Questions of two kinds arise in mathematical neuroscience: (1) How can mathematics explain observed patterns of neural activity and their response to perturbations? (2) What is the function of those patterns? These two questions are generally separable, in the sense that one can be studied without any knowledge of the other. Tying dynamics to function, one ultimate goal of this field, is possible only when the problem is sufficiently ripe. Another goal is to develop the theory with an eye to making new predictions that can be tested experimentally.

The mathematical neuroscience of weakly electric fish is one ripe problem area in which dynamics have been linked to function at many levels. The jamming avoidance response, for example, prevents neighboring fish from distorting each other’s view of the world—and is often considered the most detailed analysis of neural function, from input to behavior, performed for any animal. This article highlights recent findings from our research program on these exotic, fascinating animals, made possible by close collaboration between Len Maler’s and John Lewis’s experimental groups and the theory group I head, all at the Center for Neural Dynamics at the University of Ottawa.

Design Principles of Sensory Function

Weakly electric fish specialize in detecting minute perturbations in the spatiotemporal electric field they set up around themselves—a task to which they devote 80% of their metabolic energy. Because they are specialists, scrutiny of their neural circuitry readily yields design principles that enable them to perform sensory tasks; surprisingly, their study also provides important information for understanding vision and hearing in non-specialists, such as humans. Design principles of sensory function can also be understood in mathematical terms because the brain activity of these fish can be recorded at many successive levels, from primary receptors on the skin to higher in the brain. This allows validation of conclusions about neural function at each level: We can verify that neurons at one level extract information X by determining whether target neurons downstream care about X or not. Neuro-anatomy is also key to this pursuit, as it is known in much detail and is simpler than in mammals—yet the fish perform similar computations, such as navigating, finding prey, and communicating with one another.

Weakly electric fish use an active electric sense. The midbody of a fish oscillates like an electric dipole throughout its life. The resulting electric image can be “seen” or measured by electroreceptors that sense the voltage drop across the skin. Changes in response to stimuli, such as rocks, prey, and other fish, are seen as changes in this baseline image. The change in the electric image is the primary “neural image” that the sensory pathway detects and processes. How is all this studied?

First, to address the function of a sensory pathway, it is important to understand its natural “physical” sensory scene. Actual measurements pose huge challenges. In the meantime, one solves Poisson’s equation in conductive media for the electric field outside, as well as in the different layers inside the fish, using a finite element method, such as FEMLAB. The distortions of this field caused by other fish, prey, weeds, river bottom, and surface can then be calculated realistically. Computational mathematics allows us to estimate what electric images a fish “sees.” Using this technique, we were recently able to predict that a fish can detect prey in a cluttered background, as long as the clutter is dense and the fish is moving. The next step in this direction is to drive the receptors with these naturalistic stimuli in our models—even though these complex fields do not yield analytic solutions as simpler stimuli might.

The firing activity of electroreceptors is modulated by stimuli, but has a built-in correlation mechanism that removes low-frequency noise—it’s as though the system reduces the noise in frequency bands over which it needs to transmit a lot of information. This effect is a type of “noise-shaping,” and our group was able to elucidate its dynamical origins only after a detailed mathematical study of the statistics of firing, using Fokker–Planck analysis. These results underscore the necessity of moving beyond, where warranted, simple “renewal” models in which successive firings do not yield analytic solutions as simpler stimuli might.

We have found that these receptors are mildly synchronized when no stimuli are present. If a fish of the opposite gender shows up, synchrony increases; when these fish talk, by sending out brief electric chirps, the synchrony transiently drops. The whole story works in reverse when two fish of the same gender interact (see [2]). This points to novel forms of coding based on positive or negative synchrony changes; we are now trying to predict how target cells read out these synchrony transitions.

In the absence of input, the target cells, known as “pyramidal” cells, fire seemingly at random and independently from one another. We know,
based on the chaotic bursting activity observed in our modeling of ionic conductance data from these cells, that part of this randomness is likely of deterministic origin. Another challenge is to understand how stimuli lead to collective oscillations in neural populations. Our experiments have shown that spatially correlated stimuli can produce such oscillations, which are associated with the presence of other fish rather than of prey. This led us to formulate a model of a population of noisy neurons with delayed negative feedback. The analysis reveals how intrinsic noise fights it out with external correlated inputs; when the correlation is stronger, oscillations are seen. This correlation-based effect relies on feedback. Novel strides in linear fluctuation theory have been required to deal with the nonlinearity of the neurons and the delay of the feedback loop.

Finally, we have found a basic design principle—transmission, in parallel, of high-frequency signals and their low-frequency amplitude modulation, or “envelope.” This modulation is seen when many interacting fish need to “sort one another out.” The trick that evolution came up with is basically the Hilbert transform, which extracts the low-amplitude modulation. This signal is then fed through a slow synapse to remove the high-frequency carrier.

Outlook

Many challenges lie ahead. One is the sheer complexity crunch, as more feedback loops are added to models. Given that positive feedback favors bistability, and negative feedback (especially with delays) favors oscillations, is there some kind of calculus that can be used to figure out the dynamics of multiple loops with various polarities and delays? How can the randomness in firing that results from noise be disentangled from that caused by intrinsic chaotic behavior? The answer here will play an important role in relating dynamics to function, because the mix of noise and nonlinearity will affect response properties. There is also the question of how natural input varies over both space and time, and the associated computational problem of integrating Poisson’s equation in three dimensions to yield the time-dependent electric image on the body. The computed electric images will eventually be matched to actual recordings at the surface of the body.

What other roles might feedback play? Does feedback make possible statistical estimates of an upcoming stimulus? If so, what neural dynamics support such a computation? There is also a continual rewiring of the diagram that results from learning at synapses. We are missing this key link at the synapses from receptors to pyramidal cells, but expect to have the answer this year. And there is a fascinating social issue: How are individuals identified, and how do they listen to one another in the clutter of everyone’s electric noise? Answers here are likely to impact next-generation neural prostheses, and will surely require more mathematics!

References and Further Reading


André Longtin is a professor of physics and director of the Center for Neural Dynamics at the University of Ottawa.