

A NOTE ON THE PERSISTENCE  
OF LEADING  $N$ -WAVES OF TSUNAMI

By

M. KOVALYOV

*Department of Mathematical Sciences, University of Alberta, Edmonton, Alberta, Canada*

**Abstract.** We consider a new class of  $N$ -wave solutions for the KdV with the property that they periodically transform themselves from leading depression  $N$ -waves to leading elevation  $N$ -waves and back. We consider them as a possible model for the tsunami waves.

**Introduction.** In reports on tsunamis, descriptions of shorelines receding significantly prior to tsunami run-up on the coastline have been frequently documented. One of the first reports of a harbor emptying before a tsunami's arrival dates back to 1755 when a tsunami struck Lisbon. Of more recent observations, we can mention the Flores (December, 1992) and Nicaraguan (September, 1993) tsunamis as well as the tsunamis in Japan (July, 1993), the Philippines (November, 1994) and Mexico (October, 1995) (Ide et al., [8]; Satake et al., [22]; Synolakis et al., [25]; Tadepalli and Synolakis, [26], [27]). Until recently, all run-up models dealt with periodic waves or solitary waves and were intended to describe run-up on the beach (Briggs and Synolakis, [2]; Davletshin and Lappo, [6]; Iwasaki, [9]; Koryavov, [10]; Mazova and Pelinovskii, [16]; Meyer, [17], [18]; Pelinovski and Talipova, [21]; Synolakis, [23], [24]). In 1994, Tadepalli and Synolakis proposed a model based on a class of  $N$ -shaped waves (or simply  $N$ -waves) obtained from the Korteweg-de Vries equation (Tadepalli and Synolakis, [26], [27]). In particular, they suggested that the very phenomenon of the tsunami may be due to the internal structure of the corresponding wave rather than to the interaction of that wave with the shoreline. Their results are obtained by taking solutions of an "appropriate linearization" of the Korteweg-de Vries equation.

In this paper we show that even without preliminary linearization of the Korteweg-de Vries equation we can obtain  $N$ -shaped exact solutions of this equation whose profiles and

---

Received February 11, 1998.

2000 *Mathematics Subject Classification.* Primary 86A10, 86A99, 76B15, 35Q53.

*E-mail address:* mkovalyo@gpu.srv.ualberta.ca

behavior are similar to those described in Tadepalli and Synolakis, [26], [27]; Williams and Jordan, [29]; Carrier and Shaw, [5]. Not surprisingly, these solutions exhibit behavior similar to that of the tsunami waves as corroborated by witnesses of numerous tsunamis.

**Models.** Following Tadepalli and Synolakis, [26], we assume that at least some aspects of tsunami motion should be describable by the Kortweg-de Vries equation of waves in shallow water (Whitham, [28])

$$u_t + \sqrt{gh} u_x + \frac{3\sqrt{gh}}{2h} uu_x + \frac{1}{6} h^2 \sqrt{gh} u_{xxx} = 0,$$

where  $u$  is the surface elevation of water above sea level,  $t$  is time,  $x$  is the Cartesian coordinate on the ocean surface along the direction of motion of the traveling wave,  $h$  is the effective depth, i.e., the greatest depth where the vertical component of the motion is nonzero,  $\sqrt{gh}$  is the linear velocity, and  $g$  is the gravitational constant. After appropriate rescaling of  $t, x$ , and  $u$

$$(u_{\text{old}} = 4hu_{\text{new}}, \quad x_{\text{old}} = \frac{h}{\sqrt{6}}x_{\text{new}}, \quad t_{\text{old}} = \sqrt{\frac{h}{6g}}t_{\text{new}})$$

the equation can be rewritten in the form:

$$u_x + u_t + 6uu_x + u_{xxx} = 0. \tag{1}$$

Use of the Korteweg-de Vries equation is justifiable for two reasons: the wave length of the tsunami waves is of the order of several hundred kilometers, which is much greater than the depth of the ocean at any point, and so it is appropriate to view them as waves in shallow water; any variations in the topography of the ocean bed are small compared to the wave length and therefore their effect on the wave is negligible, allowing us to assume that the ocean bed is flat, and so the conditions of physical applicability of the Korteweg-de Vries equation are fulfilled (Whitham, [28]) at least in the ocean where the tsunamis propagate.

Following the same reasoning as that of Williams and Jordan, [29], Carrier and Shaw, [5], we look for exponentially modulated oscillating solutions of (1) that preserve their identities over transoceanic propagation distances. We already know (Gardner et al., [7]) a class of solutions of (1) that preserve their shape over long distances; they are so-called solitons

$$u(x, t) = 2\kappa^2 \operatorname{sech}^2 k(x - t - \kappa^2 t - x_0)$$

with  $x_0$  and  $\kappa$  being arbitrary real constants. To obtain exponentially modulated oscillating solutions of (1), it seems only reasonable to try taking complex  $\kappa = k + i\varepsilon$ ; unfortunately, the resulting solutions are complex rather than real valued, which in turn makes it difficult to provide them with a meaningful physical interpretation. Instead, we

try formal superposition of two complex solitons (Gardner et al., [7]):

$$\begin{aligned}
u &= -2 \frac{d^2}{dx^2} \ln \det \begin{pmatrix} 1 + \frac{e^{8\kappa_1^3 t - 2\kappa_1(x-t) + \varphi_1}}{2\kappa_1} & \frac{e^{8\kappa_2^3 t - (\kappa_1 + \kappa_2)(x-t) - \varphi_2}}{\kappa_1 + \kappa_2} \\ \frac{e^{8\kappa_1^3 t - (\kappa_1 + \kappa_2)(x-t) + \varphi_1}}{\kappa_1 + \kappa_2} & 1 + \frac{e^{2\kappa_2^3 t - 2\kappa_2(x-t) + \varphi_2}}{2\kappa_2} \end{pmatrix} \\
&= -2 \frac{d^2}{dx^2} \ln \left( 1 + \frac{e^{8\kappa_1^3 t - 2\kappa_1(x-t) + \varphi_1}}{2\kappa_1} + \frac{e^{8\kappa_2^3 t - 2\kappa_2(x-t) + \varphi_2}}{2\kappa_2} \right. \\
&\quad \left. + \frac{(\kappa_1 - \kappa_2)^2}{4\kappa_1 \kappa_2 (\kappa_1 + \kappa_2)^2} e^{8(\kappa_1^3 + \kappa_2^3)t - 2(\kappa_1 + \kappa_2)(x-t) + \varphi_1 + \varphi_2} \right) \\
&= -2 \frac{d^2}{dx^2} \ln \left( \frac{2\sqrt{\kappa_1 \kappa_2} (\kappa_1 + \kappa_2)}{(\kappa_1 - \kappa_2)i} e^{-4(\kappa_1^3 + \kappa_2^3)t + (\kappa_1 + \kappa_2)(x-t) - 0.5(\varphi_1 + \varphi_2)} \right. \\
&\quad + \sqrt{\frac{\kappa_1}{\kappa_2}} \frac{\kappa_1 + \kappa_2}{(\kappa_1 - \kappa_2)i} e^{4(\kappa_1^2 - \kappa_2^3)t - (\kappa_1 - \kappa_2)(x-t) + 0.5(\varphi_1 - \varphi_2)} \\
&\quad + \sqrt{\frac{\kappa_2}{\kappa_1}} \frac{\kappa_1 + \kappa_2}{(\kappa_1 - \kappa_2)i} e^{-4(\kappa_1^3 - \kappa_2^3)t + (\kappa_1 - \kappa_2)(x-t) - 0.5(\varphi_1 - \varphi_2)} \\
&\quad \left. + \frac{(\kappa_1 - \kappa_2)}{2\sqrt{\kappa_1 \kappa_2} (\kappa_1 + \kappa_2)i} e^{4(\kappa_1^3 + \kappa_2^3)t - (\kappa_1 + \kappa_2)(x-t) + 0.5(\varphi_1 + \varphi_2)} \right) \quad (2a)
\end{aligned}$$

with  $\kappa_1 = \frac{\varepsilon + ik}{2}$ ,  $\kappa_2 = \frac{\varepsilon - ik}{2}$ , and two arbitrary complex constants  $\varphi_1, \varphi_2$  determined by the initial conditions. Writing out this formula yields:

$$\begin{aligned}
u &= 2 \left[ \frac{-\cos k(\gamma + 3\varepsilon^2 t - k^2 t + t - x) + \cosh \varepsilon(p + \varepsilon^2 t - 3k^2 t + t - x)}{\frac{\sin k(\gamma + 3\varepsilon^2 t - k^2 t + t - x)}{k} - \frac{\sinh \varepsilon(p + \varepsilon^2 t - 3k^2 t + t - x)}{\varepsilon}} \right]^2 \\
&+ 2 \left[ \frac{k \sin k(\gamma + 3\varepsilon^2 t - k^2 t + t - x) + \varepsilon \sinh \varepsilon(p + \varepsilon^2 t - 3k^2 t + t - x)}{\frac{\sin k(\gamma + 3\varepsilon^2 t - k^2 t + t - x)}{k} - \frac{\sinh \varepsilon(p + \varepsilon^2 t - 3k^2 t + t - x)}{\varepsilon}} \right] \quad (2b)
\end{aligned}$$

$$\begin{aligned}
&= \frac{8\varepsilon^2 k^2}{[\varepsilon \sin k(\gamma + 3\varepsilon^2 t - k^2 t + t - x) - k \sinh \varepsilon(p + \varepsilon^2 t - 3k^2 t + t - x)]^2} \\
&\quad - \frac{2\varepsilon k(\varepsilon^2 + k^2) e^{\varepsilon(p + \varepsilon^2 t - 3k^2 t + t - x)} \cos(k(\gamma + 3\varepsilon^2 t - k^2 t + t - x) + \alpha)}{[\varepsilon \sin k(\gamma + 3\varepsilon^2 t - k^2 t + t - x) - k \sinh \varepsilon(p + \varepsilon^2 t - 3k^2 t + t - x)]^2} \\
&\quad - \frac{2\varepsilon k(\varepsilon^2 + k^2) e^{-\varepsilon(p + \varepsilon^2 t - 3k^2 t + t - x)} \cos(k(\gamma + 3\varepsilon^2 t - k^2 t + t - x) - \alpha)}{[\varepsilon \sin k(\gamma + 3\varepsilon^2 t - k^2 t + t - x) - k \sinh \varepsilon(p + \varepsilon^2 t - 3k^2 t + t - x)]^2}, \quad (2c)
\end{aligned}$$

where

$$\begin{aligned}
p &= \frac{\varphi_1 + \varphi_2}{2\varepsilon} + \frac{1}{\varepsilon} \ln \frac{k}{\varepsilon \sqrt{k^2 + \varepsilon^2}}, \\
\gamma &= \frac{\varphi_1 - \varphi_2}{2ik} + \frac{1}{2ik} \ln \frac{\kappa_1}{\kappa_2} + \frac{\pi}{2ik}, \\
\alpha &= \arcsin \frac{\varepsilon^2 - k^2}{\varepsilon^2 + k^2} = \arccos \frac{2\varepsilon k}{(\varepsilon^2 + k^2)}.
\end{aligned}$$

$\varepsilon = 0$  corresponds to the limit of (2) as  $\varepsilon \rightarrow 0$ . Another derivation of (2) based on a generalization of the Inverse Scattering Theory is given in Kovalyov, [11], [12], [13].

Since the denominator of  $u$  vanishes for some value of  $x$  at every moment of time  $t$ , functions (2) possess a moving pole of second order whose equation of motion is implicitly

given by

$$\frac{\sin k(\gamma + 3\varepsilon^2 t - k^2 t + t - x)}{k} - \frac{\sinh \varepsilon(p + \varepsilon^2 t - 3k^2 t + t - x)}{\varepsilon} = 0.$$

In this aspect, our model is similar to those of Meyer, [17], [18] and Carrier and Shaw, [5], which are also based on singular solutions. Presence of a singularity indicates that the equations (1) and (2) fail to provide a proper description of the process in a small neighborhood of the singularity although they seem to work adequately outside that neighborhood. Simple analysis of (2c) indicates that away from that pole, i.e., for  $|p + \varepsilon^2 t - 3k^2 t + t - x| \gg 1$ ,

$$u \approx -A(x, t) \cos k[\gamma + 3\varepsilon^2 t - k^2 t + t - x + \alpha \operatorname{sign}(p + \varepsilon^2 t - 3k^2 t + t - x)],$$

$$A(x, t) = \frac{2\varepsilon(\varepsilon^2 + k^2)}{k \sinh \varepsilon(p + \varepsilon^2 t - 3k^2 t + t - x)}.$$

This can be physically relevant (Whitham, [28]), only if the wavelength is large compared to the amplitude:

$$\frac{1}{k} \gg |A(x, t)|,$$

which is true due to the largeness of  $(p + \varepsilon^2 t - 3k^2 t + t - x)$ , provided that

$$(\varepsilon^2 + k^2) = O(1); \quad (3a)$$

the oscillatory wave moves to the right with velocity close to one:

$$3\varepsilon^2 - k^2 + 1 \approx 1, \quad (3b)$$

and the amplitude is slowly varying:

$$\left| \frac{\partial A}{\partial t} \right| + \left| \frac{\partial A}{\partial x} \right| \ll |A(x, t)|,$$

which is true only if

$$\varepsilon = o(1) \quad \text{and} \quad \varepsilon^2 - 3k^2 + 1 = O(1). \quad (3c)$$

Estimates (3) impose the following conditions of physical relevancy:

$$\varepsilon = o(1); \quad \frac{1}{3}k^2 - \varepsilon^2 = o(1). \quad (4)$$

If  $p$  is large, we can choose an arbitrary finite space interval  $(a, b)$ , whose size is small compared to its distance from the pole, but large relative to the wave length, i.e.,  $|p| \gg |b - a| \gg \frac{1}{k} \gg 1$ , and an arbitrary finite time interval  $(T_1, T_2)$ , such that all  $t \in (T_1, T_2)$  and all  $x \in (a, b)$  satisfy  $|\varepsilon^2 t - 3k^2 t + t - x| \ll p$ . Then also, for  $x \in (a, b)$ ,  $t \in (T_1, T_2)$ , we obtain  $\alpha \approx -\frac{\pi}{6}$ ,

$$u(x, t) = \frac{2(\varepsilon^2 + k^2)\varepsilon}{k \sinh \varepsilon p} \sin k(\gamma - 3k^2 t + t - x + \frac{\pi}{6}) + o\left(\frac{\varepsilon}{\sinh \varepsilon p}\right).$$

The wave in this case is essentially a linear wave plus lower-order terms resulting from the nonlinearity in (1). The dispersion relationship (Whitham, [28])  $\omega = (k + i\varepsilon) - (k + i\varepsilon)^3$  involves dependence on amplitude through the parameter  $\varepsilon$ , which drops out when  $\varepsilon = 0$ .

Even though away from the pole the wave travels to the right, the pole itself can move in either the right or left direction. The evolution of  $u(x, t)$  near the pole is shown in Fig. 1

(when it moves to the right) and in Fig. 2 (when it moves to the left). As the wave moves, an elevation wave periodically builds up, jumps over the trough, fades away and after a while reappears again. This process periodically repeats itself. On the right-hand side of the pole,  $u$  can be written as  $u_+ \approx -A(x, t) \cos k(\gamma + 3\varepsilon^2 t - k^2 t + t - x - \alpha)$  whereas on the left-hand side of the pole,  $u$  can be written as  $u_- \approx -A(x, t) \cos k(\gamma + 3\varepsilon^2 t - k^2 t + t - x + \alpha)$ . The elevation wave builds up when  $u_+$  moves faster than  $u_-$  and it fades away when  $u_+$  moves slower than  $u_-$ .

Although (2), just like the  $N$ -waves of Tadepalli and Synolakis, [26], [27], are obtained from KdV with similar profiles, there are certain differences in their behaviors. The  $N$ -waves of Tadepalli and Synolakis, [27], are assumed to be linear and non-dispersive, thereby allowing them to keep the same profile over constant depth. These  $N$ -waves are essentially modeled by solitons. The waves (2), however, arise as dispersive nonlinear solutions of KdV that preserve their oscillating “identities” as they travel, even though at different points they materialize at different stages with a different profile. The dispersion here does not lead to dispersing waves but rather to the creation of  $N$ -waves, which periodically change from leading depression  $N$ -waves (abbreviated hereafter as LDN) to leading elevation  $N$ -waves (abbreviated hereafter as LEN) to simple depression waves and back to LDN.

What is interesting is that (2) seems to provide an explanation as to why the shoreline recedes before the tsunami’s arrival, the reason for that being the ever-present leading depression  $N$ -wave. Indeed, if there were an island located at the interval [25, 30] in Fig. 1, first the leading depression part of LDN would arrive resulting in a receding shoreline, and then a large elevation wave would hit the island from the left.

**Babi Island effect.** Another observed phenomenon is illustrated in Fig. 2. If there were an island located at the interval [30, 40] in Fig. 2, the elevation wave would hit the left side of the island not the right side, notwithstanding that the wave is coming from the right. This phenomenon was observed during the 1992 Indonesian earthquake, when the villages on the south side of Babi Island were the hardest hit by a tsunami wave even though the wave came from the north (Liu, [14]). Similar phenomena are suggested from the account of the Hokkaido Island tsunami as well as in a number of other cases described in Liu. The standard explanation of this according to Briggs et al., [3], and Liu et al., [14], is that the incoming tsunami wave splits into two waves refracting around the island and that these two waves collide on the lee side into a large elevation wave which splashes up and is responsible for all the destruction. There is no denial that this phenomenon, at least partially, contributes to the Babi Island effect.

Our model suggests that another phenomenon may also be in the works. Recall that the Babi Island effect has been recorded when tsunamis hit islands whose size was considerably smaller than the wave length of the incoming wave (Babi Island itself is only 2km in diameter). As we know from the classical theory of linear waves, obstacles whose size is considerably smaller than the wave length do not affect the wave besides possibly slightly distorting it in a small neighborhood of the obstacle. (It is for that reason that the resolution of a regular microscope is inferior to that of an electron microscope.) If we assume that the same is true for the solutions (2) and the tsunami wave near an

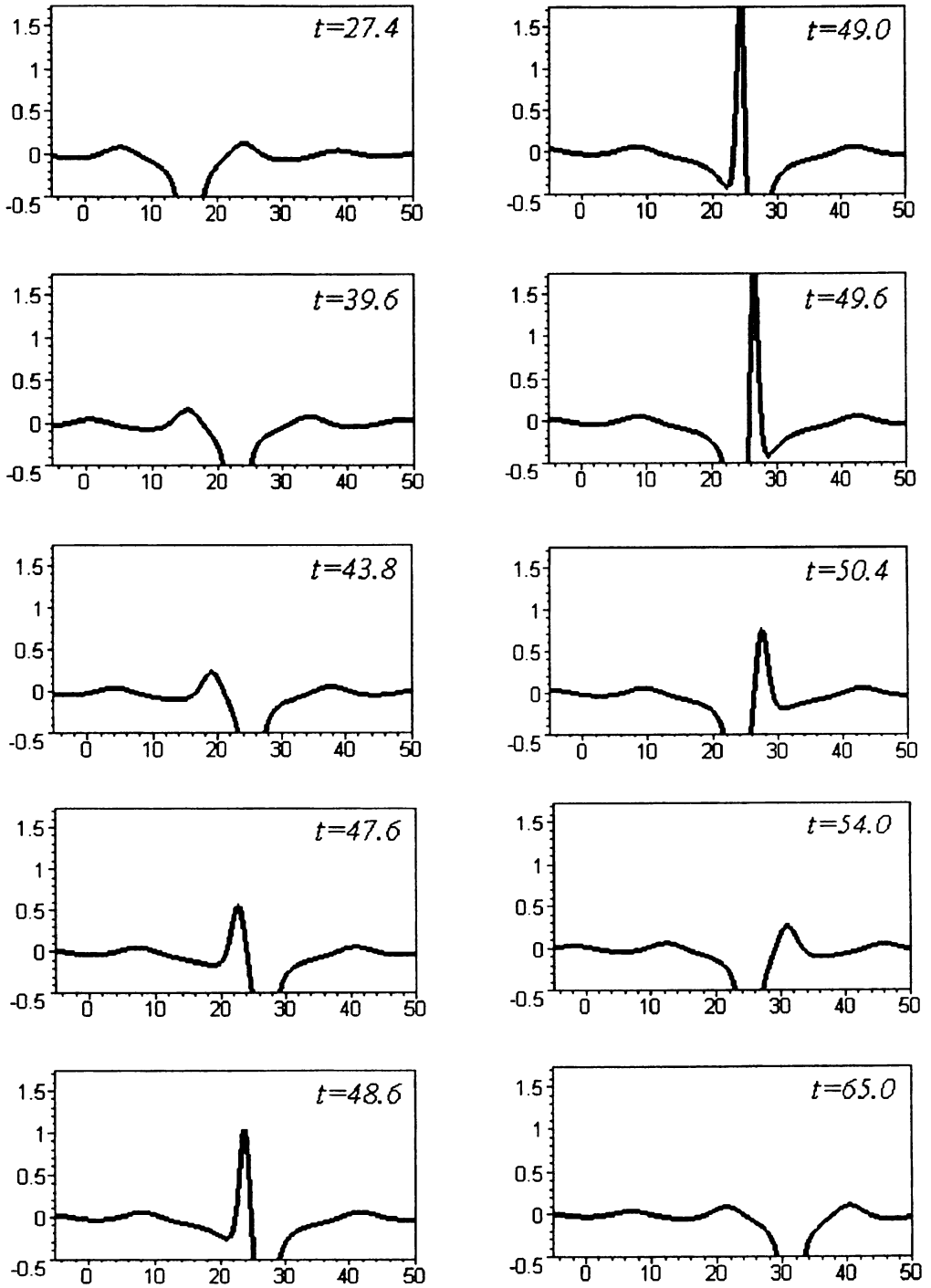


FIG. 1. Graph of  $u(x, t)$  given by (2) with  $\varepsilon = 0, \gamma = 0, p = 6.0, k = 0.45$ , plotted for selected values of time shown at the top of each frame

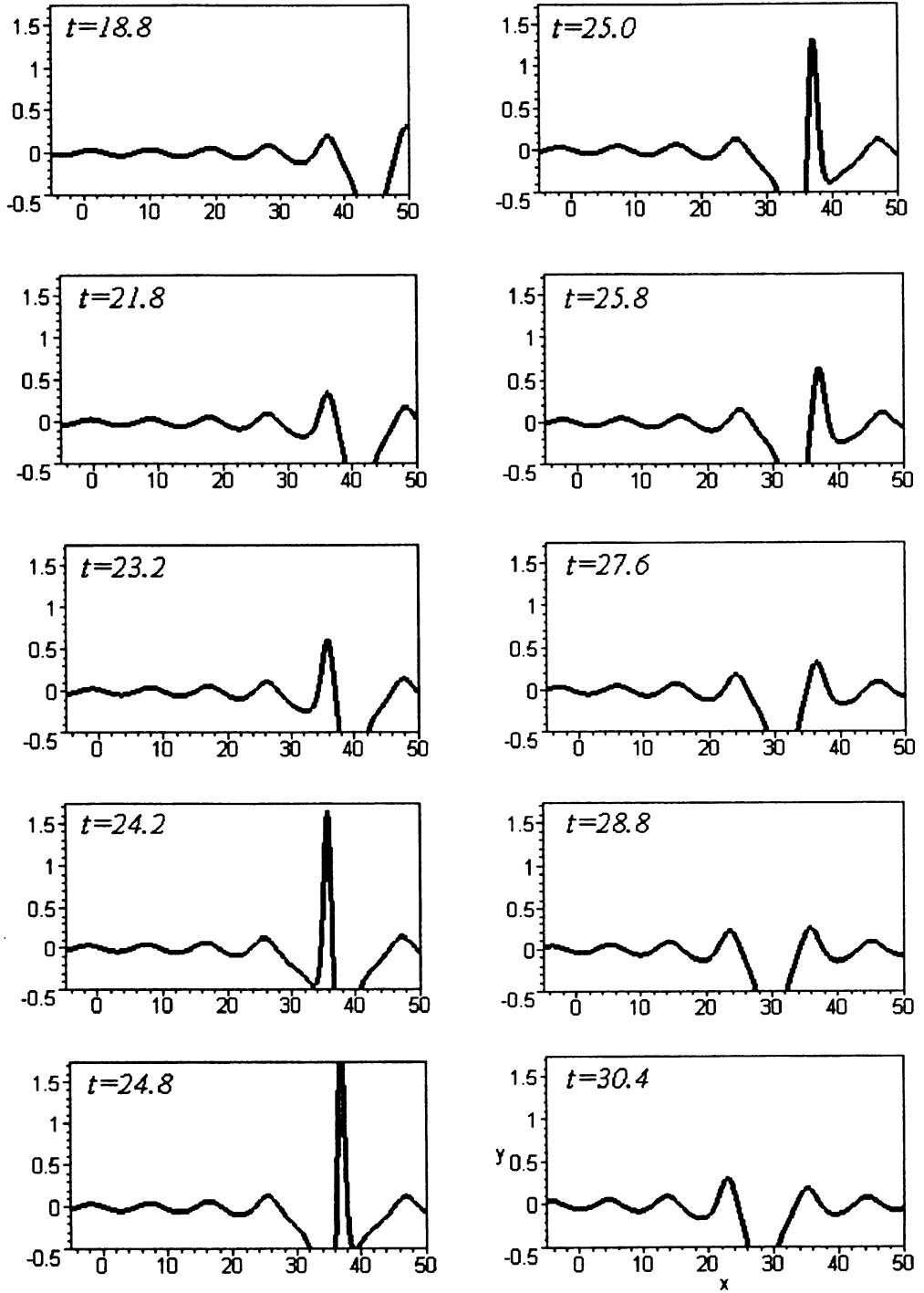


FIG. 2. Graph of  $u(x, t)$  given by (2) with  $\varepsilon = 0, \gamma = 0, p = 42.0, k = 0.7$ , plotted for selected values of time shown at the top of each frame

island is plane and given by (2), we are led to the conclusion that the location of the leading elevation  $N$ -wave and its motion will not be affected much by the presence of the island. Thus if the LEN happens to build up close to the lee side of the island and the parameters  $\varepsilon$  and  $k$  satisfy  $\varepsilon^2 - 3k^2 + 1 < 0$ , the LEN will appear to be moving towards the lee side of the island in the direction opposite to the direction of motion of the tsunami itself as shown in Fig. 2 and as observed whenever the Babi Island effect takes place. The phenomenon described is especially pronounced if the size of the island is very small compared to the wave-length  $1/k$  of the wave. Indeed, as described in Liu, [14], during the Flores and Okushiri tsunami the ratio of the island diameter to the wave length is important to the Babi Island effect. One can conjecture that three different processes contribute to this, namely collision of the trapped waves, the process we just proposed, and the influence of the ocean bed topography.

There are some regions where the elevation wave never builds up at all. Indeed, the elevation wave builds up in (2) whenever the denominator  $\varepsilon \sinh(\gamma + 3\varepsilon^2 t - k^2 t + t - x) - k \sinh \varepsilon(p + \varepsilon^2 t - 3k^2 t + t - x)$  has a double zero and that happens for  $t$  close to  $\frac{p - \gamma + 2n\pi}{2(\varepsilon^2 + k^2)}$ , in the neighborhood of the point

$$x = \frac{\gamma(3k^2 - \varepsilon^2 - 1) + (3\varepsilon^2 - k^2 + 1)(p + 2n\pi)}{2(\varepsilon^2 + k^2)},$$

where  $n$  is an arbitrary integer. If these points are sufficiently spaced out, some intervals between them may not experience elevation wave buildup at all. For example, if  $\varepsilon = 0, k = 0.75, p = 6.0, \gamma = 0$  (the same parameters as for  $u$  shown in Fig. 1), there is no elevation wave building up on the interval  $[15, 20]$ , direct numerical computations show that on this interval,  $u < 0.5$  for all values of  $t$ . So if there were an island located on  $[15, 20]$  we would still be able to witness the lowering of the water level followed only by a relatively small tidal wave. This helps perhaps to explain why a tsunami wave causing a lot of damage at some points on its path may be harmless at the other points. According to our model, the strength of the wave when it hits an island or a coastline depends mainly on at what stage the wave arrives there; the boundary conditions on the coastline and the interaction of the wave with the shoreline seem to be only of secondary nature.

One can define nonlinear superposition of waves (2) (Gardner, [7]) as follows. Let  $k_l, \varepsilon_l, l = 1, 2, \dots, n$ , be the “frequencies” of  $n$  waves of the form (2) and let  $\gamma_l, p_l, l = 1, \dots, n$ , be their “phases”. Then nonlinear superposition of these  $N$ -waves is defined to be the function

$$u_n(x, t) = -2 \frac{d^2}{dx^2} \ln \det A, \quad (5)$$

where  $A$  is a  $2n \times 2n$  matrix whose entries are

$$A_{lm} = \delta_{lm} + \frac{\beta_l e^{\mu_l^3 t - (\mu_l + \mu_m)x}}{\mu_l + \mu_m}$$

and for  $1 \leq l \leq n$ ,  $\mu_l = ik_l + \varepsilon_l$ ,  $\mu_{n+l} = -ik_l + \varepsilon_l$ ,  $\beta_l = 2\varepsilon_l e^{2ik_l \gamma_l + 2p_l \varepsilon_l}$ , and  $\beta_{n+l} = 2\varepsilon_l e^{-2ik_l \gamma_l + 2p_l \varepsilon_l}$ . It has been shown in Kovalyov, [11], [12], that a sufficiently large class of solutions of (1) that decays at infinity sufficiently fast can be approximated by a nonlinear superposition of waves (2). So it is not surprising that tsunami profiles obtained by other



authors by solving (1) or similar equations resemble profiles in Figures 1 and 2 (Tadepalli and Synolakis, [26], [27]; Williams and Jordan, [29]; Carrier and Shaw, [5]).

**Singularities.** Having discussed certain advantages of our presentation, we should also mention a weak spot: presence of a singularity indicated by breaks in the graphs. Clearly, tsunamis do not possess that kind of singularity. What they do possess is some sort of a “center” in the very close neighborhood of which the model breaks down. Such a situation is similar to the failure of the electromagnetic theory to model the electromagnetic field in the very close proximity of a moving electron; for that purpose, quantum theory is required. Closer to home analogues may be found in fluid mechanics, e.g., the theory of shock waves does not explain what exactly happens in the very close proximity of a moving shock wave; instead, it replaces it with a discontinuity, yet it provides a fairly decent description of the shock propagation. In a similar manner, our model may provide a fairly good description of the tsunami motion including an explanation of certain effects related to the behavior of the wave near the singularity, e.g., the Babi Island effect, receding shoreline. The reason is that the effects exhibit themselves long before the singularity renders solutions (2) inapplicable for modeling tsunamis. The model clearly fails in a very close proximity to the “center”, and that failure is reflected by the appearance of a singularity.

**Conclusion.** In this note we have shown that some singular solutions (2) of the Korteweg-de Vries equation seem to be able to provide a qualitative explanation to a number of aspects of tsunami waves observed by witnesses. Profiles of these waves closely resemble profiles obtained by other authors with other models. One should understand, however, that the model described is based on the one-dimensional Korteweg-de Vries equation and thus is rather limited. A better model would be provided by considering appropriate solutions of the Boussinesq equations or perhaps the two-dimensional Kadomtsev-Petviashvili equation. It is also clear that a model based on (1) or its generalizations may describe only some aspects of tsunami waves of a certain origin as it does not take into account terrain of the ocean bed and the planet’s curvature.

**Acknowledgments.** The author would like to express his gratitude to Dr. A. A. Stahlhofen of the Max Planck Institut für Metallforschung for the help and advice he provided and to the referees for careful reading of the manuscript. The author would like to thank Ms. V. Spak for typing the manuscript and for patiently making numerous changes. Finally, the author would also like to express his gratitude to Elaine W. Becker of the American Mathematical Society for carefully proofreading and editing the manuscript and making adjustments.

## REFERENCES

- [1] J. M. Abreu and Seabra Santos, *Generation and propagation of tsunami produced by the movement of the ocean bed*, Annales Geophysicae, Atmospheres, Hydrospheres and Space Sciences **10**, No. 1–2 (1992)
- [2] M. J. Briggs and C. E. Synolakis, *Large scale model tests of tsunami run-up*, EOS, Wash. **73**, 266 (1992)

- [3] M. J. Briggs, C. E. Synolakis, G. S. Harkin, and D. R. Green, *Laboratory experiments of tsunami runup on a circular island*, PAGEOPH **144**, 3/4, 569–593 (1995)
- [4] G. F. Carrier, *On-shelf generation and coastal propagation*, Proc. IUGG/IOC Internat. Tsunami Sympos., Wakayama, Japan, 1993, pp. 23–40
- [5] G. F. Carrier and R. P. Shaw, *Response of narrow-mouthed harbors to tsunami*, Tsunami in The Pacific Ocean, East-West Center Press, Proceedings of the International Tsunami Symposium (1969)
- [6] V. Davletshin and D. D. Lappo, *Tsunami forces exerted on vertical cylindrical barriers*, Fluid Mechanics - Soviet Research, **15**, No. 3 (1986)
- [7] C. S. Gardner, J. M. Greene, M. D. Kruskal, and R. M. Miura, *Korteweg-de Vries equation and generalizations. VI. Methods for exact solution*, Communications on Pure and Applied Mathematics **XXVII**, 97–133 (1974)
- [8] S. Ide, F. Imamura, Y. Yoshida, and K. Abe, *Source characteristics of the Nicaraguan tsunami earthquake of September 2, 1992*, Geophys. Res. Lett. **20**, 863–866 (1993)
- [9] T. Iwasaki, *Behaviour of tsunami waves*, Navigation **83** (1985)
- [10] V. P. Koryavov, *Electromagnetic fields associated with tsunami propagation*, Izvestiya Academy of Sciences USSR, Physics of the Solid Earth **28**, No. 2 (1992)
- [11] M. Kovalyov, *Basic motions of the Korteweg-de Vries equation*, Nonlinear Analysis, Theory, Methods and Applications **31**, 599–619 (1998)
- [12] M. Kovalyov, *Modulating properties of harmonic breather solutions of KdV*, Journal of Physics A **31**, 5117–5128 (1998)
- [13] M. Kovalyov, *On the structure of the two-soliton interaction for the Korteweg-de Vries equation*, Journal of Differential Equations **152**, 431–438 (1999)
- [14] P. L.-F. Liu, Y. S. Cho, M. J. Briggs, U. Kanoglu, and C. E. Synolakis, *Runup of solitary waves on a circular island*, Journal of Fluid Mechanics **302**, 259–285 (1995)
- [15] V. B. Matveev, *Asymptotics of the multipositon-soliton  $\tau$  function of the Korteweg-de Vries equation and the supertransparency*, J. Math. Phys. **35**, 2955–2970 (1994)
- [16] R. Mazova and E. N. Pelinovskii, *Linear theory of tsunami wave runup on a beach*, Izvestiya Akademii Nauk SSSR, Fizika Atmosfery i Okeana **18**, No. 2 (1982)
- [17] R. E. Meyer, *On the shore singularity of water waves. I. The local model*, Phys. Fluids **29**, 3152–3163 (1986)
- [18] R. E. Meyer, *On the shore singularity of water-waves theory. II. Small waves do not break on gentle beaches*, Phys. Fluids **29**, 3164–3173 (1986)
- [19] T. Nakayata, *Boundary element analysis of non-linear water wave problems*, International Journal for Numerical Methods in Engineering **19**, No. 7 (1983)
- [20] S. Novikov, S. Manakov, L. Pitaevskii, and V. Zakharov, *Theory of Solitons, The Inverse Scattering Method*, Chapter 1, §5. Contemporary Soviet Mathematics, Consultants Bureau, 1984
- [21] Y. N. Pelinovski and T. G. Talipova, *Height variations of large-amplitude solitary waves in the near-shore zone*, Oceanology **17**, No. 1 (1997)
- [22] K. Satake, J. Bourgeois, K. Abe, Y. Tsuji, F. Imamura, Y. Io, H. Katao, E. Noguera, and F. Estrada, *Tsunami field survey of the 1992 Nicaragua Earthquake*, EOS, Transactions, AGO **74**, 145, 156 (1993)
- [23] C. E. Synolakis, *Are solitary waves the limiting waves in the long wave run-up?*, Proc. ASCE, 21st Conf. Coastal Engineering, Costa del Sol, Malaga, Spain, 1988, pp. 219–233
- [24] C. E. Synolakis, *The run-up of solitary waves*, Journal of Fluid Mechanics **185** (1987)
- [25] C. E. Synolakis, F. Imamura, Y. Tsuji, S. Matsutomi, B. Tinti, B. Cook, and M. Ushman, *Damage, conditions of east Java tsunami of 1994 analyzed*, EOS, Transactions, American Geophysical Union **76** (26), 257 and 261–262 (1995)
- [26] S. Tadeballi and C. E. Synolakis, *Model for the leading waves of tsunamis*, Physical Review Letters **77**, 2141–2144 (1996)
- [27] S. Tadeballi and C. E. Synolakis, *The run-up of N-waves on sloping beaches*, Proceedings of the Royal Society of London, Series A **445**, 99–112 (1994)
- [28] G. B. Whitham, *Linear and Nonlinear Waves*, Chapter 13, Pure and Applied Mathematics, A Wiley-Interscience Publication, John Wiley and Sons, New York, 1974
- [29] J. B. Williams and J. M. Jordan, *A laboratory model of a doubled-humped wave impinging on a plane, Sloping Beach*. Tsunami in the Pacific Ocean, East-West Center Press, Proceedings of the International Symposium on Tsunami, 1969