Cross-Cultural Analysis of Students with Exceptional Talent in Mathematical Problem Solving

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At a conference held in January 2005, Lawrence Summers, then president of Harvard University, hypothesized that a major reason for the paucity of women mathematicians among the tenured faculty of elite research universities in the USA might be sex-based differences in "intrinsic aptitude" for mathematics, especially at the very high end of the distribution [36]. This commonly held belief is largely based upon data from standardized tests such as the quantitative section of the Scholastic Aptitude Test (SAT) I. This test, designed to determine mathematical proficiency of USA eleventh and twelfth graders, identifies students who have mastered grade-level material, but does not distinguish the profoundly gifted, that is, those who are four or more standard deviations above the mean, from the merely gifted who also score in the ninety-ninth percentile on this exam.

To identify students who perform above grade level, the Study of Mathematically Precocious Youth (SMPY) administered the SAT I to children younger than thirteen years of age. The SMPY defined children as highly gifted in mathematics if they scored at least 700 (on a 200 to 800 scale) on the quantitative section of this test. Using this criterion, Benbow and Stanley reported in 1980 large gender differences in "mathematical reasoning ability" [4]. They concluded that "sex differences in achievement in and attitude towards mathematics result from superior male mathematical ability . . . [it] is probably an expression of a combination of both endogenous and exogenous variables."

Since these tests lack questions that require creative thinking and insight into higher-level mathematical concepts, they do not identify children with extremely high innate ability in mathematics, that is, ones who may go on to become top research mathematicians. They cannot differentiate between profoundly and moderately gifted children, regardless of age at which the examinations are administered. Thus, the SMPY identified thousands of children who, while quite bright and ambitious, were not necessarily profoundly gifted in mathematics.

The SMPY also failed to identify many children with extreme ability in mathematics who lacked one or more of the socio-economically privileged environmental factors necessary to be recognized by this mechanism. Coincidentally, the ratio of boys to girls identified in the SMPY has dramatically declined during the past quarter century from the high of 13:1 originally reported in 1983 [5] to 2.8:1 in a 2005 report [6]. The fact that 29% of Ph.D.'s awarded to USA citizens in the mathematical sciences went to women in the 2006–2007 academic year [30] supports the idea that this latter ratio is a more accurate reflection of current

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interest and ability in mathematics among USA females. This dramatic change likely reflects in part increased educational opportunities available to USA girls since enactment in 1972 of Title IX that banned sex discrimination in schools.

While the USA has been producing many more women mathematicians in recent years, they remain poorly represented among tenured professors at the very top-ranked USA research universities and people identified as profoundly gifted in the field. This article presents for the first time a comprehensive compilation of data, including cross-cultural comparisons, regarding young people identified during the past twenty years as possessing profound aptitude for mathematics based upon their performances in extremely difficult examinations in mathematical problem solving. We show that many girls exist who possess such extremely high aptitude for mathematics. The frequency with which they are identified is due, at least in part, to a variety of socio-cultural, educational, or other environmental factors that differ significantly among countries and ethnic groups and can change over time. Girls were found to be 12%-24% of the children identified as having profound mathematical ability when raised under some conditions; under others, they were 30-fold or more underrepresented. Thus, we conclude that girls with exceptional mathematical talent exist; their identification and nurturing should be substantially improved so this pool of exceptional talent is not wasted.

Methods

To identify college and high school students who possess profound intrinsic aptitude for mathematics, we compiled complete data sets from the past ten to twenty years of the top-scoring participants in the William Lowell Putnam Mathematical Competition [40], International Mathematical Olympiad (IMO) [22], and USA Mathematical Olympiad (USAMO) [39]. These competitions consist of extremely difficult problems whose solutions require the writing of rigorous proofs. The top scorers on these examinations have truly exceptional skills in mathematical problem solving, that is, at the one-in-a-million level. Since the IMO is taken by the very top mathematics students from approximately ninety-five countries throughout the world, it provides information regarding cultural differences among countries as well.

The names of the top scorers were determined from the following sources: Putnam top twenty-five, articles published annually in the *American Mathematical Monthly* that summarize the results of the Putnam; country-by-country IMO members, the medals they won and team ranks, IMO Compendium Group and IMO official websites [21], [22]; and USAMO top twelve award winners, American Mathematics Competitions (AMC) website [39]. To look at the pipeline for these exceptional USA students, we also examined data from the following sources: the participants in the Mathematical Olympiad Summer Program (MOSP), AMC website [2]; and MATHCOUNTS® top twelve, MATHCOUNTS® website [32]. Wherever not obvious from the student's name and IMO team membership, gender, birth country, ethnicity, and sources of mathematical training were confirmed, to a large extent, by consultation with members of the Putnam Mathematical Competition Committee, IMO team leaders and contestants, staff at the American Mathematics Competitions headquarters, photographs and articles accessible via the World Wide Web, referenced literature, and people with personal knowledge of the individuals. In cases of gender-ambiguous names, the student was assumed to be male unless we had definitive evidence of female identity. For many countries, our data on girl participants in the IMO is 100% accurate; for some, we estimate the numbers presented here to be at least 80% accurate, that is, we may have occasionally mis-identified someone's gender. In the tables indicating ethnicity/race, all students were assigned to the one most consistent with their family name unless we had definitive information indicating a different country of origin or bi-racial/ethnic background. Bi-racial/ethnic students were assigned to their ethnicity that is the lesser-represented one in their country unless it is a smaller percentage of their identity. We counted Jews as a separate ethnic category from other non-Hispanic whites because they have been historically highly overrepresented among mathematicians. Data on faculty in the top five graduate mathematics departments, based upon 2008 *U.S. News & World Report* rankings [38], were determined likewise from university and faculty websites; personnel who work in these departments; personal knowledge of individuals; information available on the World Wide Web; and a 2007 report on diversity among science, technology, engineering, and mathematics (STEM) faculty [27].

Our data on race/ethnicity of the USA and Canadian females is 100% accurate. We did not determine definitively the race/ethnicity of the greater than 900 USA and Canadian males. We are confident the estimates presented for Asian and ethnic Jewish males are accurate to within 90% and 70%, respectively; they are more likely under-estimates due to intermarriage and Anglicization of family names leading to some mis-assignments. Correspondingly, the data presented for "other non-Hispanic whites" are more likely over-estimates. We also may have failed to identify one or two historically underrepresented minorities, especially if he is bi- or multi-racial.

The percentages of IMO team members who were girls varied among countries. To determine whether these differences could be due to statistical fluctuation, we tested the hypothesis that the chance each position on an IMO team is filled by a girl is a fixed probability independent of country by constructing a contingency table listing the numbers of girls and boys making the IMO teams for countries with high-ranked teams during the twenty-year period 1988–2007 and performing a chi-square test.

To determine the degree of correlation between the racial/ethnic/gender distribution of very top-ranked mathematics faculty versus USA and Canadian IMO and MOSP participants, we created, using data from 2007, two lists containing the total number of faculty versus total number of IMO and MOSP participants in each of the nine ethnic/gender/birth country categories presented below in Table 7 and calculated the correlation between them.

Table 1. Women Among Top 25 in 1992-2007 Putnam, a Mathematical Competition Written Only By Students Attending Colleges in USA and Canada.

Name	Year	Birth Country (HS[1])	IMO Medals
Olena Bormashenko	2004	Russia (Canada)	1 gold, 1 silver
Ana Caraiani	2003(**F**[2]), 2004(**F**)	Romania	1 gold, 2 silver
Ioana Dumitriu	1995,1996(**F**)	Romania	-
Julie Kerr	1992	USA	-
Suehyun Kwon	2003	South Korea	1 gold
Alison Miller	2004 - 2007	USA	1 gold
Greta Panova	2001	Bulgaria	1 gold, 2 silver
Dana Pascovici	1992	Romania	-
Melanie Wood	2001, 2002(**F**)	USA	2 silver
Wai-Ling Yee	1999	Canada	-
Inna Zakharevich	2004	Russia (USA)	USAMO top 12[3]

[1]Attended high school in country of birth except where indicated otherwise.
[2]Putnam Fellow (among top 5), indicated in bold.
[3]Scored among top 12 in USAMO, but did not qualify for USA IMO team.

Table 2. Top 26 Students[1] in 2006 Putnam Mathematical Competition.

Name (alphabetical order within category)	IMO Team (Birth Country[2])	IMO Medals
Putnam Fellows (top 5)		
Hangsheng Diao	China	1 gold
Daniel Kane	USA	2 gold
Tiankai Liu	USA (China)	3 gold
Po-Ru Loh	USA	2 gold, 1 silver
Yufei Zhao	Canada (China)	1 gold, 1 silver,1 bronze
6th – 15th		
Timothy Abbott	USAMO top 12[3]	-
Ralph Furmaniak	Canada	- (2003 Putnam Fellow)
Anders Kaseorg	USA	1 gold, 1 silver
Sung-Yoon Kim	Korea	1 gold
Yuncheng Lin	China	1 gold
Alison Miller	USA	1 gold
Kevin Modzelewski	USAMO top 24[3]	-
Andrei Negut	Romania	1 gold, 2 silver
Aaron Pixton	USA	2 gold
Eric Price	USA	1 gold
16th – 26th		
Saran Ahuja	Thailand	2 bronze
Doo Sung Park	Korea	1 gold
Shinn-Yih Huang	Taiwan	1 gold,1 silver
Matthew Ince	USA	1 silver
Theodore Johnson-Freyd	USAMO top 27[3]	-
Cedric Lin	Taiwan	1 silver
Thomas Mildorf	USA	1 gold
Xuancheng Shao	China	1 gold
Andrei Ungureanu	Romania	2 gold, 1 silver
Yeo-Il Yoon	Korea	1 gold
Rumen Ivanov Zarev	Bulgaria	2 gold, 1 silver

[1]Woman indicated in bold.
[2]Birth country presumed to be same as IMO Team except where indicated otherwise in parenthesis.
[3]Scored among top 12 Award Winners or top 25 or so Honorable Mention group on USAMO, but did not qualify for 6-member USA IMO team.

Results

Putnam. The Putnam Mathematical Competition is an inter-collegiate six-hour, twelve-problem essay examination that has been given annually (with a few exceptions) since 1938 [15], [40]. In recent years it has been taken annually by 3,500–3,750 undergraduate mathematics students who are attending colleges in the USA and Canada. Most of these already highly self-selected students are unable to solve any of the twelve problems within the allotted six-hour time period, attesting to the extreme difficulty of the examination. The top twenty-five scorers usually solve five or more of the problems; the top five scorers, designated "Putnam Fellows", typically solve eight to eleven of them.

Table 1 shows the names and countries of origin of the eleven women who ranked among the top twenty-five students in the Putnam during the past sixteen years. Three of these women even achieved Putnam Fellow, proving the existence of women with this profound ability in mathematical problem solving. Interestingly, only approximately eight women ranked among the top twenty-five during the half century prior to 1992, a number that is not known exactly because gender identification was not requested of examinees prior to that year. This increase is consistent with post-Title IX changes in participation of USA women in STEM fields [9]. However, since only three of these eleven recent Putnam top twenty-five women were born in the USA, much of this increase is actually due to exceptional foreign students matriculating to colleges in the USA and Canada following the collapse of the Soviet Bloc and the opening up of China. Approximately half of recent top-ranked men were foreign-born as well [29], (Table 2). Remarkably, two of the three women Fellows were from Romania, a country with a population only one-fifteenth that of the USA. Thus, USA-born women remain highly underrepresented among top Putnam scorers.

Almost all of these foreign-born women and men who excelled in the Putnam had received instruction in mathematical problem solving prior to coming to the USA and Canada. Many of these women (Table 1) and 100% of the 2006 non-USA/Canadian-born men (Table 2) had been members of IMO teams. For example, Ana Caraiani and Suehyun Kwon had achieved gold medals as members of the Romanian and South Korean IMO teams, respectively, before matriculating to Princeton University. After training with the Russian IMO team, Olena Bormashenko immigrated to Canada where she achieved a gold medal as the top-scoring member of Canada's 2003 IMO team. Ioana Dumitriu achieved Fellow after training with, but not quite qualifying for Romania's IMO team. Almost all of the Americans had also either been members of Canadian or USA IMO teams; the few exceptions had received similar instruction at the MOSP or elsewhere (Table 3). Thus, the Putnam has become in recent years an inter-collegiate IMO-like competition in which essentially all of the top performers are not only profoundly gifted in mathematics, but also had previously obtained extensive extra-curricular training in mathematical problem solving.

Table 3. Mathematics Education Prior to College Matriculation of USA Students Among Top 26 in 2006 Putnam.

Name	Pre-undergraduate Mathematics Education[1]
Daniel Kane	MOSP, UW-Madison, UW-MTS[2], self-studied with math Ph.D. prof. parent
Tiankai Liu	MOSP, PEA[3], BAMC[4]
Po-Ru Loh	MOSP, UW-Madison, UW-MTS, self-studied with math Ph.D. prof. parent
Timothy Abbott	MOSP, TJHSST[5]
Anders Kaseorg	MOSP, UNC-Charlotte, Home-schooled
Alison Miller	MOSP, Union College, SUNY-Albany, home-schooled with math B.A. parents
Kevin Modzelewski	MOSP, PEA, self-studied with IMO team/Putnam Fellow parent
Aaron Pixton	MOSP, SUNY-Binghamton, self-studied with math Ph.D. prof. parent
Eric Price	MOSP, TJHSST
Matthew Ince	MOSP, Washington U.-St. Louis, Home-schooled
Theodore Johnson-Freyd	Canada/USA Mathcamp, U. Oregon-Eugene
Thomas Mildorf	MOSP, TJHSST

[1]Many participated as well in a variety of summer programs for mathematically gifted students.
[2]UW-Mathematics Talent Search, a correspondence competition in mathematical problem solving.
[3]PEA, Phillips Exeter Academy, a private high school with MOSP Director/USA IMO Team Leader on staff.
[4]BAMC, Bay Area Math Circle, Sunday seminars for 7th- through 12th-grade students led by former IMO Team members.
[5]TJHSSH, Thomas Jefferson HS for Science and Technology, a Virginia public high school for gifted students that offers college-level mathematics courses and a club that provides training for the AIME and USAMO.

IMO. The International Mathematical Olympiad is a pre-collegiate nine-hour, six-problem essay style examination that has been given annually (with one exception) since 1959. In recent years as many as ninety-five countries have sent six-student teams to participate. Table 4 lists the twelve countries that have ranked among the top fifteen in the world in the IMO in at least ten of the past fourteen years. All but one of these countries is located in Asia or Eastern Europe. India also used to be a member of these top-ranked countries, but was dropped from the list in 2008 due to worse-than-fifteenth rankings in 2005 through 2008. No country from Western Europe made the list. Germany was the closest, with only one year better than tenth and six years worse than fifteenth. The USA did make the list, but with approximately half of its team members being immigrants or children of immigrants from these other eleven countries plus India. China and India likely produce top-ranked teams in part because they have huge populations of students from which to identify a few with profound mathematical ability. However, population size cannot account for why a tiny country such as Bulgaria, with only 8 million people, also consistently produces a top-ranked team. Neither can gross national product nor per capita income explain why Romania readily makes the list. Rather, what most of these countries have in common are rigorous national mathematics curricula along with cultures and educational systems that value, encourage, and support students who excel in mathematics.

Table 4. Countries Ranking Among Top 15 in at Least 10 of the 1995 - 2008 IMOs.

Region of World	Team Median Rank (Range)	1st Year Participated	No. Different Girls on Team	No. Different Girls Since 1988	Medals by Girls Since 1988
Asia (6)					
China	1st (1-2)	1985	7	5	2 gold, 2 silver, 1 bronze
Iran	8.5th (1-11,17,18)	1987	2	2	2 gold, 1 silver
Japan	11th (7-15,16)	1990	1	1	1 gold, 1 silver
South Korea	6th (3-12)	1988	5	5	5 gold, 1 silver, 3 bronze
Taiwan	9th (5-14,16,20)	1992	5	5	2 silver, 2 bronze
Vietnam	6th (3-15)	1974	6	4	1 silver, 3 bronze
Eastern Europe (5)					
Bulgaria	5.5th (1-14,21)	1959	21	6	1 gold, 9 silver, 1 bronze
Hungary	9.5th (2-12,16,17,21)	1959	7	4	1 gold, 4 silver, 2 bronze
Romania	7.5th (4-11,17)	1959	5	3	3 gold, 4 silver
Russia (USSR)	2.5rd (1-6)	1992 (1959)	15	11	6 gold, 8 silver, 1 bronze
Ukraine	12.5th (6-15,18,20,21,23)	1992	6	6	3 gold, 3 bronze
Americas (1)					
USA[1]	3rd (2-11)	1974	3	3	2 gold, 3 silver
Rest of World					
None					

[1]~ 50% of recent USA IMO Team members were 1st- or 2nd-generation immigrants from these other Asian and Eastern European countries or India, with the latter a member of this list until 2008.

Table 4 also summarizes participation in the IMO of girls from these top-ranked countries. The number of medals they won is typically greater than the number of girls because some of these girls participated more than once. Gold medals are awarded to the top-scoring one-twelfth of these outstanding students; silver medals to the next one-sixth; bronze medals to the next one-third. Chenchang Zhu of China, Maryam Mirzakhani of Iran, and Evgenia Malinnikova of Russia are among the girls who achieved perfect scores, a feat typically accomplished by only a few students per year. Three girls, Maria Ilyukhina of Russia, Sherry Gong of the USA, and Livia Alexandra Ilie of Romania, ranked among the top eleven in the 2007 IMO, a year in which none of the 522 participants obtained a perfect score. Thus, numerous girls exist who possess truly profound ability in mathematical problem solving.

The frequency with which girls are members of top-ranked IMO teams varies considerably from country to country (Table 4). For example, Bulgaria's teams have often included girls, beginning with the very first IMO competition held in 1959 (Table 5). One of them, Greta Panova, attended college at MIT where she also ranked among the top twenty in the Putnam. Russia's teams have also frequently included girls (Table 5), accounting for 20% of their members during the decade from 1988–1997 (Table 6). In contrast, Japan's teams have only included one girl, the gold-medal top-scoring member of their 1996 team, in the nineteen years in which they have participated. The USA's teams were girl-less throughout their first twenty-three competitions, with Melanie Wood finally making its 1998 team. USA IMO team members Melanie

Table 5. Girl Participants in IMO from Some Top 20-Ranked Countries: Year(s) Participated.

Bulgarian Team - 21 girls
- **Tsvetana Penkovska**: 1959
- **Nedka Ivanova**: 1959
- **Brezitsa Popova**: 1960
- **Atanaska Stoianova**: 1961
- **Rumiana Proikova**: 1961
- **Milka Popova**: 1961
- **Milena Ivanova**: 1962
- **Lidia Vashtinska**: 1965
- **Ludmila Krasteva**: 1966
- **Diana Petkova**: 1967
- **Donka Pashkuleva**: 1969
- **Virdzhinia Hristova**: 1969,1970
- **Nadezhda Ribaska**: 1977,1978
- **Milena Moskova**: 1978
- **Galia Simeonova**: 1983
- **Zvezdelina Stankova**: 1987,1988
- **Dessislava Bakardzhieva**: 1987,1988
- **Ludmila Kamenova**: 1996
- **Greta Panova**: 1999, 2000, 2001
- **Tsvetelina Tseneva**: 2005, 2006, 2007
- **Elina Robeva**: 2006, 2007

USA Team - 3 girls (1974-2008)
- **Melanie Wood**: 1998,1999
- **Alison Miller**: 2004
- **Sherry Gong**: 2005, 2007 (also 2002, 2003, 2004 on Puerto Rico's Team)

Japanese Team – 1 girl (1990-2008)
- **Sachiko Nakajima**: 1996,1997

East German/German Team – 19 girls
- **Ingrid Seidel**: 1959
- **Irene Neumann** : 1959
- **Elke Genz**: 1959
- **Mary Schleifstein**: 1961
- **Heike Wenzel**: 1961
- **Katharina Görke**: 1962
- **Ilona Zinke**: 1964
- **Monika Titze**: 1964,1965
- **Ursula Tyl**: 1970
- **Karin Gröger**: 1984
- **Angelika Drauschke**: 1985
- **Uta Hövel**: 1987
- **Ingrid Voigt**: 1990
- **Astrid Mirle**: 1990
- **Anja Wille**: 1994
- **Eva Mierendorff**: 1998
- **Annika Heckel**: 2004
- **Lisa Sauermann**: 2007, 2008
- **Jessica Fintzen**: 2008

Canadian Team - 9 girls (1981-2008)
- **Lily Yen**: 1984
- **Etsuko Amano**: 1990
- **Alyssa Ker**: 1994,1995
- **Mihaela Enachescu**: 1997,1998
- **Yin (Jessie) Lei**: 1998,1999
- **Denise Cheung**: 2001
- **Olena Bormashenko**: 2002, 2003
- **Viktoriya Krakovna**: 2006
- **Yan Li**: 2007, 2008

USSR/Russian Team – 15 girls
- **Lidia Goncharova**: 1962
- **Elena Neklyudova**: 1969
- **Tatyana Khovanova**: 1975, 1976
- **Olga Leonteva**: 1985
- **Maria Roginskaya**: 1989
- **Evgenia Malinnikova**: 1989,1990,1991
- **Yulia Pevtsova**: 1992
- **Elizaveta Rosenblum**: 1993
- **Anna Dyubina**: 1994
- **Natalia Dobrinskaya**: 1994
- **Veronika Essaoulova**: 1995, 1996
- **Elena Roudo**: 1996
- **Irina Anno**: 1997, 1998
- **Nadejda Petukhova**: 2004
- **Maria Ilyukhina**: 2007

United Kingdom Team – 11 girls (1967-2008)
- **Alison McDonald**: 1983
- **Katherine Christie**: 1989
- **Eva Myers**: 1992
- **Karen Page**: 1992
- **Catriona McClean**: 1993,1994
- **Louisa Orton**: 1995
- **Rebecca Palmer**: 1999
- **Hannah Burton**: 2001
- **Jennifer Gardner**: 2002, 2003
- **Anne Marie Shepherd**: 2004
- **Alison Zhu**: 2007, 2008

West German Team – 0 girls (1977-1990)

Table 6. Percent Girl Participants in IMO Over Time Among Top 34 Countries[1]

Country	Median Team Rank 1995-2008	% of World's Population	1st Year Participated	IMO Participants Girls/Total (% Girls) pre-1988	1988-1997	1998-2008
Asia						
China	1	20	1985	2/18 (11)	4/60 (7)	1/60 (2)
India	14.5	17	1989	-	0/54 (0)	4/66 (6)
Iran	8.5	1.1	1987	0/7 (0)	3/60 (5)	0/66 (0)
Israel	21.5	0.11	1979	0/36 (0)	0/60 (0)	3/66 (5)
Japan	11	1.9	1990	-	2/48 (4)	0/66 (0)
Kazakhstan	27	0.23	1992	-	0/30 (0)	1/66 (2)
South Korea	6	0.72	1988	-	2/60 (3)	7/66 (11)
Taiwan	9	0.34	1992	-	3/36 (8)	2/66 (3)
Turkey	17.5	1.1	1985	0/8 (0)	3/60 (5)	2/66 (3)
Vietnam	6	1.3	1974	2/55[2](4)	2/60 (3)	2/66 (3)
Europe						
Belarus	18.5	0.15	1992	-	1/31 (3)	5/66 (8)
Bulgaria	5.5	0.11	1959	19/192 (10)	3/60 (5)	8/66 (12)
Czechoslovakia	-	-	1959	4/92[2](4)	0/30 (0)	-
Czech Republic	34	0.16	1993	-	2/30 (7)	2/66 (3)
Slovakia	28	0.08	1993	-	2/30 (7)	10/66 (15)
France	31	0.96	1967	1/62[2](2)	0/60 (0)	4/66 (6)
Germany	15	1.2	1991	-	1/42 (2)	5/66 (8)
East Germany	7('84-'90)	-	1959	13/184 (7)	2/18 (11)	-
West Germany	7('84-'90)	-	1977	0/66 (0)	0/18 (0)	-
Hungary	9.5	0.15	1959	3/200 (2)	2/60 (3)	5/66 (8)
Poland	19	0.57	1959	6/191[2](3)	1/59 (2)	0/66 (0)
Romania	7.5	0.32	1959	2/107[2](2)	0/60 (0)	7/66 (11)
Russia (USSR)	2.5	2.1	1959	5/180 (3)	12/60 (20)	3/66 (5)
Serbia & Montenegro	21.5	0.16	1992	-	2/24 (8)	17/72 (24)
Ukraine	12.5	0.69	1992	-	1/35 (3)	6/66 (9)
United Kingdom	19.5	0.91	1967	1/146 (0.7)	6/60 (10)	7/66 (11)
Other						
Australia	22.5	0.32	1981	2/42 (5)	2/60 (3)	5/66 (8)
Brazil	26	2.8	1979	1/44[2](3)	2/60 (3)	3/66 (5)
Canada	19.5	0.5	1981	1/40 (3)	4/60 (7)	9/66 (14)
USA	3	4.5	1974	0/90 (0)	0/60 (0)	5/66 (8)

[1]Data not determined for Hong Kong and Singapore, other countries with median team ranks among the top 34.
[2]Data for pre-1988 years are incomplete.

Wood and Alison Miller also ranked among the Putnam top fifteen. With the Bulgarian, Russian, and USA teams as a whole and their girl members as individuals performing at similar levels, these differences in girl participation rate cannot be due to country-specific differences in difficulty in girls qualifying for the IMO. Thus, some countries routinely identify and nurture both boys and girls with profound mathematical ability to become world-class mathematical problem solvers; others, including the USA, only rarely identify girls of this caliber.

Bulgaria and Russia are not alone in producing girls with IMO medal-level ability in mathematical problem solving. South Korea, Taiwan, and the Ukraine, despite only participating in the IMO since 1988, 1992, and 1992, respectively, have already had five, five, and six different girls on their teams, respectively (Table 4). Romania has only had five girls on its teams since 1959; however, two additional Romanian girls who did not quite qualify for their country's IMO teams achieved Putnam top twenty-five, one even as Fellow (Table 1). Top-ranked (Table 4) and high-ranked (Table 6) IMO teams are clustered predominantly in Eastern Europe and Eastern Asia, but even neighboring countries can show marked disparities in whether they successfully identify and nurture their mathematically very gifted girls. Some examples are East Germany vs. West Germany, Slovakia vs. the Czech Republic, and South Korea vs. Japan. Chi-square analysis indicated it is virtually impossible that this observed country-to-country fluctuation was caused by random chance (statistic = 98.396; df = 25; p-value = <0.0001). Thus, the probability that a position on a team was filled by a girl varied from country to country. These findings suggest that socio-cultural, educational, or other environmental factors that differ among countries are significant determinants of whether (i) students are produced with exceptional skills in mathematical problem solving, and (ii) girls are frequently included among them.

These factors can change over time within a country. Like the USA, the United Kingdom IMO teams were almost girl-less from 1967 through 1988; however, they have included ten individual girls representing 11% of their participants during the past two decades (Tables 5 and 6). France's IMO teams were also almost girl-less for almost four decades

Table 7. Race/Ethnicity of USA and Canadian IMO Participants, MOSP Participants, and Top 5-Ranked Math Department Faculty

	Male				Female		Other Non-Hispanic White		
	Asian[1]	Jewish	Other Non-Hispanic White[2]	URM[3]	Asian	Jewish	European-born	Canadian- and USA-born	URM
USA IMO Team 1988-2007									
No. Students (%)	40 (33)	~25 (21)	~50 (42)	0 (0)	2 (2)	1 (0.8)	0 (0)	2 (2)	0
% Population[4]	1.5-2.3	~1.0	41-38	~12-15	1.4-2.2	~1.0	0.6[5]	38-35	~11-14
%/% Population	18	~21	~1.1	0 (0)	0.9	~0.8	0	0.05	0
Canada IMO Team 1988-2007									
No. Students (%)	60 (50)	~12 (10)	~36 (30)	0 (0)	5 (4)	0 (0)	5 (4)	2 (2)	0
% Population	5	~0.6	~43	~7	5	~0.6	~2.0[5]	~39	~7
%/% Population	10	~17	~0.7	0 (0)	0.8	0	2	~0.04	0
MOSP 1998-2007									
No. Students (%)	216 (42)	~56 (11)	~209 (40)	2 (0.4)	22 (4)	5 (1)	3 (0.6)	5 (1)	0
%/% Population	20	~11	~1.0	~0.03	2.1	~1	1	0.03	0
Profs. Top 5 USA Math Depts.[6,7,8]									
No. Tenured (%)	18 (10)	~33 (18)	~120 (67)	1 (0.6)	1 (0.6)	2 (1)	4 (2)	1 (0.6)	0 (0)
No. Jr. Faculty (%)	7 (13)	~5 (9)	~30 (55)	1 (2)	2 (4)	0 (0)	7 (13)	3 (6)	0 (0)
%/% Population	6	~16	~1.7	~0.06	0.7	~0.9	3	0.03	0

[1]Includes West Asians, Middle Easterners, Pacific Islanders, and bi-racial Asian-whites as well as East Asians.
[2]Includes Jews and Hispanics unidentified due to intermarriages or Anglicization of family names.
[3]Underrepresented minorities were Hispanic; no African- or Native-Americans were identified. Some underrepresented minorities may have been missed.
[4]Range indicates approximate change in demographics between 1988 and 2007, with girls comprising 48% of high school-age children and 50% of working-age adults.
[5]Percentage of high school-age population; 1.8% of total US population.
[6]Data determined from [**27**] and web sites for mathematics departments at Harvard, MIT, Princeton, Stanford, and UC-Berkeley; excluded faculty whose primary appointment was in mathematics education or other department.
[7]Birth countries of male faculty hired since July, 1997 were Belgium, Bulgaria, 4xCanada, 4xChina, 5xDenmark, 3xFrance, 3xGermany, Hong Kong, 2xHungary, 2xIndia, 3xIsrael, Italy, New Zealand, Poland, Portugal, 2xRomania, 7xRussia, Slovakia, Sweden, Switzerland, Taiwan, 2xUnited Kingdom, and 29xUSA, with the latter consisting of 5 Asians and 24 non-Hispanic whites (including Jews).
[8]Birth countries of female faculty were Belgium, 2xCanada, China, 3xGermany, Iran, Italy, 2xRomania, 2xRussia, Serbia, Switzerland, USSR, and 4xUSA, with the latter consisting of 1 Asian/white, 1 Jew, and 2 non-Hispanic whites.

before including four individual girls in the past four years. On the other hand, participation by girls on the USSR/Russian teams has dropped off significantly from nine individual girls during the 1988–1997 decade to only two new girls during the most recent 1998–2008 eleven-year period. Likewise, China's teams included six individual girls, two of whom had perfect scores, during their first twelve years of participation, but only one new girl since 1996. It is improbable that these fairly large changes in girl participation rate were merely due to statistical fluctuation. In some cases, they could have been due to the country's team leader altering the method used to select the team members, for example, from simply asking one's friends to suggest candidates to performing an open, nationwide search. In other cases, they probably reflect socio-cultural, educational, economic, or other environmental changes that have occurred within the country.

One can observe effects of culture by looking at IMO participation rates among children from different ethnicities raised within the same country. Because the number of USA IMO girls is too small to draw any significant conclusions, we also examined the effect of race/ethnicity on IMO participation rate among Canadian girls and USA and Canadian boys (Table 7). While native-born white girls were found to be very highly underrepresented in proportion to their percentages of their country's population, ethnic Asian girls were not underrepresented. White girls who were immigrants from the top-ranked IMO countries such as Romania, Russia, and the Ukraine were also not underrepresented. Analysis of the USA and Canadian boy participants led to a similar conclusion: Asian and ethnic Jewish boys were approximately ten- to twenty-fold more likely to become IMO participants than other non-Hispanic white boys. Historically underrepresented minorities were not identified among the 240 USA and Canadian IMO participants. Analysis of MOSP participants from the past decade, a constituency of over 500 students, led to remarkably similar findings (Table 7). Taken together, these data indicate that the scarcity of USA and Canadian girl IMO participants is probably due, in significant part, to socio-cultural and other environmental factors, not race or gender *per se*. These factors likely inhibit native-born white and historically underrepresented minority girls with exceptional mathematical talent from being identified and nurtured to excel in mathematics. Assuming environmental factors inhibit most mathematically gifted girls being raised in most cultures in most

Table 8. Birth Country[1]/Ethnicity/Race and Gender[2] of USAMO Award Winners, an Exam Taken Only by 7th-13th Graders Attending Schools in USA and Canada.

2006 Competition		2005 Competition	
Yakov Berchenko-Kogan	Russia-Jewish	Robert Cordwell	White
Sherry Gong	Chinese-American	Zhou Fan	China
Yi Han	China	**Sherry Gong**	Chinese-American
Brian Lawrence	Jewish/White[3]	Rishi Gupta	Indian-American
Tedrick Leung	Chinese-American	Hyun Soo Kim	Korea
Richard McCutchen	White	Brian Lawrence	Jewish/White
Taehyeon Ko	Korea	Albert Ni	Chinese-American
Peng Shi	China (Canada)	Natee Pitiwan	Thailand
Yi Sun	China	Eric Price	White
Arnav Tripathy	Indian-American	Peng Shi	China (Canada)
Alex Zhai	China	Yi Sun	China
Yufei Zhao	China (Canada)	Yufei Zhao	China (Canada)

[1]Resided in the USA at the time of the competition unless indicated otherwise in parenthesis.
[2]Female indicated in bold.
[3]Bi-ethnic.

countries at most times from pursuing mathematics to the best of their ability, we estimate the *lower* bound on the percentage of children with IMO medal-level mathematical talent who are girls to be in the 12%–24% range, that is, the higher of the percentages observed in the data presented in Table 6. In a gender-neutral society, the real percentage could be significantly higher; however, we currently lack ways to measure it.

We also determined likewise these percentages among tenured and junior faculty in the five highest ranked USA graduate mathematics departments (Table 7). A high correlation (0.727; p = 0.026) was found between the race/ethnicity/birth country/gender distributions for IMO and MOSP participants versus these outstanding research mathematics faculty. The only significant difference was somewhat fewer Asians, consistent with USA-citizen Asians being only slightly overrepresented among recent USA mathematics Ph.D. recipients [18], [30]. Noteworthy is the fact that only 20% of the tenured and junior female faculty in these departments, most of whom were fairly recent hires, had been born in the USA. Thirty-eight percent of their male faculty counterparts hired during the past decade were USA born. Many of the foreign-born women (see Footnote 8 of Table 7) are immigrants from countries in which girls are frequently members of IMO teams (Table 6) and women make up 20%–40% of the tenured instructors of mathematics at the university level [11]. Thus, we conclude that the mathematics faculty being hired by these very highest-ranked research universities reflects the pool of IMO medal-caliber students of mathematics coming through the pipeline. If a goal is to have research university faculty who are representative of the diversity of the USA population, we need to increase greatly both the diversity and number of USA-born students in this particular pool.

USAMO. Why is the USA not among the countries with a good record of female participation in the IMO? To begin to answer this question, we consider data from the primary competition and training camp that leads up to selection for membership on the USA IMO Team. Table 8 lists the high school students who were the 2005 and 2006 USAMO award winners, that is, the top twelve scorers, in the USA Mathematical Olympiad [39], the first of two IMO-like examinations used to select the six members of the USA IMO Team. The 250–525 students invited each year to write this examination are seventh through twelfth graders attending schools in the USA and Canada who ranked among the top scorers on two prior qualifying exams, the AMC10/12 and the American Invitational Mathematics Examination (AIME). Only one girl, Sherry Gong, a five-time IMO participant, made the list, doing so in both 2005 and 2006 and, again, in 2007. Also notable is the paucity of USA- and Canadian-born, non-Jewish white boys; they accounted for only two, one, and, one of the twelve students in 2005, 2006, and 2007, respectively. In fact, half of the 2005 and 2006 USAMO award winners were born in other top-ranked IMO countries.

The results of the 2004 USAMO were similar [1], [39]. Of the top twenty-four scorers, three were girls: Alison Miller (½ Jewish, home-schooled), Po-Ling Loh (Chinese-American child of immigrants), and Elena Udovina (Russian emigrant). The twenty-one boys were three immigrants to Canada (Chinese, Russian-Israeli Jew, Eastern European), a Thai (member of Thailand's 2003 IMO Team), eleven USA students who were immigrants or children of immigrants from China, South Korea, Russia, or India, and six USA-born whites, two of whom were home-schooled. Thus, USA-born white boys as well as girls are underrepresented among the top scorers on the USAMO; historically underrepresented minorities are almost non-existent. Except for the three home-schooled students, all were attending pre-collegiate schools in the USA or Canada at the time they took the USAMO. Thus, parental, sociocultural or other environmental factors, even more so than educational systems, play a major role in determining who excels in this examination.

MOSP. Very few USA high schools teach the advanced mathematical skills, such as writing rigorous essay-style proofs, needed to excel in the USAMO. Most USAMO award winners acquire this knowledge by a combination of self-study and participation in summer mathematics camps. The premier mathematics training camp in the USA is the Mathematical Olympiad Summer Program, which all USA IMO team members are required to attend prior to participation in that summer's IMO. Admission to MOSP is extremely selective, traditionally restricted to the top twenty-four to thirty pre-college students in the USA based upon their scores in the USAMO, AIME, and AMC10/12. Typically, one-to-three of the thirty students qualifying for the MOSP each year by these criteria are female [2]. With generous support from the Akamai

Foundation, MOSP has been expanded in recent years to include some ninth graders admitted by separate criteria. In 2002, the first year in which additional ninth graders were admitted, there were 162 mostly ninth-grade participants. Eleven of them were girls, with eight of them being ethnic Asian (Table 9). Of the three non-Asians, one was born in Russia, one was home-schooled, and one lived in Lexington, Massachusetts, a town highly populated by Boston-area academics. Less than one percent of the MOSP 2002 participants were USA-born, public school-educated, white girls; none of the girls was a historically underrepresented minority.

The data were similar for MOSPs 2003–2007 where girls were three out of thirty, six out of fifty-five, four out of fifty-five, two out of fifty-five, and three out of fifty-five of the participants, respectively (Table 9). MOSP 2008 included two ninth-grade girls out of the forty-eight participants plus ten additional girls who qualified to train for the China-sponsored Girls' Mathematical Olympiad (CGMO). Again, most of these girls were ethnic Asian. The three non-Asian girls attended special high schools for gifted students or a public high school highly populated by children of Chicago-area academics. One had been home-schooled prior to ninth grade. Even the USA-born Asian girls were mostly children of immigrants. Only 4.5% of the USA population is currently classified as Asian. Thus, by ninth grade, Asian-American girls and boys were 2.1- and twenty-fold overrepresented in proportion to their percentages of the USA population, respectively, among USA students who excelled at the highest level in these mathematics competitions (Table 7). Jewish boys and Eastern European-born girls were also overrepresented. USA-born, non-Jewish, non-Hispanic white boys were still represented in ninth grade in proportion to their percentage of the USA population, but USA-born white girls and historically underrepresented minorities were already almost non-existent. Of the thirteen girls who have represented the USA in the 2007 or 2008 CGMO based upon their being among the top eight or ten girl scorers in the 2006 and 2008 USAMO, respectively, all but one, Jennifer Iglesias, were Chinese- or Korean-American (Table 9). Socio-cultural factors, including parental expectations and beliefs (for example, see [35]), likely account for why Chinese-American girls are overrepresented among this group of mathematics students while USA-born non-Asian girls with exceptional mathematical talent are rarely identified.

Table 9. Race/Ethnicity/Birth Country of USA Girl Participants in 1998-2008 MOSP or 2007 China Girls' Mathematical Olympiad.

Name	Race/Ethnicity/ Birth Country[1]	Years at MOSP	USAMO Award Winner (Honorable Mention) *(2006,2008 Top Girl[2])*
Melanie Wood	White	1996-1999	1998,1999
Shuang You	China	2000	-
Alison Miller	Jewish/White[3]	2000-2004	2002, (2003), 2004
Inna Zakharevich	Russia	2001-2002	2002
Po-Ling Loh	Chinese-American	2002-2004	2003, (2004)
Sarah Cheng	China	2002	-
Ann Chi	Chinese-American	2002	-
Atoshi Chowdhury	Indian-American	2002	-
Rong Hu	Chinese-American	2002	-
Tiffany Ko	Chinese-American[4]	2002	-
Elizabeth Marcil	White	2002	-
Irena Wang	China	2002	-
Stephanie Zhang	Chinese-American	2002	-
Sherry Gong	Chinese-American	2003-2005, 2007[5]	(2003), 2005, 2006, 2007
Linda Liu	China	2004	-
Maria Monks	White	2004-2005	-
Livia Zarnescu	Romania	2004	-
Sway Chen	Chinese-American	2005	*(2006)*
Wendy Hou	Chinese-American	2005	*(2006)*
Patricia Li	Chinese-American	2006[6]	*(2006,2008)*
Jennifer Iglesias	White/Hispanic[3]	2008	*(2006,2008)*
Colleen Lee	Chinese-American	2008	*(2006,2008)*
Marianna Mao	Chinese-American	-[6]	*(2006,2008)*
Wendy Mu	Chinese-American	2008	*(2006,2008)*
Iris Xu	Canada/Chinese	2007,2008	*(2008)*
Lynnelle Ye	Chinese-American	2007,2008	*(2008)*
In Young Cho	Korea	2008	*(2008)*
Qian Huang	China	2008	*(2008)*
Ning (Jenny) Jin	China	2008	*(2008)*
Carolyn Kim	Korean-American	2008	*(2008)*
Katherine Rudolph	White	2008	*(2008)*
Elizabeth Synge	White[4]	2008	*(2008)*
Shijie (Joy) Zheng	Australia/Chinese	2008	*(2008)*

[1]USA born unless indicated otherwise.
[2]Qualified for 2007 China Girls' Mathematics Olympiad by ranking among top 8 pre-12th grade US citizen/permanent resident girls in 2006 USAMO or for MOSP 2008 by ranking among top 12 girls in 2008 USAMO.
[3]Bi-ethnic.
[4]Birth country not determined.
[5]Also qualified for MOSP 2006, but participated in Intl. Physics Olympiad, instead.
[6]Qualified for MOSP 2008, but participated in Research Science Institute, instead.

While Asian girls were 2.1-fold overrepresented in proportion to their percentage of the USA population overall, they were only 9.3% of the Asian participants at the MOSP during 1998-2007. Since Asian girls growing up in the USA also hear the messages "girls can't excel at math" and "only nerds enjoy math," this number, too, is only a lower bound on the percentage of children with very high intrinsic aptitude for mathematics who are girls.

AwesomeMath Summer Program (AMSP). Several summer mathematics camps aimed at exposing children to numerous areas of pre-calculus mathematics not typically taught in schools have been started in the USA during the past decade. AwesomeMath is one such program, begun in 2006 for seventh- through twelfth-graders who can demonstrate a high-level interest in learning areas of mathematics needed to perform well in the AIME and USAMO. It is run by the same instructors who direct the MOSP and coach the USA IMO Team. Of the ninety-three USA students who participated in the camp in 2006, fourteen (15%) were girls. Eleven of these girls were Asian-American, with family names originating from China, India, Korea, and Vietnam. Sixty-eight percent of the boys attending AMSP 2006 were also Asian-American. The data for 2007 were quite similar: of the 112 USA participants (excluding the seven who attended to train for the CGMO),

Table 10. Race/Ethnicity and Gender[1] of Top 12 Students in National MATHCOUNTS Competition

2006 Ranking		2005 Ranking	
Neal Wu	- Chinese	Neal Wu	- Chinese
Daniel Li	- Chinese	Mark Zhang	- Chinese
Kevin Chen	- Chinese	**Patricia Li**	- Chinese
Nathan Benjamin	- Chinese/Jewish[2]	**Karlanna Lewis**	- White
Daesun Yim	- Korean	Sergei Bernstein	- Russian/Jewish[2]
Sam Keller	- White	Nathan Benjamin	- Chinese/Jewish[2]
Brian Hamrick	- Asian/White[2]	David Benjamin	- Chinese/Jewish[2]
George Silvis	- White	Mike Jin	- Chinese
Rolland Wu	- Chinese	Andrew Ardito	- White
Andrew Ardito	- White	Alan Huang	- Chinese
George Yu	- Chinese	Pardha Ponugoti	- Indian
Arjun Puranik	- Indian	Kiran Kota	- Indian

[1]Girls indicated in bold.
[2]Bi-racial.

71% and 12% were Asian-American boys and girls, respectively, while 4% were non-Asian girls. Thus, the overrepresentation of Asians is even greater at AMSP than it is at the MOSP, while non-Asian girls are already highly underrepresented.

MATHCOUNTS®. At what age are USA-born non-Asian students with the potential to excel in mathematics being lost from the pipeline? MATHCOUNTS® is a national mathematics enrichment, coaching, and competition program that promotes high-level achievement in middle school mathematics. Thousands of middle school teachers throughout all fifty states as well as the USA territories use the MATHCOUNTS curriculum materials to supplement classroom materials or as an extracurricular activity. Students can compete individually or as part of a team in one of more than 500 competitions held nationwide each year. Top-performing students advance to the State Competition, with the top four students from each state and U.S. territory advancing to the National Competition. If bi-racial Asian/white students are counted as Asian, the top twelve scorers at Nationals included at most one, three, and four USA-born, non-Asian boys and two (including one Asian), zero, and one girls in 2005, 2006, and 2007, respectively [32]; (Table 10). Thus, USA-born white boys as well as girls and historically underrepresented minorities with high potential to excel in mathematics are already failing to be identified and nurtured in middle school.

B.A.'s and Ph.D.'s in Mathematics. In the USA, women received 48% of the bachelor degrees awarded in mathematics in 2000, with 8% and 3% of these women being Asians and non-resident aliens, respectively [18]. In 2007, women received 29% of the mathematics Ph.D.'s awarded to USA citizens, with ethnic Asians comprising only 6% of these women and 5% of the USA citizen men [30]. Thus, the overrepresentation of Asians and underrepresentation of women in mathematics among USA citizens are at most twofold at the bachelor and doctoral levels. However, USA citizens received only 43% of the mathematics Ph.D.'s awarded by USA universities in 2007. Of the Ph.D.'s awarded in mathematics by USA universities in 1993–2002, women received 27% of them, with only 32% of these women being non-resident aliens [19]. Thus, many women exist in the USA with both the interest and ability to master college- and graduate-level mathematics who are neither Asian-American nor foreign. Some of them likely possessed the intrinsic aptitude to excel in the AIME, USAMO, IMO and Putnam, but were not identified, encouraged, and nurtured to do so.

Social Stigma Associated with Mathematics. Many USA-born white and historically underrepresented minorities who are gifted in mathematics do not participate in MATHCOUNTS®, AMC examinations, or, even, school mathematics clubs and teams. When asked why, a typical response is, "Only Asians and nerds do math (extracurricularly)." In other words, it is deemed uncool within the social context of USA middle and high schools to do mathematics for fun; doing so can lead to social ostracism. Consequently, gifted girls, even more so than boys, usually camouflage their mathematical talent to fit in well with their peers. This peer group social problem has been noted in interviews with top Putnam students [29] and USA female Olympians [13], [28]. The overwhelming preponderance of foreign-born and Asian-American students in high school mathematics clubs is a nationwide phenomenon (for example, see [41]). Almost all of the girls who have achieved USAMO Award Winner or Honorable Mention (that is, top twenty-five) in this examination's thirty-five-year history were foreign-born, Asian-American, or home-schooled. Thus, we hypothesize that the extreme scarcity of USA-born non-Asian girls among the top scorers in the AIME, USAMO, and Putnam is not due to a lack of girls with profound intrinsic aptitude for mathematics; rather, it is due to their choosing to spend their free time on nonmathematical pursuits. The *substantial* overrepresentation of Asian-American and foreign-born boys indicates that USA-born non-Asian boys are also being adversely affected by the social stigma associated with doing mathematics, although not to the extreme extent it is affecting girls. Likely, some boys feel comfortable doing mathematics for fun because they are either less socially astute or less concerned about their social status than are most girls.

Discussion

One commonly held belief to explain the extreme scarcity of females who excel at the highest level in mathematics is that women simply lack sufficient aptitude for the field (for examples, see [25], [31], [36]). The data presented here neither prove nor disprove whether the frequency of occurrence of people with profound intrinsic aptitude for mathematics differs between women and men. What they do indicate, however, is that this scarcity is due, in significant part, to changeable factors that vary with time, country, and ethnic group. First and foremost, some countries identify and nurture females with very high ability in mathematics at a much higher frequency than do others. This phenomenon is observed, not only in the degree to which girls participate in the IMO (Tables 4, 5 and 6; see also [23]), but also in the representation of women among students earning Ph.D.'s in the mathematical sciences [10] and among tenured mathematics faculty employed at universities [10], [11], with fairly high positive correlations existing among these variables. A strong correlation also exists between the magnitude of measured gender difference in mathematics performance by eighth and tenth graders in a country and other measures of gender stratification such as participation in the labor force and politics [3], [16]. Second, girls perform as well if not better than boys in mathematics throughout elementary school; it is during the middle school years, an age when children begin to feel pressure to conform to peer and societal expectations, that they start to lose interest and fall behind in most, but not all countries [37], (see also [17], [20], [34] for comprehensive recent reviews on the general topic of girls and math). In some of the most gender-equal cultures, a gender gap is not observed in mathematics among fifteen-year-old students on the Programme for International Student Assessment, not only with respect to median score, but also in the ratio of girls to boys performing above the ninety-fifth and ninety-ninth percentiles [16]. Third, Asian girls and white girls who are immigrants from Eastern Europe are well represented among the very top students identified in the extremely difficult mathematics competitions discussed here; it is only USA-born white and historically underrepresented minority girls who are underrepresented, underrepresented by almost two orders-of-magnitude relative to Asian girls educated in the same school systems (Table 7). Fourth, the scarcity of females is much less pronounced in the sciences and engineering, fields that depend upon a solid understanding of mathematics. Their percentages in these other STEM fields have been steadily increasing post-Title IX [9]. In 2007 girls accounted for twenty of the forty Intel Science Talent Search finalists, including six of the top ten. However, as with the USAMO Award Winners, a survey of the 2004 Intel Finalists found that 60% of them were children of immigrants [1]. Girls accounted for seven of the sixteen 2006 Fellows of the Davidson Institute for Talent Development [12], (Table 11), a foundation that provides scholarships to USA children identified, without regard to gender, as profoundly gifted in mathematics, science, technology, literature, philosophy, or music. Again, most of the Davidson Fellows in the STEM fields were children of immigrants or Asian-American, a phenomenon consistently observed since these fellowships were first awarded in 2001. On the other hand, the Fellows in literature and music were mostly white. Profoundly gifted children are frequently multi-talented, for example, prodigious in both music and mathematics; they usually invest more of their effort in the fields that provide more positive feedback. Thus, socio-cultural factors strongly influence the fields in which profoundly gifted children are identified.

Table 11. Race/Ethnicity and Gender[1] of Davidson Institute for Talent Development 2006 Fellows by Category

Math, Science or Technology		Music or Literature	
Sheela Krishnan	Indian	**Stephanie Chen**	Chinese
Varun Kumar	Indian	Kyle Dacuyan	White/Philippino[2]
Albert Shieh	Chinese	**Heather Engebretson**	White
Adam Solomon	Jewish	Travis Johnson	White
Shivani Sud	Indian	Drew Peterson	White
Yi Sun	Chinese	**Anna Stalker**	White
Anarghya Vardhan	Indian		
Michael Viscardi	Korean/White[2]		
Xin Wang	Chinese		
Steven Wu	Chinese		

[1]Females indicated in bold.
[2]Bi-racial.

The skill sets necessary to excel in mathematical problem solving and mathematics research are not identical. Research requires the stamina to work on problems over extended periods of time without knowing whether solutions even exist; the competitions discussed here require the ability to solve difficult problems known to be solvable under timed conditions. Thus, some world-class research mathematicians exist who attempted, but did not excel in the Putnam, IMO, or its qualifying examinations. Nevertheless, a high correlation exists between exceptional ability in mathematical research and problem solving since both require outstanding mathematical intuition and creativity along with the interest in devoting considerable time and effort toward acquiring extensive knowledge in the field. Numerous Putnam Fellows have gone on to receive the Fields Medal (the so-called Nobel Prize of Mathematics) or the Nobel Prize in Physics. Some who never quite achieved Fellow status have also been awarded Nobel Prizes. Eight of the eighteen Fields medalists from 1990 through 2006 were IMO gold or silver medalists in their youth, with Grigorij Perelman, who recently resolved the Poincaré Conjecture, having achieved a perfect forty-two in the 1982 IMO. Five of the eight most recent three-time and four-time Putnam Fellows, two of whom also achieved perfect forty-two's in the IMO, were winners or honorable mention awardees of the Frank and Brennie Morgan Prize [24] for outstanding research in mathematics by an undergraduate student (with a sixth eligible for the 2009 Morgan Prize). Likewise, all three of the women Putnam Fellows were winners of the Alice Schafer Prize [33] for excellence in mathematics by an undergraduate woman; one also won the Morgan Prize. Comparison of the list of the Fellows [40] against the lists of tenured mathematics professors at the top five-ranked USA research universities indicates that less than 10% of these world-class mathematicians managed to achieve Putnam Fellow in college (albeit some of them, including most foreign-educated ones, never took this examination). Therefore, most of the women who excel in these competitions probably possess the ability to become world-class mathematics researchers and university professors if provided the educational opportunities and working environments afforded their male peers.

Conclusions and Proposals

In summary, some Eastern European and Asian countries frequently produce girls with profound ability in mathematical problem solving; most other countries, including the USA, do not. Children, including girls, of immigrants to the USA and Canada from some of the countries that excel in the IMO are overrepresented among students identified as profoundly gifted in mathematics; USA-born girls from all other ethnic/racial backgrounds, including white, are very highly underrepresented. There exist many girls with profound intrinsic aptitude for mathematics; however, they are rarely identified due to socio-cultural, educational, or other environmental factors.

Girls and boys with mathematical ability, whether profound, gifted, or merely above average, should be identified and encouraged to study mathematics beyond the high school level so they may pursue careers in STEM fields because these jobs are plentiful, well-paying, challenging, interesting, and beneficial to society. Their doing so is vital to the future of the USA economy as elegantly documented in Thomas Friedman's *The World Is Flat: A Brief History of the Twenty-First Century* [14]. *Rising Above the Gathering Storm: Energizing and Employing America for a Brighter Economic Future* [8], *Beyond Bias and Barriers: Fulfilling the Potential of Women in Academic Science and Engineering* [7], and *Foundations For Success: The Final Report of the National Mathematics Advisory Panel* [26] outline numerous steps the USA should take to ensure we have the well-educated labor force needed to fill the STEM jobs of the future. To reduce loss of mathematical talent, especially of profoundly gifted students who have a high potential to make major contributions to society, the USA urgently needs to improve how it identifies and nurtures children of both genders with aptitude for mathematics. Here is a list of some proposals, most previously suggested by others but worth reiterating, to help stimulate the national discussion necessary to induce the needed changes.

1. First and foremost, ***the myth that females cannot excel in mathematics must be put to rest.*** Teachers, guidance counselors, parents, principals, university presidents, the lay public, and, most importantly, girls themselves need to be informed about the fact that females *can* excel in mathematics, even at the very highest level. When people believe they cannot do something, it becomes a self-fulfilling prophecy. To quote Henry Ford, "If you think you can or can't, you are right."

2. We need to improve greatly the lay public's perception of mathematicians via the news media, movies, and TV shows such as *Numb3rs* so preteens and teenagers of *both* genders will feel it is socially acceptable to study and to enjoy doing mathematics. To quote the science fiction writer Jack Vance, "That which is never attempted, never transpires." Girls, especially, must be made aware that mathematicians do important, interesting work, and most of them are not nerds. Mathematics Olympians in the other top-ranked IMO countries are highly honored and similarly praised as sports Olympians. The same should be true in the USA.

3. Mathematics courses should be taught starting in sixth or seventh grade at the latest by mathematics-certified teachers who majored in the field. Foreign languages, music, art, machine shop, cooking, and physical education are usually taught by specialists in middle school. Why not mathematics and science as well?

4. There need to exist many more schools such as the Thomas Jefferson High School for Science and Technology, Stuyvesant High School, Illinois Math and Science Academy, and Davidson Academy of Nevada to provide a socially and academically supportive environment in which mathematically gifted children can pursue their interests with like-minded peers under the mentorship of highly qualified teachers. Essentially all USA-born students identified by the very high-level mathematics examinations discussed here attended a special public or elite private high school, had access to college mathematics courses, self-studied mathematics with help from a parent highly knowledgeable in the field, or were home-schooled (for examples, see Table 3). Unfortunately, only a tiny percentage of pre-college students in the USA currently have access to any of these educational opportunities. Without them, the USA is squandering a valuable resource.

5. We should facilitate the ease with which mathematically precocious pre-college students who lack access to programs for gifted children within their own schools can study mathematics above their grade level at local colleges and via correspondence and online programs such as Stanford University's Education Program for Gifted Youth, Northwestern University's Center for Talent

Development's Gifted LearningLinks, and the Art of Problem Solving Foundation.

6. We should encourage mathematically gifted children to participate in summer camps such as MathPath®, All Girls/All Math, AwesomeMath, Canada/USA Mathcamp, Hampshire College Summer Studies in Mathematics, Program in Mathematics for Young Scientists, and the Ross Mathematics Program so they can be exposed to areas of mathematics not currently taught in USA high schools and get to know other children who share their enjoyment of mathematics.

Yes, some of these proposals cost money. However, there will be a far greater cost to the future of the USA economy and our standard of living if we fail to nurture and develop the talents of the vast majority of our mathematically gifted children, boys as well as girls. Hopefully, passage by the U.S. Congress of the *America COMPETES Act*, *"10,000 Teachers, 10 Million Minds" Science and Math Scholarship Act*, and *Sowing the Seeds through Science and Engineering Research Act* will lead to funding to help enable success.

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