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CORRECTIONS TO "ON THE RIEMANN–HILBERT–BIRKHOFF INVERSE MONODROMY PROBLEM AND THE PAINLEVÉ EQUATIONS"

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In our joint paper with A. A. Bolibrukh [1] there is a technical mistake in the proof of Lemma 2. The lemma itself is correct, while its proof should be modified. First, in the definition of the norm of the space H_+ on page 142 [English page 124], $|\lambda f(\lambda, x)|$ should be replaced with $|(1+|\lambda|)f(\lambda, x)|$. Second, the part of the text containing Proposition 1 and its proof should be altered as follows.

Proposition 1. *The operators $P_\pm\big[\cdot\mathring{g}\big]$, where $\mathring{g}(\lambda,x) \equiv g(\lambda,x)-I$, are bounded operators from H to $H_\pm$.*

Proof of the proposition (an adaptation of the technique of [2]). Each ω_k can be represented as the union of the domains ω_k^+ and ω_k^-, in accordance with Figure 6. Assume that $\lambda \in \Omega_+ \setminus \bigcup_k \omega_k^+$; then

$$\frac{|\lambda|}{\operatorname{dist}}\{\lambda, \gamma_+\} \le C_\epsilon$$

for some positive constant C_ϵ. We have

$$\begin{aligned}
\lambda P_+\big[f\mathring{g}\big](\lambda,x) &= \frac{\lambda}{2\pi i}\int_{\gamma^+}\frac{f(\mu,x)\mathring{g}(\mu,x)}{\mu-\lambda}\,\mu,\\
\big|\lambda P_+\big[f\mathring{g}\big](\lambda,x)\big| &\le \frac{|\lambda|}{2\pi}\|f\|_H\|\mathring{g}\|_H\int_{\gamma^+}\frac{|d\mu|}{|\mu|^2|\mu-\lambda|}\\
&\le \frac{|\lambda|}{2\pi\operatorname{dist}\{\lambda,\gamma_+\}}\|f\|_H\|\mathring{g}\|_H\int_{\gamma^+}\frac{|d\mu|}{|\mu|^2}\\
&\le \frac{C_\epsilon}{2\pi}\|f\|_H\|\mathring{g}\|_H\int_{\gamma^+}\frac{|d\mu|}{|\mu|^2} \equiv C_+\cdot\|f\|_H.
\end{aligned}$$

Similarly, if $\lambda \in \Omega_- \setminus \bigcup_k \omega_k^-$, then

$$\big|\lambda P_-\big[f\mathring{g}\big](\lambda,x)\big| \le C_-\cdot\|f\|_H. \tag{54}$$

Suppose now that $\lambda \in \omega_k^+$. Then we can use identity (53) and rewrite $\lambda P_+\big[f\mathring{g}\big](\lambda,x)$ as

$$\lambda P_+\big[f\mathring{g}\big](\lambda,x) = \lambda f(\lambda,x)\mathring{g}(\lambda,x) - \lambda P_-\big[f\mathring{g}\big](\lambda,x). \tag{55}$$

On the other hand, $\lambda \in \omega_k^+ \implies \lambda \in \Omega_- \setminus \bigcup_k \omega_k^-$, so that we can use (54) on the right hand side of (55). Therefore (cf. [2] and the proof of Lemma A.1 in [1, Appendix 1]), we obtain

$$\big|\lambda P_+\big[f\mathring{g}\big](\lambda,x)\big| \le C'_+\cdot\|f\|_H$$

if $\lambda \in \omega_k^+$. In other words, we have the inequality

$$\big|\lambda P_+\big[f\mathring{g}\big](\lambda,x)\big| \le C''\cdot\|f\|_H, \quad C'' := \max\{C_+, C'_+\},$$

for all $(\lambda, x) \in \overline{\Omega_+} \times \mathcal{K}$. Together with the estimate

$$\begin{aligned}\big|\big(P_+^g f\big)(\lambda, x)\big| &\le \frac{1}{2\pi}\|f\|_H\|\overset{\circ}{g}\|_H \int_{\gamma^+} \frac{|d\mu|}{|\mu|^2|\mu-\lambda|}\\ &\le \frac{1}{2\pi\,\mathrm{dist}\{\lambda,\gamma_+\}}\|f\|_H\|\overset{\circ}{g}\|_H \int_{\gamma^+}\frac{|d\mu|}{|\mu|^2} \equiv \tilde{C}_+\cdot\|f\|_H, \quad |\lambda| \le \frac{\rho}{2},\end{aligned}$$

this implies

$$\big\|P_+\big[f\overset{\circ}{g}\big]\big\|_{H_+} \le C\cdot\|f\|_H, \quad C := \max\{C'', \tilde{C}_+\}.$$

Similarly,

$$\big\|P_-\big[f\overset{\circ}{g}\big]\big\|_{H_-} \le C'\cdot\|f\|_H$$

and the proposition is proved. □

The rest of the proof of Lemma 2 is unchanged. It is though worth noticing that $P_+\overset{\circ}{g} \in H_+$. This fact follows from the existence of the asymptotic expansion of the function $\overset{\circ}{g}(\lambda, x)$ over λ^{-1} as $\lambda \to \infty$, $\lambda \in \Omega_0$.

We have also noticed the following misprints in the text of the paper.

(1) In the definition of $M(\lambda, x)$ on the top of page 138 [English page 121], the matrix product $S_{k-1}\cdots S_1$ should be replaced by $S_{k-1}^{-1}\cdots S_1^{-1}$.
(2) In Figure 3, the shaded domain includes the circular hole in the middle of the picture (there should be no hole).
(3) Equation (48) on page 141 [English page 123] is, in fact, the equation for the matrix $g^{-1}(\lambda, x)$. Accordingly, in equation (49) on same page, the matrix $G_k(\lambda, x)$ should be replaced by its inverse.
(4) In Figure 7, the symbol $g_k^{-1}(\lambda, x)$ on the right part of the picture should be replaced by $g_{k-1}^{-1}(\lambda, x)$.

References

[1] A. A. Bolibruch, A. R. Its, and A. A. Kapaev, *On the Riemann–Hilbert–Birkhoff inverse monodromy problem and the Painlevé equations*, Algebra i Analiz **16** (2004), no. 1, 121–162; English transl., St. Petersburg Math. J. **16** (2005), 105–142. MR2069003 (2005d:34194)

[2] D. V. Anosov, A. A. Bolibrukh, *The Riemann–Hilbert problem*, Aspects Math., vol. E 22, Friedr. Vieweg, Braunschweig, 1994. MR1276272 (95d:32024)