

QUARTERLY  
OF  
APPLIED MATHEMATICS

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# QUARTERLY OF APPLIED MATHEMATICS

The QUARTERLY prints original papers in applied mathematics which have an intimate connection with applications. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

Manuscripts (two copies) submitted for publication in the QUARTERLY OF APPLIED MATHEMATICS should be sent to the Editorial Office, Box F, Brown University, Providence, RI 02912, either directly or through any one of the Editors. The final decision on acceptance of a manuscript for publication is made by the Managing Editor. In accordance with their general policy, the Editors welcome particularly contributions which will be of interest both to mathematicians and to scientists or engineers. Authors will receive galley proof only. The author's institution will be requested to pay a publication charge of \$30 per page which, if honored, entitles the author to 100 free reprints. Detailed instructions will be sent with galley proofs.

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## SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

**Manuscripts:** Manuscripts should be typewritten double-spaced on one side only. Marginal instructions to the typesetter should be written in pencil to distinguish them clearly from the body of the text. The author should keep a complete copy.

The papers should be submitted in final form. Only typographical errors should be corrected in proof; composition charges for any major deviations from the manuscript will be passed on to the author.

**Titles:** The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he/she prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line following his/her name.

**Mathematical Work:** As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the average typewriter should be inserted using either instant lettering or by careful insertion in ink. Manuscripts containing pencilled material other than marginal instructions to the typesetter will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter *O*, between the numeral one (1), the letter *l* and the prime (*'*), between alpha and *a*, kappa and *k*, mu and *u*, nu and *v*, eta and *n*.

The level of subscripts, exponents, subscripts to subscripts, and exponents to exponents should be clearly indicated.

Single embellishments over individual letters are allowed; the only embellishment allowed above groups of letters is the overbar.

Double embellishments are not allowed. These may be replaced by superscripts following the symbols.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol *exp* should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{[a^2 + b^2]^{1/2}}.$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos(x/2b)}{\cos(a/2b)} \text{ is preferable to } \frac{\cos \frac{x}{2b}}{\cos \frac{a}{2b}}.$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u \, du \text{ is preferable to } \int \frac{\sin u}{u} \, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in typeset formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t(a + bx).$$

**Figures:** Figures should be drawn in black ink with clean, unbroken lines; do not use ball point pen. The paper should be of a nonabsorbant quality so that the ink does not spread and produce fuzzy lines. If the figures are intended for reduction, they should be drawn with heavy enough lines so that they do not become flimsy at the desired reduction. The notation should be of professional quality and in proportion for the expected reduction size. Figures which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying figures should be written on a separate sheet.

**Bibliography:** References should be grouped together in a Bibliography at the end of the manuscript. References in text to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354–372 (1893)). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow them.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Stromung zaher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details such as ed., vol., no., chap., p.

**Footnotes:** As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

**Abbreviations:** Much space can be saved by the use of standard abbreviations such as Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c." even if this special abbreviation is defined somewhere in the text.

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*Linear Programming in Infinite-Dimensional Spaces.* By Edward J. Anderson and Peter Nash. John Wiley & Sons, New York, 1987, pp. xi + 172, \$44.95.

This is a volume in the Wiley Interscience Series in Discrete Mathematics and Optimization. It provides a fully comprehensive survey of linear programming in infinite-dimensional spaces. It includes a complete treatment of duality theory and of the fundamental theory of simplex-like algorithms for linear programs posed over vector spaces which may be infinite-dimensional. More than half of the book is devoted to a detailed investigation of various types of infinite-dimensional linear programs which occur, for example, in approximation theory, optimal control theory, dynamic networks, mass transfer problems, and structural design. The approach is inductive; specific problems and algorithms are discussed in detail, and the authors proceed from these to more general concepts and results. Chapter headings: 1. Infinite-dimensional linear programs. 2. Algebraic fundamentals. 3. Topology and duality. 4. Semi-infinite linear programs. 5. The mass transfer problem. 6. Maximal flow in a dynamic network. 7. Continuous linear programs. 8. Other infinite linear programs.

*Numerical Methods in Engineering and Applied Science: Numbers are Fun.* By B. M. Irons and N. G. Shrive. John Wiley & Sons, New York, 1987. pp. 1 + 248. \$32.95.

This is a volume in the Ellis Horwood Series in Mathematics and its Applications. This book is concerned with the application of numerical methods to solve mathematical problems in engineering and the applied sciences. The approach is simple, straightforward, and physical, explaining the matter of the methods, what is good about them, their limitations, and the conditions under which they might misbehave.

*Algorithm Design: A Recursion Transform Framework.* By Marvin C. Paull. John Wiley & Sons, New York, 1988. pp. xiv + 490. \$44.95.

This text examines the fundamentals of algorithm design and analysis, especially for the design and implementation of recursive definitions. The main emphasis of the book is the development of basic algorithms as illustrations of consistent design principles. A structured framework for description of the origin of good algorithms is developed and every algorithm presented is developed within this framework. The approach can be thought of as generalized recursion removal, similar to approaches in which the origin of algorithms is modeled as a transformation from an initial problem formulation as a recursive definition to a good algorithm, where the transformation used depends on properties of the definition. It is the choice of properties and transformations which complete the authors' adopted framework of recursive definition to good algorithm.

*Differential Geometry Applied to Curve and Surface Design—Volume I: Foundations.* By Anthony W. Nutbourne and Ralph R. Martin. John Wiley & Sons, New York, 1988, pp. 1 + 282.

This work shows how differential geometry can be used to design curves and surfaces, and how it can be made more accessible and more understandable to engineers. It presents a unified approach to the synthesis of curves and surfaces using vector and matrix algebra. The author introduces the cyclidal patch as a new element for surface synthesis. The synthesis of curves is based on curvature and torsion profiles with special attention to composite curves called biarcs. These consist of two circular arcs, usually of different radii, that lie in different planes but join together with a common tangent. The synthesis of surfaces is based on principal patches. These are four-sided patches formed by the two sets of lines of curvature that intersect at right angles. The geometry of a single patch is studied in detail. The authors set up general frame-matching and position-matching equations, and solve them for special cases where each boundary curve is planar. The simplest such case is a cyclide surface. The cyclidal patch is very versatile as it can be part of a plane, sphere, cylinder, cone, or torus and other shapes.

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*Bifurcation of Extremals in Optimal Control.* By Jacob Kogan. Springer-Verlag, New York, 1986. pp. viii + 106. \$12.80.

This is volume 1216 of Lecture Notes in Mathematics. The author shows that in optimal control conjugate points exist even under a natural generalization of the sufficiency criterion of the calculus of variations. He discovers that the set of the conjugate points has a simple and elegant structure. The first chapter of the study is an overview of the concepts, definitions, methods and results, the last, however, without proofs. The remainder of the work contains full proofs of the results. Chapter headings: 1. Overview. 2. Branching points in linear control problems. 3. Branching pairs in linear control problems. 4. The nonlinear case. 5. Linear systems with vector valued performance index. 6. Nonlinear control systems with vector cost. 7. Optimal control problems with constraints. 8. Appendix.

*Probability and Banach Spaces.* Edited by J. Bastero and M. San Miguel. Springer-Verlag, New York, 1986. pp. x + 222. \$19.40.

This is volume 1221 of Lecture Notes in Mathematics. It is the proceedings of a seminar held in Zaragoza, Spain, June 17–21, 1985. The texts of eight lectures are included.

*Stability Problems for Stochastic Models.* Edited by V. V. Kalachnikov, B. Penkov, and V. M. Zolotarev. Springer-Verlag, New York, 1987. pp. vi + 223. \$19.40.

This is volume 1233 of Lecture Notes in Mathematics. It is the proceedings of the 9th International Seminar held in Varona, Bulgaria, May 13–19, 1985. A central item among the talks was that every approximate problem can be considered as a stability problem in the framework of a relevant characterization model. Closely connected to this approach is the use of metrics in the space of random variables and their distributions, which lead naturally to a closer consideration of different questions concerning probability metrics. There are 23 papers.

*Separation of Variables for Riemannian Spaces of Constant Curvature.* By E. G. Kalnins. John Wiley & Sons, New York, 1986. pp. 1 + 172. \$56.95.

This is volume 28 of the series Pitman Monographs and Surveys in Pure and Applied Mathematics. It is its aim to show how all the actual inequivalent separable coordinate systems can be computed for the Hamilton–Jacobi and Helmholtz equations on real positive-definite Riemannian spaces of constant curvature. The results necessary for the solution of this problem are developed in the text. Proofs that are central to the computation of all the inequivalent coordinate systems mentioned above are given in full; the more general results of the theory are often quoted, suitable references being given. Chapter headings: 1. Introduction. 2. Historical outline of the separation of variables. 3. Separation of variables on the  $n$ -sphere  $S_n$ . 4. Separation of variables in Euclidean  $n$ -space  $E_n$ . 5. Separation of variables on  $H_n$ . 6. Separation of variables on conformally Euclidean spaces. 7. Separation of variables for the heat equation. 8. Other aspects of variable separation.

*Economics with Many Agents: An Approach Using Nonstandard Analysis.* By Salim Rashid. Johns Hopkins University Press, Baltimore, Md. 1987. pp. xii + 160. \$30.00.

This book aims primarily at introducing nonstandard analysis to economics. The author believes that nonstandard analysis is a (relatively) new mathematical tool that is at once intuitive, simple, and powerful. The economic topic chosen to illustrate the author's thesis is economics with many agents. The principal problems are named after their originators and are called Farrell's conjectures and Edgeworth's conjectures. They are dealt with at length after the reader has been introduced to nonstandard analysis and nonstandard economies.

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*Calculation of Special Functions: The Gamma Function, the Exponential Integrals and Error-Like Functions.* By C. G. van der Laan and N. M. Temme. Mathematisch Centrum, Amsterdam, 1985. pp. iv + 231. Dfl. 33.30.

This is volume 10 of the series CWI Tract. The main scope of these notes is to review and to discuss several aspects of implementations for the numerical computation of special functions. The authors consider functions which are related to the Euler gamma function, the exponential integrals and the error functions. For each of these groups they give 1. definitions, analytic properties and fundamental formulas; 2. algorithms, implementations, error analysis, references to tabulated coefficients, and testing aspects.

*The Numerical Analysis of Ordinary Differential Equations.* By J. C. Butcher. John Wiley & Sons, New York, 1987. pp. xv + 512. \$75.95.

The scope of this book is indicated by its subtitle: Runge-Kutta and General Linear Models. There are four chapters. Chapter 1. Mathematical and computational introduction. 2. The Euler method and its generalizations. 3. Analysis of Runge-Kutta methods. 4. General linear models. Chapter 1 contains introductory sections on a number of mathematical topics. These range from differential calculus on a vector space to graphs and combinatorics. Chapter 2 is intended to provide an introduction to numerical methods for solving initial value problems by dealing with particular classes of methods as generalizations of the method of Euler. In Chapter 3, Runge-Kutta methods are studied in considerable detail. The author claims that the potential efficiency of Runge-Kutta methods in their own right has not been fully realized and that Runge-Kutta methods provide a theoretical framework for understanding general linear methods where the potential may be even greater. It is to general linear methods that Chapter 4 is devoted. These provide a means for studying a wide range of intersecting methods in a unified manner. The theory for this large class of methods can be applied immediately to linear multistep methods and some of the conclusions arrived at through the traditional approach. The book contains an extensive bibliography of nearly 100 pages of the subject up to the year 1982.

*Free Boundary Problems: Applications and Theory-Volume III.* Edited by A. Bossavit, A. Dammlamian, and M. Fremond. Pitman Advanced Publishing Program and John Wiley & Sons, Boston, 1985. pp. 1 + 303. \$44.95.

This is volume 120 of Research Notes in Mathematics. There are 31 papers in this volume, divided into three groups: 1. Stefan problems. 2. Generalized Stefan problems. 3. Porous media. They constitute the proceedings of an International Colloquium held in Maubisson (France), June 7-16, 1984.

*Free Boundary Problems: Applications and Theory-Volume IV.* Edited by A. Bossavit, A. Dammlamian, and M. Fremond. Pitman Advanced Publishing Program and John Wiley & Sons, Boston, 1985. pp. 305 + 613. \$44.95.

This is volume 121 of Research Notes in Mathematics. The 31 papers are divided into 5 groups: 1. Solid mechanics. 2. Fluid mechanics. 3. Reaction diffusion. 4. Control. 5. Mathematical methods—Numerical methods. They constitute the proceedings of an International Colloquium held in Maubisson (France), June 7-16, 1984.

*Fractal Calculus.* Edited by A. C. McBride and G. F. Roach. Pitman Advanced Published Program and John Wiley & Sons, Boston, 1985. pp. 1 + 214. \$38.95.

This is volume 138 of Research Notes in Mathematics. It constitutes the proceedings of a seminar held at Ross Priory, University of Strathclyde in August 1985. There are 16 papers.

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*The Rapid Evaluation of Potential Fields in Particle Systems.* By Leslie F. Greengard. MIT Press, Cambridge, 1988. pp. iv + 90. \$25.00.

This is a volume in the series ACM Distinguished Dissertations. This thesis presents an algorithm for the rapid evaluation of the potential and force fields in large systems of particles. In order to evaluate all pairwise Coulombic interactions of  $N$  particles to within round-off order, the algorithm requires an amount of work proportional to  $N$ , and this estimate does not depend on the statistics of the distribution. In practice, speedups of three to four orders of magnitude may be expected in a system of a million particles, rendering previously prohibitive simulations feasible.

*Team Theory.* By K. H. Kim and F. W. Roush. John Wiley & Sons, New York, 1987. pp. viii + 246. \$54.95.

This is a volume in the Ellis Horwood Series in Mathematics and Its Applications. Team theory as a discipline was initiated by Jacob Marschak whose book with Roy Radner, *The Economic Theory of Teams*, remains the standard work on the subject. Marschak and Radner's teams are groups of individuals with a common goal but with individual information and actions. Often an organization of specially separated agents is used for example. The authors' aim in this book is to extend the theory of teams to questions of assignment, efficiency of cooperation, search, coordination, and team games. Chapter headings: 1. Elements of decision theory. 2. Group decision theory. 3. Information. 4. Basic concepts of teams. 5. Teams as systems. 6. Pure coordination teams. 7. Games played by teams. 8. Implementation, incentives, and teams.

*Optimization Using Personal Computers: With Applications to Electrical Networks.* By Thomas R. Cuthbert. John Wiley & Sons, New York, 1987. pp. vxi + 474. \$44.95.

The first goal of this book is to explain the mathematical basis of optimization, using iterative algorithms on a personal computer to obtain key insights and to learn by performing the computations. The second goal is to acquaint the reader with the more successful gradient optimization techniques, especially Gauss-Newton and quasi-Newton methods with nonlinear constraints. The third goal is to help the reader develop the ability to read and comprehend the essential content of the vast amount of optimization literature. Many important topics in calculus and matrix algebra will be reinforced in that preparation. The last goal is to present programs and examples that illustrate the ease of obtaining exact gradients (first partial derivatives) for response functions of linear electrical networks and their analogues in the physical sciences.

*Advances in Nuclear Science and Technology: Volume 18.* Edited by Jeffrey Lewins and Martin Becker. Plenum Press, New York, 1986. pp. xii + 421. \$79.50.

This volume contains five articles, on light water reactors, multidimensional two-phase flow modeling, and fast breeder reactors.

*Operational Research.* By D. J. White. John Wiley & Sons, New York, 1985. pp. xiii + 361.

This text concentrates on certain mathematical aspects of Operations Research. Chapter headings: 1. Introduction. 2. Selected techniques. 3. Sensitivity, parametric, and post-optimality analysis. 4. Problem formulation. 5. Combined use of techniques. 6. Measurement in operational research. Chapter 2 covers a selection of techniques, some of which relate to dynamic problems and have common characteristics, and others of which relate to some techniques for solving problems.

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*Programs and Solutions in Electromagnetic Theory.* By C. M. Lerner. John Wiley & Sons, New York, 1985. pp. x + 614.

This is a book consisting of 425 problems in Electromagnetic Theory with completed, thoroughly worked out solutions for every problem. The level of the problems varies from that of a freshman level (approximately one-third of the book is on this level), to that of an advanced graduate level (approximately one-third of the book is on this level). The other third of the book consists of the intermediate level problems, which deal with work commensurate with typical junior-senior level courses in the field. The intent of this book is to explain the physical principles and ideas of electromagnetic theory through the use of problem solving.

*Canonical Forms in Finitely Presented Algebras.* By Philippe Le Chenadec. John Wiley & Sons, New York, 1986. pp. 1 + 201. \$22.95.

This is a volume in the series Research Notes in Theoretical Computer Science. This monograph is an introduction to rewriting systems of algebras defined by a finite set of identities. Rewriting systems, developed in the last two decades, attempts to solve the equality problem between algebraic structures via computation of normal forms. This notation of rewriting or simplification has emerged as a crucial component in software performing symbolic computations. Chapter headings: 1. Equational varieties. 2. Canonical forms. 3. Monoids and groups. 4. Rings, modules, and algebras. 5. Dehn algorithm and the symmetrization. 6. Complete group presentations.

*Categorical Combinators, Sequential Algorithms and Functional Programming.* By P. L. Curien. John Wiley & Sons, New York, 1986. pp. 1 + 300. \$24.95.

This is a volume in the series Research Notes in Theoretical Computer Science. The aim of this monograph is to give a concrete approach to the semantics of sequential programming languages, with application to the design and implementation of programming languages. The motivation behind the author's constructions was to ensure that any two program pieces have the same meaning as soon as they behave the same way on a computer. Restricting this known problem to sequential programming languages implied giving a semantic account of sequentiality. Chapter headings: 1. Categorical combinators. 2. Sequential algorithms on concrete data structures. 3. CDSO: The kernel of a functional language. 4. The full abstraction problem. 5. Conclusion. 6. Mathematical prerequisites.

*Studies in Complexity Theory.* Edited by Ronald V. Book. John Wiley & Sons, New York, 1986. pp. 1 + 226. \$22.95.

This is a volume in the series Research Notes in Theoretical Computer Science. It consists of these papers: Applying techniques of discrete complexity theory to numerical computation, by Ker-I Ko; Sparse sets and reducibilities, by Stephen R. Mahaney, and Models of arithmetic and complexity theory, by Kenneth McAloon.

*Multilevel Models in Educational and Social Research.* By Harvey Goldstein. Oxford University Press, New York, 1987. pp. viii + 98. \$29.95.

This book sets out to familiarize students and researchers with the essential features of multilevel models by a general methodological discussion, illustrated with practical examples. These examples include topics concerned with "school effectiveness", progress in reading achievement, child growth, and social attitudes. Chapter headings: 1. Introduction. 2. The basic multilevel linear model. 3. The general multilevel linear model with random coefficients. 4. Longitudinal and repeated measures data. 5. Multivariate data. 6. Proportions as responses. 7. Further topics.

*Riemannian Foliations*. By Pierre Molino. Birkhäuser, Boston, 1988. pp. xii + 339. \$44.00.

This is volume 73 in the series Progress in Mathematics. It is a translation from the French by Grant Cairns. It presents the first elementary exposition of basic results in Riemannian foliations obtained in the last ten years. The structure theorems for Riemannian foliations on closed manifolds are described in a self-contained, detailed way with exercises at the end of each chapter so that the book may be used as a graduate text.

*Proceedings Seminar 1981–1982: Mathematical Structures in Field Theories*. Edited by E. M. de Jager and H. G. J. Pijls. Mathematisch Centrum, Amsterdam, 1984. pp. iii + 217. Dfl. 31.

This is volume 2 of the series CWI Syllabus. The program of the Seminar was during this year directed towards differential geometry and gauge field theory. The texts of five lectures are given here.

*System Theoretic Description of Physical Systems*. By A. J. van der Schaft. Mathematisch Centrum, Amsterdam, 1984. pp. xv + 256. Dfl. 36.90.

This is volume 3 of the series CWI Syllabus. The basic motivation underlying this monograph is two-fold. Firstly, it is thought that the framework of mathematical system theory can contribute to the modeling of dynamical systems as encountered in physics. Secondly, it is thought that system (and control) theory can benefit from a closer study of the natural structures possessed by physical systems. Chapter headings: 1. Systems with external variables. 2. Smooth dynamical systems. 3. Hamiltonian systems. 4. Symmetries, conservation laws and time-reversability. 5. Gradient systems. 6. Optimal control and Hamiltonian systems.

*Colloquium Topics in Applied Numerical Analysis: Volume 1*. Edited by J. G. Verwer. Mathematisch Centrum, Amsterdam, 1984. pp. vi + 253. Dfl. 36.90.

This is volume 4 of the series CWI Syllabus. The colloquium was held at the Centre for Mathematics and Computer Science, Amsterdam, during the academic year 1983/1984. The aim of this colloquium was to draw attention to the widespread use of numerical mathematics in scientific real-life problems, as well as to foster cooperation between mathematicians working in an academic environment and representatives from industries and institutes where the numerical solution of real-life problems is studied. There are twelve lectures in this first volume.

*Colloquium Topics in Applied Numerical Analysis: Volume 2*. Edited by J. G. Verwer. Mathematisch Centrum, Amsterdam, 1984. pp. vi + 483. Dfl. 33.30.

This is volume 5 of the series CWI Syllabus. There are twelve lectures in this second volume (see above for first volume).

*Proceedings Seminar 1982–1983: Mathematical Structures in Field Theories*. Edited by E. M. de Jager and H. G. J. Pijls. Mathematisch Centrum, Amsterdam, 1985. pp. iii + 249. Dfl. 35.70.

This is volume 6 of the series CWI Syllabus. There are three lectures on quantum field theory and four lectures on mathematical methods in connection with Yang–Mills theories.

*Radically Elementary Probability Theory.* By Edward Nelson. Princeton University Press, New Jersey, 1987. pp. ix + 97. \$40.00 hardcover, \$15.00 paperback.

This is number 117 of Annals of Mathematics Studies. It is an attempt to lay new foundations for probability theory, using nonstandard analysis. The mathematical background required is little more than that which is taught in high school, and it is the author's hope that it will make deep results from the modern theory of stochastic processes readily available to anyone who can add, multiply, and reason. What makes this possible is the author's decision to leave the results in nonstandard form. Mathematicians will ask whether the results developed here are as powerful as the conventional results, and whether it is worth their while to learn nonstandard methods. These questions are addressed in the appendix, which assumes a much greater level of mathematical knowledge than does the main text. But the main text stands on its own.

*Manifolds and Mechanics.* By Arthur Jones, Alistair Gray, and Robert Hutton. Cambridge University Press, New York, 1987. pp. 1 + 166. \$39.95 hardcover, \$13.95 paperback.

This is volume two in the Australian Mathematical Society Lecture Series. It is the authors' aim to make some of the basic ideas about manifolds readily available to applied mathematicians and theoretical physicists while at the same time exhibiting applications of an important area of modern mathematics to mathematicians. Classical texts use ideas such as "virtual work" and "infinitesimal displacements" in their derivation of Lagrange's equations. By contrast the modern texts jump straight to Hamiltonian systems and lose the physical motivation. In these notes, the derivation of Lagrange's equations makes direct appeal to geometrical principles. Chapter headings: 1. Calculus preliminaries. 2. Differentiable manifolds. 3. Submanifolds. 4. Differentiability. 5. Tangent spaces and maps. 6. Tangent bundles and manifolds. 7. Partial derivatives. 8. Deriving Lagrange's equations. 9. Form of Lagrange's equations. 10. Vector fields. 11. Lagrangian vector fields. 12. Flows. 13. The spherical pendulum. 14. Rigid bodies.

*Principles of Antennas: Wire and Aperture.* By T. S. M. Maclean. Cambridge University Press, New York, 1986. pp. x + 360. \$79.50.

This book is intended both as a students' textbook at final year undergraduate and postgraduate level, and for use by engineers in industry who wish to ask the question 'why', rather than simply 'how', a particular approach is used in constructing different antennas. The book is not intended, however, to teach commercial antenna design. A deliberate choice has been made to devote approximately equal amounts of space to wire antennas and aperture antennas. Most texts tend to concentrate rather heavily on one or the other, but here the author has attempted to treat both fields from as unified a standpoint as possible. The book is quantitative rather than descriptive. Chapter headings: 1. Planar sources of uniform plane waves. 2. Current element sources. 3. Dipole and monopole antennas. 4. Computer solutions of dipole and monopole antennas. 5. Loop antennas. 6. Helical antennas. 7. Yagi-Uda antennas. 8. Frequency independent and logarithmically periodic antennas. 9. Noise power delivered by wire antennas. 10. Aperture antennas. 11. Angular spectrum of plane waves. 12. Waveguide radiators. 13. Paraboloidal reflector. 14. Receiving paraboloidal reflector with feed. 15. Analysis of transmitting paraboloids. 16. Cassegrain and offset reflector analysis.

*Computational Limitations for Small-Depth Circuits.* By Johan Torkel Håstad. MIT Press, Cambridge, 1987. pp. 1 + 82. \$20.00.

This is a volume in the series ACM Doctoral Dissertation Awards. The goal of complexity theory is to determine the amount of computational resources needed to perform certain computational tasks. In particular the author does this for small-depth circuits, which contain AND gates, OR gates and negations, similar to general circuits but have the restriction that any path between any input node and the output node is short compared to the length of the input. The main result of the thesis establishes close to optimal lower bounds on the size of small-depth circuits computing functions such as parity and majority.