

QUARTERLY  
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## FOREWORD

This issue of the Quarterly of Applied Mathematics is dedicated to the memory of our distinguished colleague Ulf Grenander. In a career spanning more than four decades, Ulf pioneered a holistic foundation for the analysis of high-dimensional signals that he termed *pattern theory*. A personal introduction to these ideas and a brief testament to Ulf as a person are contained in the article by our colleague David Mumford. The remaining articles in this issue are scientific papers that discuss many current aspects of pattern theory.

I express my gratitude to Michael Miller, Anuj Srivastava, and Song-Chun Zhu for their enthusiastic appreciation for Ulf's work, their assistance in editing this issue, and their persistent efforts to bring it to completion.

Govind Menon  
Associate Managing Editor

## SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

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### **Submission of Accepted Manuscripts:**

Submission of manuscripts electronically-prepared in L<sup>A</sup>T<sub>E</sub>X is required, with a strong preference for  $\mathcal{A}\mathcal{M}\mathcal{S}$ -L<sup>A</sup>T<sub>E</sub>X. To assist authors in preparing electronic manuscripts, the AMS has prepared author packages. The author package includes instructions for preparing electronic manuscripts, the *AMS Author Handbook*, samples, graphic creation instructions, and a style file. When choosing a style file for the *Quarterly of Applied Mathematics*, choose the QAM-specific journal package, made available by the American Mathematical Society at [www.ams.org/distribution/qam/qamauthorpac.html](http://www.ams.org/distribution/qam/qamauthorpac.html). For more technical information, please visit [www.ams.org/authors/author-faq.html](http://www.ams.org/authors/author-faq.html).

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