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CORRIGENDUM FOR "ENERGY BALANCE FOR VISCOELASTIC BODIES IN FRICTIONLESS CONTACT"

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Abstract. The proof in "Energy balance for viscoelastic bodies in frictionless contact" of an energy balance has a crucial flaw that renders the proof incorrect. At present no correction for this flaw is known.

In [1], a proof was given claiming to show an energy balance for the dynamic frictionless Kelvin–Voigt viscoelastic contact problem

$$\frac{\partial^2 u}{\partial t^2} = -\mathcal{A}u - \mathcal{B}\frac{\partial u}{\partial t} + f(t,x)$$

over a domain $x \in \Omega \subset \mathbb{R}^d$ and $t \geq 0$ with Signorini contact conditions

$$\begin{aligned} 0 \leq N(t,x) \quad &\perp \quad n(x)\cdot u(t,x) - \varphi(x) \geq 0, \\ \sigma(t,x)\cdot n(x) \quad &= \quad N(t,x)\, n(x), \end{aligned}$$

over the boundary $\partial\Omega$, and initial conditions $u(0,x) = u_0(x)$ and $\partial u/\partial t(0,x) = v_0(x)$ for $x \in \Omega$. Here $\sigma(t,x)$ is the stress tensor, which is assumed to be a linear function of the linearized strain tensor $\varepsilon(t,x) = \frac{1}{2}\left(\nabla u(t,x) + \nabla u(t,x)^T\right)$. The elasticity operator $\mathcal{A}$ and the viscosity operator $\mathcal{B}$ are second-order partial differential operators that are semi-elliptic in the sense that $\mathcal{A} + \alpha I$, $\mathcal{B} + \alpha I \colon H^1(\Omega) \to H^{-1}(\Omega)$ are elliptic operators for any $\alpha > 0$.

The crucial flaw in the argument for the energy balance was the assumption that $H^1(\Omega)' = H^{-1}(\Omega)$, although implicitly $H^{-1}(\Omega)$ is the dual of $H_0^1(\Omega)$. Lemmas 2.1 and 2.2 of the paper are correct. What is incorrect is their use in Section 3. The flaw is fatal for the proof since it is shown that the acceleration $\partial^2 u/\partial t^2 \in L^2(0,T;\, H^{-1}(\Omega))$, and we need the duality pairing between $\partial^2 u/\partial t^2$ and the velocity $\partial u/\partial t \in L^2(0,T;\, H^1(\Omega))$. The problem is that the acceleration may not be sufficiently regular on the boundary in order to form the duality pairing of these two functions. Without it, the proof cannot be carried through.

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The use of the variational inequality formulation hides the normal contact force so that we know essentially nothing about it. However, it reappears indirectly in $\partial^2 u/\partial t^2$, and knowledge of its regularity appears to be crucial for futher progress on this problem.

References

[1] David E. Stewart. Energy balance for viscoelastic bodies in frictionless contact. *Quarterly of Applied Mathematics*, 64(4):735–743, 2009. MR2588233