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The QUARTERLY prints original papers in applied mathematics which have an intimate connection with application in engineering. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unneccessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\surd$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\,[(a^2 + b^2)^{1/2}] \text{ is preferable to } e(a^2 + b^2)^{1/2}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos\,(\pi x/2b)}{\cos\,(\pi a/2b)} \text{ is preferable to } \frac{\cos \frac{\pi x}{2b}}{\cos \frac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u \, du \text{ is preferable to } \int \frac{\sin u}{u} \, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n] \cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n) \cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354–372(1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

CONTENTS

—BOOK REVIEW SECTION—

Statistical design and analysis of experiments. By Peter W. M. John. The Macmillan Company, New York, 1971. xii + 356 pp. $14.95.

This book is written primarily for a scholarly audience; it will serve well as either a text for a graduate-level course in linear models and experimental design for students of statistics, or as a reference work for researchers in design of experiments. The author has had considerable experience in practical applications of this methodology to the oil industry, and the examples are thus partially freed from the field's agricultural origins. The reader interested mainly in the practical considerations of experimental design, however, will find the text unsuited for his purposes because of its level of mathematics and emphasis on theory.

Chapter 1 is a very readable introduction and provides good motiviation for the subject matter to follow. Chapters 2–5 discuss in detail linear models, quadratic forms, and one-, two-, and multi-way layouts. These chapters could constitute the heart of a one-semester graduate course that might be entitled "Linear models and analysis of variance". Although the subject matter in these sections is standard, the presentation is clear and crisp. There is a nice interweaving of examples and theory indicating how varying the assumptions alters the analysis.

Chapters 6–12 in turn constitute the heart of a one-semester course in design of experiments. Chapter 6 consists of a thorough discussion of Latin squares, while Chapters 7–9 are devoted to confounded and fractional factorials. Much of the material on fractional factorials appears in a text for the first time. A discussion of response surfaces, Chapter 10, is included primarily for completeness. Chapters 11 and 12 deal with incomplete block designs, balanced, partially balanced, and in general. These sections tend to get slightly bogged down in mathematics and read more slowly than the preceding ten chapters. I find it hard, nevertheless, to see how I would improve the presentation of this material.

Chapters 13–15 are intended for a super-select audience, researchers in the area of incomplete block designs. The author indicates that these three chapters "call for considerably more mathematical maturity than the others", taking "the reader to the frontier of research in the construction of incomplete block designs". The description is valid, and those with such interest will be pleased to find material which is scattered through the journals gathered in one text.

A criticism of this book, and of most, if not all of its competitors, is the lack of attention to complete and varied analysis of the data obtained through experiments. Residual analysis, transformations, graphical procedures, and style of reporting results of the analysis are not mentioned, or mentioned all too briefly.

In spite of these drawbacks above, this book is a welcome and up-to-date addition to the library of any student of the design of experiments.

BARRY H. MARGOLIN (*New Haven*)

Tensor analysis and continuum mechanics. By W. Flügge. Springer-Verlag, Berlin, Heidelberg, New York, 1972. vii + 207 pp. $14.60.

It was fashionable some years ago amongst applied mathematicians to argue whether tensors were a concept or "merely" a notation, and also to argue the question whether the index-free notation (favored by pure mathematicians) or the older index notation (favored by physicists) was more incisive.

Whatever the truth, it is hardly deniable that the tensor notation was a most powerful invention: it was, surely, the theory of relativity which proved its value—covariance, contravariance, invariance of equations fell out automatically if only you handled the indices wisely, and consequently it was obvious whether or not an equation could or could not be a law of nature.

Professor Flügge has, for the first time, and very clearly, expounded continuum mechanics in terms of general (i.e. not cartesian) tensors. It is a profoundly geometric approach—as is, of course, relativity theory—which emphasizes coordinate-free representation and invariance. The generality of this framework is additionally shown to be very useful when particular coordinate systems (not necessarily orthogonal) are introduced and special problems are solved.

The emphasis is not on the discussion of general constitutive equations—for that (also in terms of tensors) one must refer to other works. It is a classical book, in the best sense of the term.

W. FREIBERGER (*Providence*)

Applications of group theory in quantum mechanics. By M. I. Petrushen and E. D. Trifanov. Translated by S. Chomet, edited and revised by J. L. Martin. The M.I.T. Press, Cambridge, Mass, 1971. vi + 318 pp. $12.00.

This book is intended for theoretical physicists who wish to understand the value of modern group-theoretical methods in quantum theory. The theory of groups and their matrix representations are developed to the level required for applications, and their relevance to the invariance properties of a physical system is established.

The theory is applied to a variety of typical physical situations, usually quantum mechanical in nature, although attention is also given to classical systems, concepts and methods being illustrated by results from analytical mechanics.

The text reads most fluently and the reader is not aware of the translation and editing process that produced it: an unusual achievement.

W. FREIBERGER (*Providence*)

An introduction to the theory of multipliers. By Ronald Larsen. Springer-Verlag, New York, Heidelberg, Berlin, 1971. XX + 282 pp. $24.10.

The classical theory of multipliers deals with Fourier coefficients $\hat{f}(n) = 1/2\pi \int_{-\pi}^{\pi} f(t) \exp(-int)\, dt$. Given classes $\mathfrak{A}$ and $\mathfrak{B}$ of functions f on $[-\pi, \pi]$ and a function $\{c(n)\}_{n=-\infty}^{\infty}$ on the integers, when is it the case that $c(n)\hat{f}(n)$ has the form $\hat{g}(n)$ for some $g \in \mathfrak{B}$ whenever $f \in \mathfrak{A}$? If this condition holds, then c is called an $(\mathfrak{A}, \mathfrak{B})$ *multiplier.*

The notion of multiplier can be generalized in many directions. For example, there is the now classical theorem of J. G. Wendel. Let G be any locally compact group with [left] Haar measure λ, and the associated function spaces $\mathfrak{L}_p(G)$ $(1 \leq p < \infty)$. Let T be a bounded linear transformation of $\mathfrak{L}_1(G)$ into itself. The following properties are equivalent.:

(i) there is a measure μ on G such that $T(f) = \mu_* f$ for all $f \in \mathfrak{L}_1(G)$;

(ii) $T(f_a) = (T(f))_a$ for all $f \in \mathfrak{L}_1(G)$ $[g_a(x) = g(xa)$ for any function g on $G]$;

(iii) $T(f_* g) = T(f) * g$ for all $f, g \in \mathfrak{L}_1(G)$ $[f * g(x) = \int_G f(xy)g(y^{-1})\, d\lambda(y)]$;

(iv) $T(f * \nu) = T(f) * \nu$ for all measures ν on G.

For the case of the circle, Wendel's theorem asserts that $(\mathfrak{L}_1, \mathfrak{L}_1)$ multipliers are exactly Fourier-Stieltjes transforms.

The volume under review contains an exhaustive study of multipliers for Banach algebras and topological linear spaces of functions on locally compact Abelian groups, the definitions of "multiplier" being taken from the appropriate part of Wendel's theorem. There are an extensive bibiography and copious references for further study. The reader interested in the compact non-Abelian case should perhaps also consult the book *Abstract harmonic analysis,* Volume II, by the reviewer and Kenneth A. Ross [same Grundlehren, Volume 152], Secs. 35–37.

EDWIN HEWITT (*Seattle*)