with in his work can be seen and discussed. Rounding errors in computations are frankly presented, but means of minimizing them are also discussed. One point, not often realized, that could be brought out is that lack of numerical accuracy arising from ill-conditioned matrices is of little importance in regression analysis, since the coefficients being determined will then have large standard errors and need not be accurately determined anyway.

The bias arising from fitting an incorrect model, which has been studied in some detail by Box and his co-workers on response surfaces, is discussed fully.

Some books on numerical analysis present a great variety of methods, between which the user is

left to choose as best he can. A valuable feature of the present book is that, together with various alternative methods of selecting regression in Chapter 6, the authors give their opinion of the merits or demerits of each one.

In the same spirit, when listing references in the eight-page general bibliography at the end of the text, they give short notes on the useful points in each reference.

With the increasing capacity of modern computers, more and more complicated regression analyses are becoming practicable. Indeed, regression analysis seems likely to become merely a branch of computer technique. This book makes full use of the power of the computer, but it avoids the temptation to carry out computations blindly. Computer programs for regression enable residuals to be printed out, and the importance of a thorough examination of these residuals is stressed. First, and most obviously, residuals can reveal the presence of outliers; in addition, they will often suggest new regression variables that may be included to improve the relation.

It will be interesting to see how long it is before "examination of residuals" is relegated to the computer, along with the other tedious numerical details. At the present time the reviewer has found that an examination of residuals is often useful to check the working of the computer program.

The few errors and misprints noted were minor, and unlikely to be misleading. On page 63, the term ϵ should be omitted from the equation. On page 68, line 12, $S_1 - S_2$ does not require the divisor $p - q$; and 6 lines from the foot $X'X$ is to be singular (not nonsingular). On page 119, 5 lines from foot, the first figure should be 19.6361, as in the table.

It should have been made clear on page 116 that a significant predictor is not always a good predictor, as many experimenters have found to their cost.

In places the style could be improved. For instance, if the sign "=" is eschewed in the text, "that is", is preferable to "equals" (pages 68, 94).

On page 132, postulating a mean of unity for the random variable is one way of ensuring that the mean of its logarithm is *not* zero; what is essential here (doubtless assumed though not stated) is that ϵ be always nonnegative.

This book lives up to its title admirably. It provides the most up-to-date techniques for regression calculation and assessment in a wide range of practical situations, and should enable the reader to cope with many of the problems he may meet in applied statistics. It can be recommended unreservedly to all those interested in regression analysis as a practical tool.

E. J. WILLIAMS (*Parkville, Vic., Australia*)

Essays in mathematical economics (in honor of Oskar Morgenstern). Edited by Martin Shubik. Princeton University Press, New Jersey, 1967. xx + 475 pp. Price \$12.50.

The essays assembled in this volume are a handsome tribute to a great mathematical economist. Had all practitioners of this science been invited to contribute who in their own work have been influenced by personal exposure to Oskar Morgenstern, the circle would have been much wider. If we consider the set of all persons whose professional thinking has been stimulated by the writings of Oskar Morgenstern, it would have to include the entire economics and operations research professions.

The contributions cover a wide range of subjects and little more can be done in this review than to describe them briefly.

The reader's expectation to find important contributions to game theory is not disappointed. In Part I, Game Theory, "A Survey of Cooperative Game Theory Without Side Payments," by Aumann delineates the directions that this body of thought has taken by relaxing the von Neumann-Morgenstern assumption that side payments are always possible and that utility is always transferable. Harold Kuhn develops a new approach to "Games of Fair Division" with his usual flair for lucidity and interesting examples. M. Davis and M. Maschler examine "The Existence of Stable Payoff Configurations for Cooperative Games" where the desire for stability is sufficiently relaxed to assure the existence of a stable payoff configuration which is Pareto optimal, for all superadditive games. Shapley considers "Solutions that Exclude One or More Players." In "Concepts and Theories of Pure Competition" by L. Shapley and M. Shubik, it is shown that as the number of players is increased three possible solutions to the bilateral monopoly game converge to one, although their rationales remain distinct.

In Part II, Mathematical Programming, S. Noble introduces output coefficients and demonstrates the usefulness in "A Property and Use of Output Coefficients of a Leontief Model." G. Thompson gives

a very readable survey of "Some Approaches to the Solution of Large-Scale Combinatorial Problems" including the traveling salesman and the job shop scheduling problem. Tornquist examines the relationship between "Minimaxing and Optimal Programming."

Part III, Decision Theory, opens with Mayberry's analysis of "Alternate Prior Distributions in Statistical Decision Theory" in terms of an application to missile testing. H. Mills considers "Smoothing in Inventory Processes" where smoothing is defined as the "ability of a requisition policy to induce stability in the derived time series of requisitions and inventory levels." "A Bayesian Approach to Team Decision Problems" is developed by K. Miyasawa. In "Capital Flexibility and Long Run Cost Under Stationary Uncertainty" D. Orr discusses the far-reaching implications of alternative assumptions about the structure of production costs.

Part IV, Economic Theory, opens with Baumol's demonstration that "even when the firm's capital is effectively rationed and it is operating in a point input-point output situation the Ricardo effect need not follow." In "The Economics of Uncertainty" K. Borch shows that under risk firms can gain from exchanging shares or from more general profit-loss sharing arrangements or by "trading" stochastic processes. K. Menger re-examines the St. Petersburg paradox in the "The Role of Uncertainty in Economics." M. H. Peston analyzes "Changing Utility Functions." J. Pfanzagl gives a rigorous treatment of "Subjective Probability Derived from the Morgenstern-von Neumann Utility Concept."

In Part V, Management Science, D. Stern presents "Some Notes on Oligopoly Theory and Experiments." T. Whitin discusses and illustrates "The Role of Economics in Management Science." Part VI, International Trade, opens with an analysis of "Competition of American and Japanese Textiles in the World Market" by A. Y. C. Koo. E. Marcus considers "Moderating Economic Fluctuations in the Underdeveloped Areas". Part VII, Econometrics, contains a definitive analysis of "The Cost of Living Index" by S. Afriat. M. D. Godfrey and H. Karreman present a detailed "Spectrum Analysis of Seasonal Adjustment." C. W. J. Granger discusses "New Techniques for Analyzing Economic Time Series and Their Place in Econometrics." M. Hatanaka develops "A Theory of the Pseudospectrum and Its Application to Nonstationary Dynamic Econometric Models." K. Mizutani proposes "New Formulas for Making Price and Quantity Index Numbers." There is no index. The book opens with a sensitive editorial essay "The Contribution of Oskar Morgenstern" followed by the impressive "Bibliography of the Work of Oskar Morgenstern." This Festschrift should give an interested applied mathematician a good idea of the state of thinking in some important areas of mathematical economics.

MARTIN BECKMANN (*Providence, R. I.*)

Distributions and the boundary values of analytic functions. By E. J. Beltrami and M. R. Wohlers. Academic Press, New York and London, 1966. xiv + 116 pp. \$6.50.

One of the most striking examples of the success of distribution theory in extending and rounding out classical mathematics occurs in the study of the boundary values of analytic functions. This monograph by Beltrami and Wohlers discusses most of the presently available results in this field, at least for the one-dimensional case. The first half of the book develops those parts of distribution theory that are needed. Here one finds a concise presentation of Schwartz distributions, convolution, Sobolev spaces, and the generalized Fourier and Laplace transforms.

The second half of the book is concerned mainly with the problem of characterizing functions $F(s)$ that are analytic on a half-plane, say $\sigma = \operatorname{Re} s > 0$ by their boundary values $(g\omega) = \lim_{\sigma \rightarrow 0+} F(\sigma + i\omega)$. Thus, for example, a distributional analogue to Fatou's theorem, due to the authors, is proven; it states necessary and sufficient conditions for a bounded measurable function $g(\omega)$, $\omega = \operatorname{Im} s$, to be the boundary value in a certain weak sense of a function that is bounded and analytic on the half-plane $\operatorname{Re} s > 0$. Actually, the sufficiency part of this theorem has no classical analogue and is an example of how distribution theory answers many previously open questions in classical mathematics.

Applications to mathematical system theory (in particular, to the characterization of causal and passive operators) to generalized Hilbert transforms and to certain questions of analytic continuation are also given. The monograph concludes with some supplementary remarks concerning partial differential equations, analytic functions of several complex variables, and the "edge of the wedge theorem." Those wishing a ready introduction to the above topics would do well to refer to this book.

A. H. ZEMANIAN (*Stony Brook, N. Y.*)

Lectures on Riemann surfaces. By R. C. Gunning. Princeton University Press, New Jersey, 1966. iv + 254 pp. \$3.75.

In this book the author develops the elementary theory of Riemann surfaces using the technical apparatus that has proved useful in the theory of several complex variables. Thus, the book is a useful addition to the literature on closed Riemann surfaces since it is the only book at present to attack the subject from this point of view. It is also useful to the student of the theory of several complex variables in that it presents an extended treatment of the one-dimensional case. However, the book is of little use to a reader who does not have some background either in a traditional approach to Riemann surfaces or in the subject of several complex variables.

The first chapter gives definitions and a few examples of Riemann surfaces. A reader finding essentially new material in this first chapter should read no further, for he clearly lacks the background for what follows.

Chapters 2 through 6 present the theory of sheaves, cohomology with coefficients in sheaves, line bundles, differential forms and the Serre duality theorem. The presentation is fairly complete and quite readable. (The author defines a line bundle as an element in the first cohomology group of the manifold with coefficients in the sheaf of germs of nowhere vanishing analytic functions, period. A further "interesting geometric interpretation" is omitted so that sections in a line bundle are defined in a rather cumbersome non-geometric manner.)

Chapter 7 gives a proof of the Riemann-Roch theorem based on the Serre duality theorem. To the traditionally oriented reader, the theorem is a little unrecognizable:

$$\dim H^0(M, \mathcal{O}(\xi)) - \dim H^1(M, \mathcal{O}(\xi)) - c(\xi) = 1 - g.$$

Hereafter, the order of treatment is more traditional. Weierstrass points are discussed and the author proves the theorem of Hurwitz which characterizes a hyperelliptic surface by having the minimum number of Weierstrass points.

Chapter 8 discusses the Picard variety and the Jacobi variety of a Riemann surface. The proof that these two varieties are isomorphic follows. The Riemann bilinear relations and Abel's theorem are proved in rather unfamiliar form, but the latter theorem is given a more traditional recasting. The Jacobi inversion problem is not solved, a surprising omission since one would expect the modern apparatus to make quick work of this theorem.

Chapter 9, entitled "Uniformization," might better be called "an approach to uniformization" for, in fact, the uniformization theorem is not proved. The best title would omit the word "uniformization" since the author does not make clear what the subject matter of this chapter has to do with the uniformization problem. Nevertheless, the chapter is interesting and presents material which is new, at least to the reviewer. The author considers structures on a Riemann surface which are finer than analytic structures. Roughly speaking a projective (affine) structure on a Riemann surface is a collection of local analytic homeomorphisms h_α where $h_\alpha \circ h_\beta^{-1}$, where defined, is a fractional linear transformation (affine map). An equivalence of projective (affine) structures is introduced and the chapter devotes itself in large part to classifying the equivalence classes of projective and affine structures on a given surface. The study lends itself very nicely to the technical apparatus introduced earlier. Also discussed are the Eichler cohomology groups for a given projective structure, but unfortunately no applications are given so the uninitiated do not know what is going on.

Chapter 10 reverts to traditional topics including: Riemann surfaces as coverings of the Riemann sphere; the field of meromorphic functions on a closed surface; Riemann surfaces as plane algebraic curves (very lightly done); the canonical embedding of non-hyperelliptic surfaces in projective space; the embedding of a Riemann surface in its Jacobian; and assorted facts about hyperelliptic surfaces.

This paperback book appears in a series entitled "Preliminary informal notes of university courses and seminars in mathematics." The tentative nature of this series presumably accounts for "some of the surprising omissions" the author admits in the preface. Hopefully, a more complete exposition of the theory from this point of view will appear in the future in which the ultra-modern approach is reconciled more fully to the more traditional approach.

Homology and Feynman integrals. By Rudolph C. Hwa and Vigdor L. Teplitz. W. A. Benjamin, Inc., New York and Amsterdam, 1966. xi + 331 pp. \$12.50.

This book opens with an introductory chapter on Landau singularities, which includes an unusually clear description of the nature of nodes and cusps. The next two chapters deal with simplicial homology and the Picard-Lefschetz theorem, the ideas being amply illustrated by specific examples. In Chapter 4 such concepts as differential forms on smooth manifolds, the de Rham cohomology, the generalized Stokes theorem and Leray's residue formula are introduced, and it is explained how the Picard-Lefschetz theorem and the residue formula may be combined to give a simple expression for the discontinuity of an integral round one of its Landau singularities. Chapter 5 is a brief but useful survey of some of the concepts of modern algebraic topology. It is perhaps unfortunate that singular homology is not mentioned until Chapter 5 and then only cursorily, because it is in terms of the C^∞ singular homology rather than in terms of simplicial homology that integrals over smooth manifolds can most naturally be defined. In Chapter 6 the isotopy and decomposition theorems of Fotiadi, Froissart, Lascoux and Pham are stated, the homological treatment of single-loop graphs is briefly discussed, and the difficulties that arise for graphs with more than one loop are mentioned. The book concludes with a well chosen selection of original papers.

This eminently readable book is a useful introduction to a field in which the mathematical techniques are not well known to most physicists. It will have served its purpose if it stimulates some of its readers to seek a deeper understanding from the self-contained and lucid exposition presented in Pham's thesis (Reference 17 in the bibliography).

J. B. BOYLING (*Cambridge, England*)

Linear systems of ordinary differential equations with periodic and quasi-periodic coefficients. By Nikolay P. Erugin. Translation Editor Richard Bellman. Academic Press, New York, London, 1966. xxi + 271 pp. \$12.00.

This monograph deals extensively with the application of the theory of analytic functions of several matrices to the study of linear systems of ordinary differential equations. The methods presented are an outgrowth of those developed by Lappo-Danelevskiy in his study of linear analytic systems in a neighborhood of a pole or essential singularity. The series solutions obtained for this problem are used directly and as models for obtaining series solutions of periodic systems. These series solutions are used in turn to study questions of existence of periodic solutions, the stability of solutions and the reducibility of periodic and quasi-periodic systems.

The monograph contains detailed information on the Floquet representation, Liapunov's first method as it applies to Hill's equation and other closely related topics. Also the text contains a large list of references to the literature almost all of which is in Russian.

Although some questions are treated with great care, other equally important questions are handled lightly and this leads to some difficulty in reading. Also there is a scarcity of worked examples which further detracts from the readability.

These faults are minor and the vast store of detailed information recommends this monograph to the specialist.

KENNETH R. MEYER (*Providence, R. I.*)

Iterative solution of elliptic systems and applications to the neutron diffusion equations of reactor physics. By Eugene L. Wachspress. Prentice-Hall, Inc., Englewood Cliffs, N. J., 1966. xiv + 299 pp. \$12.95.

As the title suggests, this book studies the theory of iterative methods for solving matrix equations,

with applications to reactor physics. The author is a recognized contributor in the area of matrix iterative methods, and he is particularly experienced with respect to the practical applications of iterative methods to the numerical solution of neutron group diffusion equations of reactor physics. This book, which is not intended as a textbook, roughly divides itself evenly between purely matrix theoretical material and applications to reactor physics. As such, it gives a complete treatment of the mathematics underlying neutron diffusion theory, plus a very valuable treatment of the practical aspects of actual machine solutions of such problems.

The chapter titles are: 1. Mathematical preliminaries; 2. Formulation and solution of discrete boundary value problems; 3. The group diffusion equations of reactor physics; 4. Successive overrelaxation; 5. Residual polynomials; 6. Alternating direction implicit iteration; 7. The positive eigenvector; 8. Numerical studies for the diffusion equation; 9. Variational techniques for accelerating convergence. A description of the contents follows.

In Chapter 1, elementary matrix theory concepts, such as vector norms, the Gerschgorin circle theorem, etc. are introduced. Then, the Perron-Frobenius theorem on nonnegative irreducible matrices is stated. With an eye toward the particular applications to reactor theory, the useful definition of an S -matrix is made. An S -matrix turns out to be a special case of an M -matrix, but such connections are not pursued here.

In Chapter 2 the discretization of elliptic boundary value problems of second order, such as the Poisson problem, is made in four different ways, via Taylor's series, Green's Theorem, the standard variational formulation of this problem, and Selengut's variational formulation by means of discontinuous trial functions. Each approach has its advantages. The matrix properties of the associated matrices are studied, but the question of convergence of discrete solutions to the continuous with successively finer meshes is not discussed. Rather, the treatment of internal interfaces and various boundary conditions, important in practical applications, is considered in detail.

In Chapter 3, the Habetler-Matino theory [Proc. AMS Symp. Applied Math. 11 (1961): 127-39; MR 26 #2280] for the group diffusion equations of reactor physics is summarized, and then the analogous discrete space matrix theory form due to Birkhoff and Varga [J. SIAM 6 (1958): 354-77; MR 20 #7407] is discussed in depth. This sets the stage for the discussion of matrix iterative methods, to be used in the so-called inner-iteration loop of neutron group diffusion programs.

Chapter 4 gives an up-to-date account of the theory of successive overrelaxation (SOR) for basically S -matrices, including the application of successive overrelaxation to matrices with complex eigenvalues. The author gives a particularly good treatment of the practical problem of estimating the optimum acceleration parameter ω_s . A few errors need, however, to be corrected in this chapter. First, the free-wheeling theorem of Ostrowski (Theorem 4.3) needs the hypothesis that the acceleration parameters ω_i satisfy $0 < \epsilon \leq \omega_i \leq 2 - \epsilon$ for some $1 > \epsilon > 0$. Next, the matrix $\mathcal{L}_{\sigma,1}$ can in certain cases have a diagonal Jordan normal form (cf. p. 109), and $\mathcal{L}_{\sigma,1}$ is always reducible (cf. p. 110).

Chapter 5 discusses Lanczos' method and the application of Chebyshev polynomials to the problem of speeding up basic iterative methods. Later, these methods are proposed in conjunction with other basic iterative methods for practical use in solving neutron group diffusion equations.

Chapter 6 gives an extremely up-to-date treatment of alternating direction implicit iteration methods (ADI). It includes the author's own ingenious solution to the problem of finding $t = 2^m$ optimum acceleration parameters [J. SIAM 10 (1962): 339-50; MR 27 #921], the more general theory of W. B. Jordan based on elliptic function theory for the case of arbitrary optimum acceleration parameters, as well as matrix conditioning which gives commuting matrices in some cases. All in all, this chapter is the high-point of the book, as no other published book is as complete on the subject of ADI.

The next three chapters are addressed to the practical problem of solving neutron group diffusion problems. For workers in this area, the book is obviously required reading. In Chapter 8, six numerical studies are made in depth which compare SOR and ADI variants as applied to group diffusion equations. Detailed comparisons of the two basic methods are made with respect to convergence rates, sweep time, etc., and generally ADI methods are superior. It is unfortunate that comparisons with the two-line successive overrelaxation variant (S2LOR) were not made, since this latter iterative method is widely used in practice.

The analytic S matrix: a basis for nuclear democracy. By Geoffrey F. Chew. W. A. Benjamin, Inc., New York and Amsterdam, 1966. ix + 103 pp. \$7.50.

S matrix theory, in the narrow sense of the term, grew out of the study of dispersion relations between relativistic scattering and reaction cross sections. Both make use of analytic continuations to complex energies and momentum transfers of scattering amplitudes given (more or less directly by experimental observations) for real energies and momentum transfers. General analytic and asymptotic properties of the continued functions are then embodied in contour integrals (such as Hilbert transforms, for example) which constitute the experimentally verifiable predictions. But whereas "old-fashioned" dispersion theorists based, or hoped to base, their analyticity statements on fundamental properties of quantum field theories from which these scattering amplitudes were expected to be calculable, *S matrix theory* is meant to stand on its own feet. The author of the present little book is the protagonist of this school of thought.

Apart from Lorentz invariance and some very general structural elements of quantum mechanics, the theory uses the most important properties of the *S* matrix (in terms of which all scattering and reaction experiments can be described): its *unitarity* and its *connectedness structure*. The former expresses the conservation of energy in the large; the latter, the fact that the probability for each multiparticle reaction must contain as partial constituents the probabilities for reactions involving fewer particles as intermediate steps. Added to this is the assumption that there exists an analytic function of the particle energies, defined in their complex planes, whose value for real, physical energies is the physical *S* matrix. The fourth ingredient is "crossing". This amounts to the assumption that the substitution rule of field theory, which for example, connects the amplitude for electron-electron (Møller) scattering with that for electron-positron (Bhabha) scattering by a simple substitution of different momentum variables, is the result of an analytic continuation of these variables from one physical region to another.

Now one adds what in this theory is called "dynamics". Particles manifest themselves as simple poles of the *S* matrix, on the real energy axis if stable, at complex energies if unstable. Crossing and unitarity then lead to branch cuts, both in the energy and in the momentum transfer variables. The *principle of maximal analyticity* (of the first degree) is the assumption that there are no other singularities. The *nuclear democracy* of the subtitle of this book is the supposition that all particles (stable or unstable) exist on an equal footing, none more fundamental than the others. The final hope is that there exists one, and (to within a single scale parameter) only one function that embodies these conditions, the *S* matrix of Nature (at least if electromagnetism and gravitation are turned off). Thus the masses and coupling constants of all particles would be determined. A solitary philosopher could, by pure thought, construct the only mathematically consistent world and predict the masses, spins, and lifetimes of all stable and unstable particles and nuclei without performing a single experiment. Only the optical spectra, chemistry, and gravitational phenomena would possibly elude him.

The present book presents a clear, authoritative, and nonrigorous introduction to the kinds of arguments on which both the experimental comparisons and the fundamental ideas of *S matrix theory* are based at the present time. Both mathematicians and physicists will find (different) occasions to raise their eyebrows. But this being one of the most actively alive schools of thought in particle physics today, it merits their attention.

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Classical mechanics, quantum mechanics, field theory. By Amnon Katz. Academic Press, New York and London, 1965.

This short book stemmed from introductory portions in a set of graduate lectures on field theory, given by the author at the Weizmann Institute. Accordingly, the title is a little misleading, because none of the named topics is treated exhaustively. Rather, as is made clear in the Preface, the aim is to formulate the oldest and simplest topic, classical nonrelativistic mechanics of particles, in such a way that successive steps to nonrelativistic quantum mechanics, to fields of identical nonrelativistic particles, and to relativistic quantum mechanics and fields, are made by reasonable generalizations of preceding theory. Thus the historic violent jumps made by Heisenberg, Schrödinger, Dirac, and others are intentionally avoided, although these approaches are treated briefly and related to the author's main line of argument.

At each transition, the principle is to retain the broad structure previously built up, yet generalize it properly to yield the necessary advance in theory. As one guide to proper formulation and ensuing generalization, appeal is made to a few salient experimental facts—e.g., the constants of motion in classical mechanics; the nonvanishing and definite value of Planck's constant and the demonstrable significance of deBroglie waves in quantum mechanics. The other principal guides are proper choices of variables (classical coordinates versus quantum wave functions, for example) and of the symmetry group (invariance to Newtonian-Galilean transformations, or to Lorentz transformations).

The main generalization leading from the classical form to quantum mechanics is replacement of trajectory integrals, in the action, by Feynman's functional integrals. Schwinger's variational and source methods of quantization are also examined briefly. The next step, to particle fields, involves as its generalization summing over topologies of Feynman paths. Finally, the step to relativistic cases is achieved by generalizing from Galilean to Lorentz transformations.

The last chapter, on relativistic mechanics and fields, is treated more briefly than the others, once the effect of Lorentz transformations, including introduction of spin, is accomplished.

Throughout, the treatment appears to be clear and orderly, although decidedly succinct. The author has succeeded in his aim of displaying fundamental unity, progressively generalized, among the four examples. Two minor criticisms could be offered, with particular relevance to the central part of the book. First, enough errors in spelling and lapses in punctuation occur to arouse fear that the many and complex equations may not have had thorough proof-reading either. Second, in a number of sections the sentences, although clear in meaning, are so short and choppy as to become irritating to the reader. It would appear that in these sections highly abbreviated lecture notes have been converted into the starkest of declarative statements (in one or two cases, not even converted), with no rewriting thereafter.

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