Mathematics in Energy Production

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The energy field is full of mathematics

It takes a lot of engineering, and a substantial amount of mathematics, to get the petrol from the oil reservoir into your car, or the electricity from renewable sources or fossil fuel power plants to your light switch.

Mathematics is used, for example, to estimate the volumes of oil and gas in reservoirs, to optimize the performance of wells and pumps that get the oil and gas to the surface, to improve the quality of the fossil fuel projects, and to minimize tanker transportation and final delivery costs. It is also essential in the design of modern wind turbine blades and in optimizing wind turbine placement, as well as in the design of solar photovoltaic (PV) systems or concentrated solar power systems. Enhanced geothermal systems will not be built before their design is passed through extensive mathematical software. In other words, without mathematics, we would not be able to drive this very energy demanding world.

A closer look at oil and gas production

The field of oil and gas production is filled with partial differential equations, nonlinear systems of equations, Monte Carlo simulations, uncertainty analysis and statistics.

Imagine you are an engineer working for an oil company on a large field. Oil and gas are produced out of reservoirs that lay 10,000 to 15,000 feet deep below the surface. The field has been produced for several decades, and as a result the pressure in the reservoir has declined. The lower pressure difference between the deep reservoir and the surface is making it harder to extract the oil from the tiny pore spaces in the reservoir rocks. The company decides that it is time to re-pressurize the reservoir by injecting gas. It obtains carbon dioxide from a nearby power plant, and asks you to find the placement of the injection wells so that production of valuable oil is maximized. This means you have to figure out how gas and oil and any water in the reservoir will flow and interact. You use reservoir simulators that model the flow of these various liquids and the chemical behavior of their mix through rocks. The models consist of thousands and thousands of unknowns and equally many equations. The equations are all strongly coupled and nonlinear and well placement optimization requires you to take into account all the uncertainties associated with the reservoir. For example, you do not know exactly what this reservoir looks like. Your geologist colleagues have given you several hundred possible reservoir structures that you will need to assess to come up with an expected behavior of the reservoir. Simulation requires hundreds of hours on the computer, but at the end it is clear where to drill the new injection wells and the production process can start.

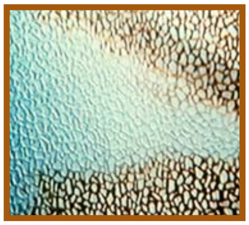

Figure 1. Carbon dioxide is injected in a "mini-reservoir" in a laboratory. Rock grains are visible as white spheres. Brown oil is seen in the pores of the rock, but where the carbon dioxide flows, the oil is no longer in the pores: it has been produced and the rock has been swept clean. This is what oil engineers hope to achieve below the subsurface also. The only possible way to assess this process is through mathematical modeling.

Figure 2. Simulation of carbon dioxide injection (from the left) into a two-dimensional reservoir on a computer. Yellow indicates 100% gas and black 100% oil. The gas flows through the rock. Since it prefers the paths of least resistance (high permeability paths) it is seen to "finger through" the domain. Where it goes, the oil is swept out of the pore space and produced (on the right).

And what about wind energy?

Mathematics is also critical in the wind energy field. Modern wind turbine blades are designed using computational fluid dynamics and structural analysis. As in oil reservoir fluid flow, the simulators are based on discretizations of partial differential equations. But placement of wind turbines also relies heavily on mathematics. Large weather models are used to find suitable locations for wind farms, and optimization strategies are applied to minimize installation and also transmission costs.

References:

General information on energy:

www.iea.org (International Energy Agency)
www.smartenergyshow.com (Margot Gerritsen's website)

Reservoir simulation:

M. Gerritsen and L. Durlofsky, Modeling fluid flow in oil reservoirs, Annual Review of Fluid Mechanics, Vol 37:211-238, 2005
www.spe.org (very large library database of papers that are accessible to everyone)

Wind energy:

MM5, community weather model: www.mmm.ucar.edu/mm5
http://firstlook.3tiergroup.com/