A CORRECTION TO "PROPERTIES OF FUNCTIONS $f(x, y)$ OF BOUNDED VARIATION"*

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Some time after our paper was written it came to our attention that the partial derivatives of a measurable function $f(x, y)$ need not be measurable, in contradiction to a Lemma of Burkill and Haslam-Jones.† Trivial examples suffice to show this; indeed such an example, due to Hahn, has been given by Neubauer.‡ The proof of Theorem 18 of our paper, which made use of this lemma, is therefore unsound. Whether or not this theorem and its Corollary 2 are true we have been unable to determine. Corollary 1, however, to the effect that a function $f(x, y)$ in class $\bar{T} \cdot M$ has an approximate total differential almost everywhere, whose proof was our main objective, can readily be established as follows. Since f is in M, by a theorem of Saks§ the approximate partial Dini derivatives (or derivative numbers) are measurable functions; since f is in $\bar{T}$, the approximate partial derivatives are then measurable functions and are finite almost everywhere. The approximate total differentiability of f may then be inferred from a theorem of Stepanoff.||

* Received by the editors October 21, 1939. Cf. these Transactions, vol. 36 (1934), pp. 711–730.

† *Notes on the differentiability of functions of two variables*, Journal of the London Mathematical Society, vol. 7 (1932), pp. 297–305, Lemma 2.

‡ *Über die partiellen Derivierten unstetiger Funktionen*, Monatshefte für Mathematik und Physik, vol. 38 (1931), pp. 139–146, §1.

§ Saks, *Théorie de l'Intégrale*, Warsaw, 1933, p. 226, Theorem 2.

|| See, for example, Saks, loc. cit., p. 228, Theorem 3.

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