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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Manuscripts should be typewritten double-spaced on one side only. Marginal instructions to the typesetter should be written in pencil to distinguish them clearly from the body of the text. The author should keep a complete copy.

The papers should be submitted in final form. Only typographical errors should be corrected in proof; composition charges for any major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he/she prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line following his/her name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the average typewriter should be inserted using either instant lettering or by careful insertion in ink. Manuscripts containing pencilled material other than marginal instructions to the typesetter will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts, and exponents to exponents should be clearly indicated.

Single embellishments over individual letters are allowed; the only embellishment allowed above groups of letters is the overbar.

Double embellishments are not allowed. These may be replaced by superscripts following the symbols.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp[(a^2+b^2)^{1/2}] \text{ is preferable to } e^{[a^2+b^2]^{1/2}}.$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos(x/2b)}{\cos(a/2b)} \text{ is preferable to } \frac{\cos\frac{x}{2b}}{\cos\frac{a}{2b}}.$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1}\sin u\,du \text{ is preferable to } \int \frac{\sin u}{u}\,du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in typeset formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a+bx)\cos t \text{ is preferable to } \cos t(a+bx).$$

Figures: Figures should be drawn in black ink with clean, unbroken lines; do not use ball point pen. The paper should be of a nonabsorbant quality so that the ink does not spread and produce fuzzy lines. If the figures are intended for reduction, they should be drawn with heavy enough lines so that they do not become flimsy at the desired reduction. The notation should be of professional quality and in proportion for the expected reduction size. Figures which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying figures should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References in text to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354–372 (1893)). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow them.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Stromung zaher Flüssigkeiten.*

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details such as ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations such as Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c." even if this special abbreviation is defined somewhere in the text.

CONTENTS

Vol. L, No. 2 **June 1992**

Let Newton Be. John Fauvel, Raymond Flood, Michael Shortland, and Robin Wilson (editors). Oxford University Press, 1988. 272 pp., $29.95.

This splendidly produced and illustrated book explores the richness of Newton's life, labours, and legacy. The 12 chapters, by various hands, explore questions such as: What kind of scientist was Newton? What connections are there between the different strands of research he undertook? Why were his contemporaries so in awe of him? How has Newton's work left its mark on our understanding of the world? The question where we in this century stand in relation to Newton is explored in a final chapter by Sir Hermann Bondi, Master of Churchill College, Cambridge.

Nonlinear Regression. By G. A. F. Seber and C. J. Wild. John Wiley & Sons, 1989. xx + 768 pp., $59.95.

This is a volume in the Wiley Series in Probability and Mathematical Statistics. The authors have endeavoured to cover both the applied and the theoretical ends of the spectrum of this difficult subject, and to appeal to a wide audience. As well as discussing practical examples, they have tried to make the theoretical literature more available to the reader than it has so far been, without becoming entangled in too many details. Most chapters are self-contained. Chapter 1 provides a short introduction to nonlinear modeling and outlines the various types of models that can arise. Chapter 2 examines a wide range of estimation techniques including least squares, quasi-likelihood, and Bayesian methods. Chapter 3 discusses some of the problems associated with estimation, such as ill-conditioning, which have received little attention in the literature up to date. Chapters 4 and 5 present new and important material relating to the concept of curvature and its growing role in statistical inference. Chapter 6 reviews nonlinear models with dependent errors. Chapters 7, 8, and 9 cover growth, compartmental and multiphase (including spline) models, respectively, and emphasise the limitations involved in fitting these models. Chapter 10 discusses errors-in-variables models for both explicit and implicit nonlinear models, and Chapter 11 gives a brief overview of nonlinear multivariate models. Chapter 12 explains the underlying assumptions of the asymptotic theory, and Chapters 13–15 provide an introduction to the growing literature on algorithms for optimization and least squares; they also include practical advice on the use of such programs. There are five appendices, an author index, an extensive list of references, and a subject index.

Fundamentals of Differential Equations. By R. Kent Nagle and Edward B. Saff. Second edition. The Benjamin/Cummings Publishing Company, 1989. xv + 682 pp., $46.95.

This is the second edition of a text first published in 1986. The most substantial change is the inclusion of a chapter on partial differential equations, which covers the method of separation of variables, Fourier series, and the heat, Laplace, and wave equations. An instructor's guide is available.

Solid Shape. By Jan J. Koenderink. The MIT Press, 1990. xii+697 pp., $65.00.

This monograph is written to give engineers and applied scientists access to the mathematical literature on three-dimensional shapes. It adopts an intuitive visual approach designed to develop heuristic tools of use in applied contexts. It is a text on extrinsic differential geometry in three-dimensional space, stressing intuitive understanding and heuristics rather than mathematical rigour and elegance for its own sake. The twelve chapters are divided into six groups: 1. Prologue (Introduction, Shape and space); 2. Space (Euclidean space, Curved submanifolds); 3. Smooth entities (Curves, Local patches); 4. Static shape (Global patches, Application to ecological optics); 5. Dynamic shape (Morphogenesis, Shape in flux); 6. Epilogue (Shape models, How to draw and use diagrams).

Continued on page 256

Continued from page 234

The Geometry of Genetics. By A. M. Findley, S. P. McGlynn, and G. L. Findley. John Wiley & Sons, 1989. xi + 156 pp., $ 44.95.

This is a volume of the Wiley-Interscience Monographs in Chemical Physics. It is an interdisciplinary monograph on molecular genetics and evolutionary biology, relying heavily on mathematical arguments and being motivated by physics in so far as the authors emphasize a search for "hidden symmetries" in biology. After a general introduction, the monograph is divided into three parts: structure, statics, and dynamics. Each of those parts is further subdivided into a presentation of the relevant mathematics, and a mathematical reformulation of the biological problem. *Structure* (Chapters 3–5) concerns the genetic code, *statics* (Chapters 6–8) represents a realization of the basic processes of molecular genetics, and *dynamics* (Chapters 9–11) provides a differential geometric treatment of biological evolution, phrased at the molecular level. The authors address this book to biologists who have an interest in the construction of fundamental theories of biological phenomena. Chapter headings: 1. General overview; 2. Biological overview; 3. Sets and their structure; 4. The genetic code; 5. Genetic code symmetries; 6. Linear spaces; 7. Transformation of macromolecules; 8. Realization of molecular genetics in a linear space; 9. Differential geometry; 10. Macromolecular evolution; 11. Realization of molecular genetics as a differential geometry.

Numerical Methods for Fluid Dynamics III. Edited by K. W. Morton and M. J. Baines. Oxford University Press, 1988. xvii + 529 pp., $95.00.

This is a volume in The Institute of Mathematics and its Applications Conference Series. It is based on the proceedings of a conference organized by The Institute for Computational Fluid Dynamics of The Universities of Oxford and Reading in association with the Institute of Mathematics and its Applications on Numerical Methods for Fluid Dynamics, held in Oxford in March 1988. The aim of the conference—the third in a series—was to bring together mathematicians and engineers working in the various fields of computational fluid dynamics and aerodynamics, to review recent advances in mathematical and computational techniques, and to promote the cross fertilisation of ideas across the differing application areas. Three main themes were selected: Numerical algorithms specific to CFD together with studies of their behaviour and performance; grid generation techniques, adaptive grids and domain decomposition; unsteady flows, such as those which occur in aircraft flutter, tidal forcing, blade row interaction in turbines, vortex flows, separation, etc. The volume contains the texts of fourteen invited papers and thirty-two contributed papers.

A treatise on the Analytical Dynamics of Particle and Rigid Bodies, with an Introduction to the Problem of Three Bodies. By E. T. Whittaker. With a Foreword by Sir William McCrea, FRS. Cambridge University Press, 1988. xvii + 456 pp., $24.95.

This is a reprint of the fourth, 1937, edition of Sir Edmund Whittaker's incomparable classic. In an illuminating foreword, Sir William McCrea sketches the history of the work and traces the influence it—and Whittaker's *Modern Analysis*—exerted on the physics of the day: *Modern Analysis* is the mathematics needed for Wave Mechanics and *Analytical Dynamics* is the transformation theory of dynamics needed for Dirac's Quantum Mechanics. He also points to the encyclopaedic character of the work: it seems that the author mentions all significant work in the field up to the time of the 1937 edition. The first eight chapters are an account of Lagrangian dynamics and Chapter 9 formulates dynamics by means of variational principles. Chapters 10–12 are an account of the Hamiltonian development of dynamics upon which the rest of the book is based, leading to the transformation theory. Chapters 13 and 14 are devoted to the problem of three bodies as an application of Hamiltonian dynamics. Chapter 15 is a general theory of orbits of dynamical systems.

Continued on page 290

Continued from page 256

Classical Equilibrium Statistical Mechanics. By Colin J. Thompson. Oxford University Press, 1988. ix + 213 pp., $42.50.

This book is intended as an introductory text in statistical mechanics. The only prerequisites are advanced calculus and an elementary knowledge of probability theory, linear algebra, and classical mechanics. No prior knowledge of thermodynamics or statistical mechanics is assumed. The aim of the subject is to relate the microscopic properties of matter to the macroscopic behaviour of systems by providing a statistical procedure—due to Gibbs—for calculating equilibrium thermodynamic quantities of systems whose microscopic behaviour is governed by the laws of classical mechanics. In Chapter 1 the author discusses laws of thermodynamics and shows in Chapter 2, on the Gibbs distribution, that the quantities derived by means of it satisfy these laws. Model systems are introduced in Chapter 3 and several one-dimensional examples of fluid and magnetic systems are treated exactly according to the Gibbs procedure. The classical theories of van der Waals, and Curie and Weiss are discussed in Chapter 4. Chapter 5 discusses the classical Ornstein-Zernike theory of correlations, fluctuation relations for fluid and magnetic systems, the decay of correlations, and the Griffiths spin-correlation inequalities. Some exactly solved models which exhibit phase transitions are given in Chapter 6. The final chapter discusses scaling theory and the renormalization group from a precise mathematical standpoint.

Handbook of Differential Equations. By Daniel Zwillinger, Academic Press, 1989. xx + 673 pp., $49.95.

This book is a compilation of the most important and widely applicable methods for solving and approximating ordinary and partial differential equations. It is divided into four parts. The first part is a collection of transformations and general ideas about differential equations and how they are solved. The second part is a collection of exact, and the third part of approximate analytical solution techniques. The fourth part is concerned with the most important methods for finding numerical solutions of common types of differential equations. All the techniques included are elementary in the usual mathematical sense. The book has been designed both as a handbook and as a complement to a text on differential equations and if used as such will introduce students to more techniques than they usually see in a differential equations course. Each technique described is accompanied by several current references for further study. Chapter headings: I A. Definitions and concepts; I B. Transformations; II. Exact analytical methods; II A. Exact methods for ODEs. II B. Exact methods for PDEs. III. Approximate analytical methods. IV A. Numerical methods: concepts. IV B. Numerical methods for ODE's. IV C. Numerical methods for PDEs.

Biological Delay Systems: Linear Stability Theory. By N. MacDonald. Cambridge University Press, 1988. vii + 235 pp., $59.50.

This is Volume 8 in the series Cambridge Studies in Mathematical Biology. The author's aim is to present a unified account of recent methods in local stability theory, which are mostly available only in the primary literature. His intention is to provide a guide to these techniques for graduate students and research workers concerned with mathematical models in biology, who encounter problems with delays and seek an up-to-date account of local stability analysis. Chapter headings: 1. How delays arise and what effect they have; 2. Ordinary differential equations: the polynomial characteristic equation; 3. Functional differential equations: the transcendental characteristic equation; 4. Hurwitz polynomials; 5. First- and second-order systems with a discrete delay; 6. Higher-order systems and systems with two delays; 7. Reducing a discrete delay problem to one with a polynomial characteristic equation; 8. Stability independent of delay; 9. Distributed delay; 10. Reducible delays and linear subsystems.

Continued on page 304

Continued from page 290

Toposes and Local Set Theories—An Introduction. By J. L. Bell. Oxford University Press, 1988. xii + 267 pp., $75.00.

This is Volume 14 in the series Oxford Logic Guides. The concept of *topos*, invented by Lawvere and Tierney, unites a number of important notions from algebraic geometry, set theory and intuitionist logic, and has led to the forging of new links between classical and constructive mathematics. This book is a selfcontained introduction to the presentation of toposes as the models of theories formulated within a typed intuitionist logic. The category theory necessary for understanding the text is developed in the first two chapters. In subsequent chapters the author introduces local set theories, sheaves, locale-valued sets, and natural and real numbers in local set theories. The Epilogue contains some thoughts on the foundational and philosophical significance of topos theory.

Mathematics for the Biosciences. By Ann C. Menell and Michael J. Bazin, Halsted Press: A Division of John Wiley & Sons, 1988. 231 pp., $39.95.

This is a volume in the Ellis Horwood Series in Mathematics and its Applications. The goal of this text is to introduce those aspects of mathematics that are likely to be of value to high school and undergraduate biology students.

Computational Mechanics of Nonlinear Response of Shells. Edited by W. B. Krätzig and E. Onate. Springer-Verlag, 1990. viii+405 pp., $89.50.

This is a volume in the Springer Series in Computational Mechanics. It contains a selection of 18 papers presented at the ICES '88 Conference in April 1988 in Atlanta, Georgia. They are divided into four groups: 1. Large deformation and large rotation theories; 2. Instability and nonlinear responses of shells; 3. New finite element derivations for nonlinear shell analysis; 4. Specific applications of nonlinear shell concepts.

Superstring Theory. By M. B. Green, J. H. Schwarz, and E. Witten. Cambridge University Press, 1988. *Volume* I: *Introduction*, viii +470 pp., $19.95; *Volume* II: *Loop Amplitudes, Anomalies and Phenomenology*, xii +596 pp., $24.95.

This two-volume work—in the series Cambridge Monographs on Mathematical Physics—attempts to fill the need for a systematic exposition of superstring theory and its applications, accessible to as wide an audience as possible. The first volume is intended to provide a self-contained introduction to superstrings. It begins with an elementary treatment of the bosonic string. Succeeding chapters describe the incorporation of additional degrees of freedom—fermionic degrees of freedom leading to gauge interactions. There follows a detailed discussion of the evaluation of tree approximation scattering amplitudes. The second volume contains many topics from recent research papers, concerned largely with the evaluation of one-loop amplitudes, the study of anomalies and 'phenomenology.' The low energy effective field theory analysis of anomalies and the emergence of the gauge groups $E_8 \times E_8$ and $SO(32)$ are explained. An extensive exposition is made of the four-dimensional physics that arises by compactification of six extra dimensions. This discussion requires many geometric concepts, which are presented in considerable detail.

Continued on page 342

Continued from page 304

The Riemann Problem and Interaction of Waves in Gas Dynamics. By Tung Chang (Tong Zhang) and Ling Hsiao (Lin Xiao). Longman Scientific & Technical, copublished in the U.S. with John Wiley & Sons, 1989. ix + 272 pp., $68.95.

This is Volume 41 in the series Pitman Monographs and Surveys in Pure and Applied Mathematics. It presents the authors' own research since 1963, which had as its aim to make the work of, say, Courant and Friedrich (in their classic book *Supersonic Flow and Shock Waves*) rigorous by developing the method of analysis in phase space. Most of the material in the first three chapters was published in Chinese journals, whilst that in the last chapter consists of recent, previously unpublished, results, Chapter headings: 1. The simplest model; 2. One-dimensional isothermal flow; 3. One-dimensional adiabatic flow; 4. Two-dimensional flow.

An Introduction to Chaotic Dynamical Systems. By Robert L. Devaney. Second Edition. Addison-Wesley Publishing Company, Inc., 1989. xvi + 336 pp., $41.95.

This is the second edition of a text first published in 1986. New material on the orbit diagram and a new section on the Mandelbrot set have been added, and the treatment of elementary bifurcation theory and Sarkovskii's theorem have been revised. The basic goal of the text remains: to introduce the reader to the field of dynamical systems on simple spaces such as the real line and the plane, where one can exhibit all the chaotic and interesting behaviour occurring on more general manifolds, without requiring prerequisites such as the theory of differentiable manifolds and advanced analysis, making the text accessible to junior and senior mathematics majors.

An Introduction to Probabilistic Modeling. By Pierre Brémaud. Springer-Verlag, 1988. xiii + 207 pp., $34.00.

This is a volume in the series Undergraduate Texts in Mathematics. It is designed to provide prerequisite material for courses in Physics, Electrical Engineering, Operations Research, and other fields of applied science where probabilistic models are used. The emphasis in the text is therefore on modeling and computation. Each chapter concludes with a number of examples illustrating applications of stochastic processes and statistics (e.g., Mendel's law and Hardy-Weinberg's theorem; Bertrand's paradox; Galton-Watson processes; information theory; Buffon's needle; chi-square test; filtering). Chapter headings: 1. Basic concepts and elementary models; 2. Discrete probability; 3. Probability densities; 4. Gauss and Poisson; 5. Convergence.

Viscous and Compressible Fluid Dynamics. By M. E. O'Neill and F. Chorlton. Halsted Press: a Division of John Wiley & Sons, 1989. 395 pp., $79.95.

This is a volume in the Ellis Horwood Series in Mathematics and its Applications. It forms a companion volume to the authors' *Ideal and Incompressible Fluid Dynamics*, also published in this series. Its scope is indicated by the chapter headings: 1. Vector and tensor methods; 2. Real fluid motion; 3. Exact solutions for incompressible flow; 4. Flow at low Reynolds number; 5. Flow at high Reynolds number; 6. Elements of thermodynamics and the dynamics of gas flows; 7. Shock waves; 8. Method of characteristics; 9. Mathematical methods in gas dynamics.

Continued on page 388

Continued from page 342

Direct Methods in the Calculus of Variations. By Bernard Dacorogna. Springer-Verlag, 1989. ix + 308 pp.

This is Volume 78 in the series Applied Mathematical Sciences. It is addressed to readers who have some elementary notions of functional analysis and Sobolev spaces, but most of the needed results are summarized in the second chapter. Chapter 3 is concerned with minimization problems involving only scalar functions, while Chapter 4 deals with vector valued functions. In Chapter 5 the authors study the relaxation of nonconvex problems in the scalar as well as the vectorial case. In an appendix they give some applications to nonlinear elasticity and optimal design of the theory developed earlier.

Topics in Nonsmooth Mechanics. Edited by J. J. Moreau, P. D. Panagiotopoulos, and G. Strang. Birkhäuser Verlag, 1988. xii + 329 pp., $61.60.

This volume contains the following articles: 1. Bounded variation in time; 2. Hemivariational inequalities and their applications; 3. Optimal design of a two-way conductor; 4. Contact with adhesion; 5. Yield theory in physics; 6. Quasi-static evolution and bifurcation analysis in standard plasticity and fracture; 7. On the numerical treatment of the inclusion $0\varepsilon\partial f(x)$; 8. Mathematical models of hysteresis.

Stochastic Processes with a Multidimensional Parameter. By M. Dozzi. Longman Scientific and Technical, copublished in the U.S. with John Wiley & Sons, 1989. 198 pp., $49.95.

This is Volume 194 in the Pitman Research Notes in Mathematics Series. The author's object is: (i) to show where new concepts are necessary for the analysis of multiparameter processes and how the results compare with classical results for one-parameter processes, (ii) to apply results for multiparameter processes to stochastic processes with values in function spaces, and (iii) to show how results from the classical and the multiparameter stochastic calculus are used for explicit results on more special processes.

Treatise on Analysis, Volume VII. By J. Dieudonné. Academic Press, 1988. xiv + 366 pp., $69.95.

This is Volume 10-VII in the series Pure and Applied Mathematics (that is, it is the seventh volume of the Treatise, the whole of which is denoted Volume 10 in the series). This volume was first published in the French language under the title "Éléments d'analyse, tome 7" by BORDAS, Paris, in 1978. This Volume 10 consists of Part One (Pseudodifferential operators) of Chapter 23 (Linear functional equations) of the treatise.

Navier-Stokes Equations. By Peter Constantin and Ciprian Foias. The University of Chicago Press, 1989. ix + 190 pp., $34.95 cloth, $14.95 paper.

This volume in the Chicago Lectures in Mathematics is based on lecture notes of graduate courses given by the authors, who aim here to give an almost self-contained treatment of the topics discussed. The text covers a variety of classical topics, including existence, regularity, and uniqueness results, vanishing viscosity limits, analyticity, backward uniqueness, and eigenvalue asymptotics. The authors also present some recent developments, concerning the universal attractor, global Lyapunov exponents, and inertial manifolds. Simple, elementary proofs are used wherever possible and nondimensionality is emphasised.