PLATE AND SHELL INDICIAL AERODYNAMICS*

BY C. S. VENTRES, (*Princeton University*)

In two papers published in this journal, Dowell [1], [2] has calculated the generalized aerodynamic forces on plates and shells undergoing arbitrary transient motion. It is the purpose of this note to point out that certain terms arising from plate or shell motions that are discontinuous at $t = 0$ are missing from his results.

In [1] the Fourier transform of the fluid velocity potential, ϕ^*, is presented in the form of a convolution integral involving the Fourier transform of the upwash f^* on the plate:

$$\phi^* = \int_0^t K(t - \tau)f^*(\tau)\, d\tau; \tag{1}$$

$$K(\tau) \equiv -aJ_0[a(\alpha^2 + \gamma^2)^{1/2}\tau], \qquad f^* \equiv \left(\frac{\partial}{\partial\tau} + i\alpha U\right)w^*(\tau), \tag{2}$$

where w^* is the Fourier transform of the plate deflection. (See [1] for the definitions of all other symbols.) In fact, Eq. (1) is correct only if f^* is bounded (finite) at $t = 0$. This will be the case only if $w(0^+) = 0$. (By assumption $w^* = 0$ and therefore $f^* = 0$ for $t < 0$.) The correct result is obtained quite simply if a continuous plate motion $w^*(\tau)$ is assumed in which w^* increases rapidly during a short time interval $0 < \tau < \epsilon$. The integral in (1) is conveniently broken up into two parts:

$$\phi^* = \lim_{\epsilon \to 0} \left[\int_0^\epsilon K(t - \tau)f^*(\tau)\, d\tau + \int_\epsilon^t K(t - \tau)f^*(\tau)\, d\tau\right].$$

Using the definition of f^* in the first of these and integrating by parts, it is apparent that

$$\lim_{\epsilon \to 0} \int_0^\epsilon K(t - \tau)f^*(\tau)\, d\tau = w(0^+)K(\tau).$$

The equation for ϕ^* valid for arbitrary plate motions is therefore

$$\phi^* = w(0^+)K(t) + \int_0^t K(t - \tau)f^*(\tau)\, d\tau.$$

This result, with $K(\tau)$ as defined in Eqs. (2), replaces Eq. (5) of [1]. Using the same reasoning, the terms

$$-\rho a w(0^+)a(\alpha^2 + \gamma^2)^{1/2}J_1[at(\alpha^2 + \gamma^2)^{1/2}]$$

and

$$a_{mn}(0^+)I_{mn\nu q}(s)$$

are added to the right-hand sides of Eqs. (7) and (10) of [1].

* Received January 8, 1971. The author would like to acknowledge helpful discussions with E. H. Dowell.

Similar terms must be added to analogous equations in [2]. Using now the notation of that reference, the terms

$$\frac{RU}{L} w(0^+)H(s)$$

$$-\frac{\rho U^2 R}{L^2} w(0^+)\left(\frac{\partial}{\partial s} + i\bar{\alpha}\right)H(s)$$

$$a_m(0^+)I_{mr}(s)$$

$$-\frac{\rho U^2 R}{L^2} w(0^+)\left(\frac{\partial}{\partial s} + i\bar{\alpha}\right)H(s)$$

and

$$\frac{1}{M} a_m(0^+)[X_{mr}(s) + \dot{Y}_{mr}(s)].$$

are added to the right-hand sides of Eqs. (5), (7), (10), (16), and the equation immediately following (16), respectively.

All of these terms go to zero as $t \to \infty$, so they are of significance only in the initial stages of the plate or shell motion.

References

[1] E. H. Dowell, *Generalized aerodynamic forces on a flexible plate undergoing transient motion*, Quart. Appl. Math. 24, 331–338 (1967)

[2] E. H. Dowell, *Generalized aerodynamic forces on a flexible cylindrical shell undergoing transient motion*, Quart. Appl. Math. 26, 343–353 (1968)