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BOOK REVIEWS

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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Manuscripts should be typewritten double-spaced on one side only. Marginal instructions to the typesetter should be written in pencil to distinguish them clearly from the body of the text. The author should keep a complete copy.

The papers should be submitted in final form. Only typographical errors should be corrected in proof; composition charges for any major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he/she prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line following his/her name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the average typewriter should be inserted using either instant lettering or by careful insertion in ink. Manuscripts containing pencilled material other than marginal instructions to the typesetter will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts, and exponents to exponents should be clearly indicated.

Single embellishments over individual letters are allowed; the only embellishment allowed above groups of letters is the overbar.

Double embellishments are not allowed. These may be replaced by superscripts following the symbols.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp[(a^2+b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}.$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos(x/2b)}{\cos(a/2b)} \text{ is preferable to } \frac{\cos\frac{x}{2b}}{\cos\frac{a}{2b}}.$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1}\sin u\,du \text{ is preferable to } \int \frac{\sin u}{u}\,du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in typeset formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a+bx)\cos t \text{ is preferable to } \cos t(a+bx).$$

Figures: Figures should be drawn in black ink with clean, unbroken lines; do not use ball point pen. The paper should be of a nonabsorbant quality so that the ink does not spread and produce fuzzy lines. If the figures are intended for reduction, they should be drawn with heavy enough lines so that they do not become flimsy at the desired reduction. The notation should be of professional quality and in proportion for the expected reduction size. Figures which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying figures should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References in text to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237: *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354-372 (1893)). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow them.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Stromung zaher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details such as ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations such as Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c." even if this special abbreviation is defined somewhere in the text.

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—BOOK REVIEW SECTION—

Theory of viscoelasticity: an introduction. By R. M. Christensen. Academic Press, Inc., New York, 1971. xi + 245 pp. $13.50.

The theory of linear viscoelasticity seems to be an unusually difficult subject on which to write a textbook. Although several attempts have been made to meet a need which certainly exists for a text at the introductory level, none of these appears to have found general acceptance. The book under review is intended primarily as a text for graduate-level instruction. Its plan is attractive, and can best be outlined by listing the chapter headings, as follows: I. Viscoelastic stress-strain constitutive relations; II. Isothermal boundary-value problems; III. Thermoviscoelasticity; IV. Wave propagation; V. General theorems and formulations; VI. Nonlinear viscoelasticity; VII. Determination of mechanical properties.

This selection of topics seems admirable for a book of this type. The author's approach to the subject is in the modern spirit through the use of creep and relaxation functions rather than of mechanical models or differential operators, although the relation between these approaches is discussed. A sensible compromise is maintained between a rigorous mathematical treatment of the subject and the discussion of its practical aspects as a theory which attempts to describe the behavior of real materials. The illustrative examples are for the most part well chosen, although they are sometimes described in excessive detail.

The book cannot, however, be commended for style and clarity. It abounds with distracting and confusing errors of grammar and spelling. This is unfortunate, because more care in this respect on the part of the author and publisher would have resulted in a much clearer and more readable book, and one which could more readily be recommended to students. As it is, although the book has virtues and fills some gaps in the textbook literature, this reviewer feels that there is still no text on the theory of viscoelasticity which could be recommended, without reservations, for a course.

A. J. M. Spencer (*Providence*)

Introduction to stochastic control. By Harold Kushner. Holt, Rinehart and Winston, Inc., New York, 1971. xvii + 390 pp. $14.95.

The book gives what, to my mind, is a very successful introduction to stochastic control, as well as to the areas of filtering and stochastic systems. I especially like the emphasis on discrete-time systems and explanation of the continuous-time analogues in terms of the discrete results. The choice of topics is quite good and the student learning from this book will certainly have a good introduction to a large number of important problems and questions in the area of applied stochastics, especially so if he does some of the problems. The references are complete enough to lead the reader to a good many of the advanced results.

On the debit side, I thought the first two chapters would be somewhat hard to digest in terms of notation and emphasis for the student. In particular, if the reader did not avail himself of a careful reading of some of the books referred to, he might not grasp the meaning or setting of some of the results. Also, I personally would have liked to see more emphasis on random times as a central theme, both in these chapters and later.

In summing up, the book is the best I have seen as an introduction to stochastic control. The author is to be congratulated on writing an introductory book that lifts the reader to "where the action is" instead of writing a book describing where the action was ten to twenty years ago.

Richard S. Bucy (*Los Angeles*)

Tellegen's theorem and electrical networks. By Paul Penfield, Jr., Robert Spence and Simon Duinker. MIT Press, Cambridge, 1970. xv + 143 pp. $7.50.

This short monograph belongs to the shelf of every professional circuit theorist and every graduate student who studies circuits. It should be read by anyone interested in circuits and by anyone who works in a field where circuit models are useful.

This reviewer read this monograph with great enthusiasm and curiosity. This monograph is remarkable because (1) it uses a single tool: a recently discovered sophisticated version of Tellegen's theorem due to the authors; (2) it derives a very large number of results using techniques understandable to seniors; (3) it covers a very wide variety of applications: arbitrary networks, linear networks, nonlinear networks, variable networks (i.e. with switches or adjustable elements), sensitivity results, network optimization; (4) it ends by sketching the application of Tellegen's theorem to other lumped and continuous systems. A consequence of the unusual international troika of authors is a very long list of references from the U. S., British, Dutch, German, Italian, etc. literature. Unfortunately, the seminal paper by F. H. Branin, Jr. in the 1966 Symposium on Generalized Networks, Brooklyn Polytechnic Institute is not mentioned.

The enthusiasm of the authors for a sophisticated understanding of Tellegen's theorem is well justified. To give just one example, Prigogine's Evolution Principle has recently been derived from Tellegen's theorem! (see G. F. Oster and C. A. Desoer, *Tellegen's theorem and thermodynamic inequalities*, Jour. Theor. Biology, in press).

The author should be congratulated for their contribution to circuit theory.

C. A. Desoer (*Berkeley*)

Handbook of the Poisson distribution. By Frank A. Haight. John Wiley & Sons, Inc., New York, 1967. $10.00.

The definition of the Poisson distribution is very simple. We say that a random variable X has a Poisson distribution with parameter a where a is a positive number if

$$P\{X = k\} = e^{-a}a^k/k!$$

for $k = 0, 1, 2, \cdots$. The Poisson distribution appears frequently in the theory of probability either as an approximating distribution or as an exact distribution.

In 1837 Poisson showed that if we consider n Bernoulli trials with probability p for success and suppose that $n \to \infty$ and $p \to 0$ in such a way that $np \to$ a where a is a positive number, then the limiting distribution of the number of successes is a Poisson distribution with parameter a. The Poisson distribution appears as an approximating distribution in the problems of matching, occupancy, sampling and several others. It arises also as a limiting distribution of sums of random variables and of Markov chains.

Perhaps the most important appearance of the Poisson distribution is in the theory of stochastic processes. If random events occur singly and the number of events occurring in disjoint intervals are independent random variables, then the number of events occurring in any interval has a Poisson distribution. Such phenomena are the radioactive disintegration, telephone traffic, electron emission, accidents and many others.

In this book the author surveys a large number of mathematical results related to the Poisson distribution. His work is based on the substance of more than 700 papers. In the book only final results are stated and proofs are entirely omitted. A brief description of the contents follows.

Chapter 1 deals with some elementary properties, the moments and the generating function of the Poisson distribution. Chapter 2 discusses various problems leading to the Poisson distribution. Chapter 3 deals with some generalizations of the Poisson distribution such as the Erlang distribution, the Pólya-Eggenberger distribution, the compound Poisson distribution, and the multivariate Poisson distribution. Chapter 4 covers some special theorems, e.g., Raikov's theorem. Chapters 5 and 6 deal with statistical problems, namely estimating parameters and testing hypotheses. Chapter 7 covers various applications in industry, agriculture, ecology, biology, medicine, telephone traffic, accidents, commerce, queuing theory, sociology, demography, traffic flow theory, military, particle counting and others. Chapter 8 contains information about available tables and computer programs. Chapter 9 contains a brief account on the works of Simeon Denis Poisson (1781-1840), Ladislaus von Bortkiewicz (1868-1931) and other pioneer researchers.

Probabilists and statisticians will find the book a very useful reference work.

The author made thorough research in collecting the material of the book; however, on several occasions he used second-hand references and failed to trace back to the original works. Here are a few such examples. On pp. 4-5 and on p. 77 he failed to refer to Jordan [5, 6, 7] concenring the properties of the G-polynomials and the Jordan expansion. The problem of matching is discussed on pp. 19 and on 22; however, it should be added that already in 1713 Bernoulli and Montmort (see [8]) found that

$$P_k(n) = \frac{1}{k!} \sum_{j=0}^{n-k} \frac{(-1)^j}{j!}$$

is the probability that exactly k matches occur if we draw all the n cards from a box which contains n cards numbered $1, 2, \cdots, n$. Furthermore, both Euler [3] and DeMoivre [2] observed that the sum in the expression of $P_k(n)$ tends to $1/e$ if $n \to \infty$. It is interesting to point out that preceding Poisson by nearly a century, Euler and DeMoivre discovered, in essence, an instance of the Poisson distribution. On pp. 52-55 the author discusses the compound Poisson distribution without mentioning that the compound Poisson process was already discovered in 1921 by Fujiwara [4]. On p. 74 the author gives the distribution of $X - Y$ where X and Y are independent random variables having Poisson distributions. He fails to notice that this distribution was already found in 1911 by Bateman [1]. On p. 72 "Cramér's theorem" is mistakenly attributed to Cramér and in any case the theorem is not true unless we assume that X and Y are independent random variables.

References

[1] H. Bateman, *Some problems in the theory of probability*, Phil. Mag. Sixth Ser. 21 (1911), 745–752.

[2] A. DeMoivre, *The doctrine of chances*, third edition, London, 1756.

[3] L. Euler, *Calcul de la probabilité dans le jeu de recontre*, Memoires de l'Académie des Sciences de Berlin 7 (1751), 255–270.

[4] M. Fujiwara, *Ein Problem in der Wahrscheinlichkeitsrechnung*, Tohoku Math. Jour. 19 (1921), 48–50.

[5] Ch. Jordan, *Développements nouveaux pour l'application du theorème de Bernoulli*, Comptes Rendus Acad. Sci. Paris 182 (1926), 303–305.

[6] Ch. Jordan, *Sur la probabilité des épreuves répétées, le théorème de Bernoulli et son inversion*, Bull de la Société Math. de France 54 (1926), 101–137.

[7] Ch. Jordan, *Statistique mathématique*, Gauthier-Villars, Paris, 1927.

[8] P. R. Montmort, *Essai d'analyse sur les jeux de hazard*, second edition, Paris, 1714.

L. Takács (*Cleveland, Ohio*)