

Curious what shape the curve you just stitched forms?

We consider the general equation for a line that connects two points in the relationship where the coordinate of the number on the vertical axis and the coordinate of the number on the horizontal axis always sum to the same value. For simplicity we will assume that the sum is ONE.

The equation that describes the entire family of lines drawn in this way is given below. Expanding the equation and noting that it is quadratic in "a" which is also the particular y-intercept for the line, we can solve to find out which values for a produce points on the curve.

$$ax + (1 - a)y = a(1 - a)$$

$$ax + y - ay = a - a^2$$

$$a^2 + (x - y - 1)a + y = 0$$

If a point (x, y) is on some stitched line then the point lies on the curve or in the region between the curve and the positive x and y axes. So there is an "a" value for each point that lies on a stitched line. Thus, $a^2 + (x - y - 1)a + y = 0$ has a solution.

That means that its discriminant is non-negative. So $(x - y - 1)^2 - 4y \geq 0$ and in fact, when the discriminant is exactly zero, then the point (x, y) is ON the boundary (and thus on the stitched curve).

Examining the equation that results from the zero discriminant gives us the equation for the curve.

$(x - y - 1)^2 - 4y = 0 \rightarrow x^2 - 2xy + y^2 - 2x - 2y + 1 = 0$ is an equation for a "tilted" parabola. Hence the curve that the stitching produces is part of a parabola.