

The Art of Mathematics

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In the traditional dichotomy between Art and Science, mathematics sits warily between the two. Hermann Weyl said that, in his mathematical work, he always strived after beauty and truth, and we might regard these as being the contrasting characteristics of Art and Science. Mathematicians try to understand the physical world, to unearth the secrets of nature, to search for truth. They do this by creating intellectual edifices of great subtlety and beauty, guided by their aesthetic judgement. Seen this way, mathematics links Art and Science in one great enterprise, the human attempt to make sense of the universe.

We mathematicians can appreciate this grand philosophical unification, but to the layman, unversed in our secrets, Science and Art seem diametrically opposed. Science deals with the hard facts of existence while the Arts exist only in the human mind; “beauty lies in the eye of the beholder.” Science is objective, Art is subjective, the two dwell in parallel planes and never meet.

This naïve distinction fails to grasp the nature of science. Poincaré said that science is no more a collection of facts than a house is a collection of bricks. The facts have to be ordered or structured, they have to fit a theory, a construct (often mathematical) in the human mind. The choice of a theory is a human choice; we prefer the theory which appeals to us best, the simplest or most beautiful. We employ Occam’s razor, which tells us to make the least assumptions. The success of science seems to indicate that the beauty which we humans search for in mathematical theories does capture aspects of truth, that the universe is indeed built on principles which harmonize with the human mind, that in the words of Keats: “Truth is beauty, beauty truth—that is all ye know on earth and all ye need to know.”

While poets have the insight to reach such an understanding, there are few who can see how to reconcile truth and beauty. Mathematics may be

art, but to the general public it is a black art, more akin to magic and mystery. This presents a constant challenge to the mathematical community: to explain how art fits into our subject and what we mean by beauty.

In attempting to bridge this divide I have always found that architecture is the best of the arts to compare with mathematics. The analogy between the two subjects is not hard to describe and enables abstract ideas to be exemplified by bricks and mortar, in the spirit of the Poincaré quotation I used earlier.

In architecture one finds a variety of function (from churches to railway stations), a variety of materials (from bricks to glass), and beauty at all levels (from fine detail to great vision). A mathematical theory exhibits similar variety except that the technology is now intellectual rather than physical and the beauty is a more difficult taste to acquire.

Fortunately there are many detailed ways in which art and beauty appear in mathematics, and some of these can be appreciated by the general public. The articles that follow will illustrate this by many specific examples drawn from different areas and show different facets of beauty in mathematics.

Perhaps I can end by reproducing the only poetic passage I have ever written. It is entitled “Dreams” and appears in “The Unravelers”,¹ a book produced by the IHES.

In the broad light of day mathematicians check their equations and their proofs, leaving no stone unturned in their search for rigour. But, at night, under the full moon, they dream, they float among the stars and wonder at the miracle of the heavens. They are inspired. Without dreams there is no art, no mathematics, no life.

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¹*The French original “Les Déchiffreurs” had a French version which some may prefer. Published by A. K. Peters.*