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ABSOLUTELY CONTINUOUS FUNCTIONS ON IDEMPOTENT SEMIGROUPS IN THE LOCALLY CONVEX SETTING

BY

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ABSTRACT. Let E be a locally convex space and let T be a semigroup of semicharacters on an idempotent semigroup. It is shown that there exists an isomorphism between the space of E-valued functions on T and the space of all E-valued finitely additive measures on a certain algebra of sets. The space of all E-valued functions on T which are absolutely continuous with respect to a positive definite function F is identified with the space of all E-valued measures which are absolutely continuous with respect to the measure m_F corresponding to F. Finally a representation is given for the operators on the set of all E-valued finitely additive measures on an algebra of sets which are absolutely continuous with respect to a positive measure.

Introduction. Functions of bounded variation and absolutely continuous functions have been studied by several authors including [1], [2], [5], [6], [7], [8] and [9]. In [6] a representation is obtained of the linear functionals on $AC(I)$ which are continuous in the bounded-variation norm in terms of the υ-integral. In [8] and [9] the concepts of bounded variation and absolutely continuous are developed on idempotent semigroups. In [1] and [2] the results of [8] and [9] are used to extend the υ-integral characterization of functionals in [6] to a υ-integral characterization of normed vector space-valued operators on normed vector space-valued absolutely continuous functions on an idempotent semigroup.

In this paper we study the space of functions from a semigroup T of semicharacters on an idempotent semigroup S into a locally convex space E. We identify E-valued functions on T with E-valued finitely additive measures on a certain algebra of subsets of S, and then represent operators on this set of finitely additive measures. To this end we adopt the notation and development in [8] and [9].

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1. **Definitions.** Let S be an abelian idempotent semigroup and let T be a semigroup of semicharacters on S containing the identity semicharacter. Let A denote the algebra of subsets of S generated by the sets J_f, $f \in T$ (see [8]). Assume that E is a real locally convex Hausdorff space and let $\{p: p \in I\}$ be a generating family of continuous seminorms on E which is directed, i.e., given p_1, p_2 in I there exists $p \in I$ such that $p \geqslant p_1, p_2$ (pointwise). For each $p \in I$, let $BV(T, E)_p$ denote the space of all functions $G: T \longrightarrow E$ for which

$$\|G\|_{BV,p} = \sup \sum_{\tau} p(L(Z, \tau)) < \infty$$

where the supremum is taken over the collection of all finite subsets Z of T. Set

$$BV(T, E) = \bigcap_{p \in I} BV(T, E)_p.$$

Clearly $BV(T, E)$ is a real vector space. Let θ denote the locally convex topology on $BV(T, E)$ generated by the family of seminorms $\{\|\cdot\|_{BV,p}: p \in I\}$. If G is a real-valued function on T and $x \in E$, then Gx is defined on T by $(Gx)(f) = G(f)x$. Clearly

$$\{Gx: G \in BV(T), x \in E\} \subset BV(T, E).$$

Let now F be positive definite. We denote by $AC(T, E, F)$ the θ-closure of the space spanned by Gx, $x \in E$ and G in the space $AC(T, E)$ of all real functions on T which are absolutely continuous with respect to F. If $G \in AC(T, E, F)$, we say that G is absolutely continuous with respect to F and write $G \ll F$.

2. **Finitely additive E-valued measures on A.** Let $p \in I$. We denote by $M_p(A, E)$ the collection of all finitely additive E-valued measures m on A for which $\sup \Sigma_{j=1}^n p(m(B_j)) = \|m\|_p < \infty$ where the supremum is taken over all finite partitions $\{B_j\}$ of S into sets in A. Since every element of A is a disjoint union of sets of B-type, it suffices to take the supremum over partitions of S into sets of B-type. Let

$$M(A, E) = \bigcap_{p \in I} M_p(A, E).$$

We denote by ω the locally convex topology on $M(A, E)$ generated by the seminorms $\{\|\cdot\|_p: p \in I\}$.

Let now μ be a real-valued finitely additive set function on A and let m be an E-valued finitely additive measure on A. Let Σ denote the algebra of subsets of $S \times S$ generated by the sets $B_1 \times B_2$, B_1 and B_2 in A. Every element of Σ can be written as a finite disjoint union of sets of the form $B_1 \times B_2$ with B_1, B_2 in A. It is not hard to show that there exists a unique E-valued finitely additive measure $\mu \times m$ on Σ such that $\mu \times m(B_1 \times B_2) = \mu(B_1)m(B_2)$ for all B_1, B_2 in A. Let $\pi: S \times S \longrightarrow S$ be defined by $\pi(s, t) = st$. If $B \in A$, then $\pi^{-1}(B) \in \Sigma$. We define the convolution $\mu * m$ on A by $\mu * m(B) = \mu \times m(\pi^{-1}(B))$. It follows easily that $\mu * m$ is finitely additive. Moreover, we have the following:

LEMMA 1. *If μ is bounded and $m \in M_p(A, E)$, then $\mu * m \in M_p(A, E)$.*

PROOF. Let $Z = \{f_1, \cdots, f_n\} \subset T$ and $B_\tau = B(Z, \tau)$. By [8] we have

$$\pi^{-1}(B_\tau) = \bigcup\{B_{\tau_1} \times B_{\tau_2}: \tau_1, \tau_2 \text{ in } T_n, \tau_1 \wedge \tau_2 = \tau\}.$$

Hence

$$p(\mu * m(B_\tau)) \leqslant \sum_{\tau_1 \wedge \tau_2 = \tau} |\mu(B_{\tau_1})| p(m(B_{\tau_2})).$$

Thus

$$\sum_{\tau \in T_n} p(\mu * m(B_\tau)) \leqslant \sum_{\tau_1} |\mu(B_{\tau_1})| \sum_{\tau_2} p(m(B_{\tau_2}))$$
$$\leqslant \|m\|_p \sum_{\tau_1} |\mu(B_{\tau_1})| \leqslant \|m\|_p \|\mu\|.$$

This implies that $\|\mu * m\|_p \leqslant \|\mu\| \, \|m\|_p$.

For μ a real-valued finitely additive measure on A and for x in E we define μx on A by $(\mu x)(B) = \mu(B)x$. If μ is bounded, then $\|\mu x\|_p = \|\mu\| p(x)$ for each p in I. Let now m be a nonnegative finitely additive measure on A. A real-valued bounded finitely additive measure μ on A is said to be absolutely continuous with respect to m, and write $\mu \ll m$, if for every $\epsilon > 0$ there exists $\delta > 0$ such that $m(B) < \delta$ implies $|\mu|(B) < \epsilon$. An E-valued measure λ on A is said to be absolutely continuous with respect to m, and we write $\lambda \ll m$, if λ is in the ω-closure of the subspace of $M(A, E)$ spanned by μx, where $x \in E$ and μ a bounded real-valued measure on A with $\mu \ll m$.

3. Relationship between E-valued functions on T and finitely additive measures on A. For each finitely additive E-valued measure m on A, we define $\hat{m}: T \longrightarrow E$, $\hat{m}(f) = m(A_f) = \int f\,dm$. This gives us a map from the set of all finitely additive measures on A into the set of all functions from T into E. The following theorem gives the properties of this map.

THEOREM 1. *The map $m \longrightarrow \hat{m}$, from the space of all finitely additive E-valued measures on A into the space of all E-valued functions on T, is linear one-to-one and onto. Moreover the following hold:*

(a) $m \in M_p(B, E)$ *iff* $\hat{m} \in BV(T, E)_p$.

(b) $\|m\|_p = \|\hat{m}\|_{BV,p}$.

(c) $m \ll a$ *iff* $\hat{m} \ll \hat{a}$.

(d) $\widehat{\mu x} = \hat{\mu}x$ *for every real measure* μ *on* A *and every* $x \in E$.

(e) $\widehat{\mu * m} = \hat{\mu}\hat{m}$.

(f) *If* h: $(M(A, E), \omega) \longrightarrow (BV(T, E), \theta)$, $h(m) = \hat{m}$, *then* h *is a topological isomorphism.*

PROOF. The proof that the map $m \longrightarrow \hat{m}$ is linear, one-to-one and onto is similar to the one in the scalar case (see [8]). Since $m(B(Z, \tau)) = L(Z, \tau)\hat{m}$ for each set $B(Z, \tau)$ of B-type, it follows easily that $\|m\|_p = \|\hat{m}\|_{BV,p}$ and hence $m \in M_p(B, E)$ iff $\hat{m} \in BV(T, E)_p$. Also, from the $\widehat{\mu x}(f) = (\mu x)(A_f) = \mu(A_f)x = \hat{\mu}(f)x$, it follows that $\widehat{\mu x} = \hat{\mu}x$ for each real-valued measure μ on A and each $x \in E$.

Assume next that a is a positive measure on A and m an E-valued measure on A. Suppose that $m \ll a$ and let $p \in I$ and $\epsilon > 0$. There are bounded real-valued measures $\mu_1, \cdots, \mu_n$ on A, with $\mu_i \ll a$, and $x_1, \cdots, x_n \in E$ such that $\|\Sigma\mu_i x_i - m\|_p < \epsilon$. Thus $\|\Sigma\hat{\mu}_i x_i - \hat{m}\|_{BV,p} = \|\Sigma\mu_i x_i - m\|_p < \epsilon$. Since $\hat{\mu}_i \ll \hat{a}$, this shows that $\hat{m} \ll \hat{a}$. The proof of the converse is similar. Finally, from

$$\widehat{\mu * m}(f) = \mu * m(A_f) = \mu \times m(\pi^{-1}(A_f)) = \mu \times m(A_f \times A_f)$$
$$= \mu(A_f)m(A_f) = \hat{\mu}(f)\hat{m}(f),$$

it follows that $\widehat{\mu * m} = \hat{\mu}\hat{m}$.

For G a real-valued or E-valued function on T we denote by m_G the measure on A such that $\hat{m}_G = G$.

Let now F be a positive definite function on T. For each E-valued simple function s we define v_s: $A \longrightarrow E$ by

$$v_s(B) = \int_B s\, dm_F.$$

Clearly $v_s \ll m_F$. By Theorem 1 we have $\hat{v}_s \ll F$. The function $\hat{v}_s = p_s$ is called a polygonal function.

THEOREM 2. *If* $AC(T, E, F)$ *is equipped with the* θ*-relative topology, then the collection of all polygonal functions is dense.*

PROOF. Let $G \in AC(T, E, F)$, $p \in I$ and $\epsilon > 0$. There exist $G_1, \cdots, G_n \in AC(T, F)$ and $x_i \in E$ such that $\|\Sigma G_i x_i - G\|_{BV,p} < \epsilon/2$. Each G_i is of the form $G_i = \hat{\mu}_i$, $\mu_i \ll m_F$. By Darst [3], there are simple functions s_i such that

$$\sum\|\mu_i - \lambda_i\|p(x_i) < \epsilon/2$$

where λ_i is defined on A by $\lambda_i(B) = \int_B s_i \, dm_F$. Let $s = \Sigma s_i x_i$. Then for the polygonal function p_s we have

$$\left\|\sum G_i x_i - p_s\right\|_{BV,p} = \left\|\sum \mu_i x_i - v_s\right\| \leqslant \sum \|\mu_i x_i - v_{s_i x_i}\|_p$$
$$\leqslant \sum \|\mu_i - \lambda_i\| p(x_i) < \epsilon/2.$$

By the triangle inequality we have $\|G - p_s\|_{BV,p} < \epsilon$ which completes the proof.

We omit the proof of the following easily established lemma.

LEMMA 2. *If E is complete, then $(M(A, E), \omega)$ is complete.*

THEOREM 3. *Let $\hat{E}$ be the completion of E. Then the completion of the space $AC(T, E, F)$ under the relative θ-topology is the space $AC(T, \hat{E}, F)$.*

PROOF. First of all we observe that $AC(T, \hat{E}, F)$ is complete as a closed subspace of the complete space $(BV(T, \hat{E}), \theta)$. For p in I let $\hat{p}$ denote the unique continuous extension of p to $\hat{E}$. Let $G = \Sigma_{i=1}^n G_i x_i$, with $x_i \in \hat{E}$ and $G_i \ll F$, and let $\epsilon > 0$ and $p \in I$. Since E is dense in $\hat{E}$ there exist $y_1, \cdots, y_n$ in E such that

$$\sum \|G_i\|_{BV} \hat{p}(x_i - y_i) < \epsilon.$$

If $H = \Sigma G_i y_i$, then $\|G - H\|_{BV,\hat{p}} \leqslant \Sigma \|G_i\|_{BV} \hat{p}(x_i - y_i) < \epsilon$. This shows that G is in the closure of $AC(T, E, F)$ in $AC(T, \hat{E}, F)$. Since the set $\{\Sigma G_i s_i \colon G_i \in AC(T, F), s_i \in \hat{E}\}$ is dense in $AC(T, \hat{E}, F)$, the result follows.

4. Representation of continuous operators on spaces of vector measures. By Theorem 1, the space $AC(T, E, F)$ can be identified with the space of all E-valued measures on A which are absolutely continuous with respect to the positive measure m_F. Therefore, to describe continuous operators on $AC(T, E, F)$ it suffices to describe continuous operators on the space of measures which are absolutely continuous with respect to m_F. In this section we will study the problem more generally.[1] Let Σ be an algebra of subsets of a set X. Denote by $M(\Sigma, E)$ the space of all E-valued measures m on Σ such that for each $p \in I$ we have $\|m\|_p = \sup \Sigma p(m(F_i)) < \infty$ where the supremum is taken over the class of all finite partitions $\{F_i\}$ of X into sets in Σ. Let ω denote the locally convex topology on $M(\Sigma, E)$ generated by the family of seminorms $\{\|\cdot\|_p \colon p \in I\}$. Let $\mu \neq 0$ a fixed nonnegative finitely additive measure on Σ and let $AC(\Sigma, E, \mu)$ denote the ω-closure in $M(\Sigma, E)$ of the space spanned by the class of all measures of the form λx where $x \in E$ and λ runs through the family of all bounded real-

(1) The author wishes to thank the referee for suggesting that he look into the problem in this general form.

valued measures on Σ which are absolutely continuous with respect to μ. Let τ be the relative ω-topology on $AC(\Sigma, E, \mu)$. We will represent the continuous linear operators from $(AC(\Sigma, E, \mu), \tau)$ into a locally convex space H.

For s an E-valued simple function, we define $m_s: \Sigma \longrightarrow E$ by $m_s(F) = \int_F s\, d\mu$. It is clear that $m_s \in AC(\Sigma, E, \mu)$.

LEMMA 3. *The class of all measures of the form m_s is τ-dense in $AC(\Sigma, E, \mu)$.*

PROOF. Let $m \in AC(\Sigma, E, \mu)$, $p \in I$ and $\epsilon > 0$. By definition there are real-valued measures $\mu_1, \cdots, \mu_n$, which are absolutely continuous with respect to μ, and $x_i \in E$ such that $\|\Sigma\mu_i x_i - m\|_p < \epsilon/2$. By Darst [3] there are real-valued simple functions s_i such that $\Sigma\|\mu_i - \lambda_i\| p(x_i) < \epsilon/2$ where λ_i is defined on Σ by $\lambda_i(B) = \int_B s_i\, d\mu$. Let $s = \Sigma s_i x_i$. Then

$$\|m_s - m\|_p \leqslant \left\|m - \sum \mu_i x_i\right\|_p + \sum \|\lambda_i - \mu_i\| p(x_i) < \epsilon.$$

The lemma is proved.

Let now H be another real locally convex space. We will represent the continuous linear operators from $(AC(\Sigma, E, \mu), \tau)$ into H.

DEFINITION. Let K be a function on Σ with values in the space $L(E, H)$ of all linear operators from E into H. Then K is called convex relative to μ if, whenever $\{B_j\}$ is a finite partition of B into sets in Σ, then $K(B) = \Sigma\alpha_j K(B_j)$ where $\alpha_j = \mu(B_j)/\mu(B)$. We take $0/0 = 0$. According to this convention it follows that $K(B) = 0$ whenever $\mu(B) = 0$. The function K is called bounded if, for each continuous seminorm q on H, there exists $p \in I$ such that

$$\sup q(K(B)x) = \|K\|_{p,q} < \infty$$

where the supremum is taken over the family of all sets B in Σ and all $x \in E$ with $p(x) \leqslant 1$.

DEFINITION. Let K be a convex (relative to μ) bounded $L(E, H)$-valued function on Σ. The υ-integral with respect to K of an E-valued measure m on Σ is defined to be the lim $\Sigma K(B_i)m(B_i)$, when it exists, where the limit is taken over the collection of all finite Σ-partitions (i.e., partitions into members of Σ) $\{B_i\}$ of X. In this case we say that m is υ-integrable with respect to K and we denote the integral by $\upsilon\int m\, dK$.

We omit the proof of the following easily established lemma.

LEMMA 4. *Let K be convex and bounded and let s be an E-valued simple function. Then m_s is υ-integrable. Moreover there exists a finite Σ-partition $\{B_i\}$ of X such that*

$$v\int m_s\,dK = \sum K(F_j)m_s(F_j)$$

for every finite Σ*-partition* $\{F_j\}$ *of* X *which is a refinement of* $\{B_i\}$.

LEMMA 5. *Let* K *be convex and bounded and assume that* H *is complete. Then*:

(1) *Every* m *in* $AC(\Sigma, E, \mu)$ *is* v*-integrable with respect to* K.

(2) *The map* $m \longrightarrow \phi_K(m) = v\int m\,dK$, *from* $(AC(\Sigma, E, \mu), \tau)$ *into* H, *is a continuous linear operator. Furthermore, if* $\|K\|_{p,q} < \infty$, *then*

$$\|K\|_{p,q} = \sup\{q(\phi_K(m)) \colon m \in AC(\Sigma, E, \mu),\ \|m\|_p \leqslant 1\}.$$

PROOF. First of all we observe that, if m is v-integrable and if $\|K\|_{p,q} < \infty$, then

$$(*) \qquad q(\phi_K(m)) \leqslant \|K\|_{p,q}\|m\|_p.$$

Let now $m \in AC(\Sigma, E, \mu)$, q a continuous seminorm on H and $\epsilon > 0$. Let $p \in I$ be such that $\|K\|_{p,q} = d < \infty$. Let s be an E-valued simple function such that $\|m_s - m\|_p < \epsilon/2d$.

By Lemma 4, there exists a finite Σ-partition $\{B_i\}$ of X such that $\phi_K(m_s) = \Sigma K(F_j)m_s(F_j)$ for each finite Σ-partition $\{F_j\}$ of X which is a refinement of $\{B_i\}$. Now if $\{F_j\}$ and $\{G_l\}$ are both Σ-partitions of X which are refinements of $\{B_i\}$, then

$$\begin{aligned} &q\Big(\sum K(F_j)m(F_j) - \sum K(G_l)m(G_l)\Big) \\ &\quad \leqslant q\Big(\sum K(F_j)m(F_j) - \sum K(F_j)m_s(F_j)\Big) + q\Big(\sum K(G_l)m_s(G_l) - \sum K(G_l)m(G_l)\Big) \\ &\quad \leqslant d\|m - m_s\|_p + d\|m - m_s\|_p < \epsilon. \end{aligned}$$

This shows that the net $\{\Sigma K(F_j)m(F_j)\}$ is a Cauchy net in H and hence convergent. This proves (1).

The inequality $(*)$ implies that ϕ_K is a continuous linear map and that

$$\|K\|_{p,q} \geqslant \|\phi_K\|_{p,q} = \sup\{q(\phi_K(m)) \colon m \ll \mu,\ \|m\|_p \leqslant 1\}.$$

Let now $\epsilon > 0$ be given. There exists $F \in \Sigma$ and $x \in E$, with $p(x) \leqslant 1$, such that

$$q(K(F)x) > \|K\|_{p,q} - \epsilon = a.$$

Let $s = \chi_F x$. Then $\|m_s\|_p = p(x)\mu(F) \leqslant \mu(F)$. Also $K(F)m_s(F) = \mu(F)K(F)x$. Thus

$$q(\phi_K(m_s)) = \mu(F)q(K(F)x) \geqslant a\mu(F).$$

Hence $\|\phi_K\|_{p,q} \geqslant a = \|K\|_{p,q} - \epsilon$. Since $\epsilon > 0$ was arbitrary, the result follows.

LEMMA 6. *If ϕ is a continuous linear operator from $AC(\Sigma, E, \mu)$ into H, then there exists a unique convex bounded function K such that $\phi = \phi_K$.*

PROOF. For each F in Σ, let λ_F be defined on Σ by

$$\lambda_F(B) = \mu(B \cap F)/\mu(F).$$

Clearly λ_F is a bounded real-valued measure on Σ and $\lambda_F \ll \mu$. We define K on Σ by

$$K(F)x = \phi(\lambda_F x), \qquad x \in E.$$

Then K is an $L(E, H)$-valued function. It is easy to see that K is convex relative to μ. Also K is bounded. In fact, let q be a continuous seminorm on H. Since ϕ is continuous there exist $p \in I$ and $M > 0$ such that $q(\phi(m)) \leqslant M$ whenever $\|m\|_p \leqslant 1$. Thus $q(K(B)x) \leqslant M$ whenever $p(x) \leqslant 1$ since $\|\lambda_B x\|_p \leqslant 1$ whenever $p(x) \leqslant 1$. This proves that K is bounded. Next we show that $\phi = \phi_K$. To this end, it suffices, by Lemma 3, to show that $\phi(m_s) = \phi_K(m_s)$ for each E-valued simple function s. Since $m_{s_1+s_2} = m_{s_1} + m_{s_2}$, it suffices to prove the claim for any s of the form $s = \chi_F x$, $F \in \Sigma$, $x \in E$. But, for $s = \chi_F x$, we have $m_s = \mu(F)\lambda_F x$. Thus

$$\phi(m_s) = \mu(F)\phi(\lambda_F x) = \mu(F)K(F)x = K(F)m_s(F) = \phi_K(m_s).$$

It follows that $\phi = \phi_K$. Finally, assume that K_1 is another convex bounded function such that $\phi = \phi_{K_1}$. If $K_2 = K - K_1$, then $\phi_{K_2} = 0$. We claim that $K_2 = 0$. Assume the contrary and let $F \in \Sigma$ be such that $K_2(F) \neq 0$. Choose $x \in E$, with $K_2(F)x \neq 0$, and set $s = \chi_F x$. Then $0 = \phi_{K_2}(m_s) = \mu(F)K_2(F)x \neq 0$ since $\mu(F) \neq 0$. This contradiction completes the proof.

Combining the preceding lemmas we get the following representation theorem.

THEOREM 4. *If H is complete, then the map $K \longrightarrow \Phi_K$ is a one-to-one linear map from the space of all convex (relative to μ) bounded $L(E, H)$-valued functions onto the space of all continuous linear operators from $AC(\Sigma, E, \mu)$ into H.*

BIBLIOGRAPHY

1. Richard A. Alò and André de Korvin, *Functions of bounded variation on idempotent semigroups,* Math. Ann. **194** (1971), 1–11. MR **45** #2454.

2. R. A. Alò, André de Korvin and R. J. Easton, *Vector valued absolutely continuous functions on idempotent semigroups,* Trans. Amer. Math. Soc. **172** (1972), 491–500. MR **46** #9283.

3. R. B. Darst, *A decomposition of finitely-additive set functions,* J. Reine Angew. Math. **210** (1962), 31–37. MR **25** #1257.

4. N. Dinculeanu, *Vector measures,* Hochschulbücher für Math. Bd. 64, VEB Deutscher Verlag, Berlin, 1966. MR 34 #6011a.

5. N. Dunford and J. T. Schwartz, *Linear operators.* I: *General theory*, Pure and Appl. Math., vol. 7, Interscience, New York, 1958. MR 22 #8302.

6. J. R. Edwards and S. G. Wayment, *Representations for transformations continuous in the BV norm,* Trans. Amer. Math. Soc. 154 (1971), 251–265. MR 43 #466.

7. T. H. Hildebrandt, *Linear continuous functionals on the space (BV) with weak topologies*, Proc. Amer. Math. Soc. 17 (1966), 658–664. MR 33 #1710.

8. S. E. Newman, *Measure algebras and functions of bounded variation on idempotent semigroups,* Trans, Amer. Math. Soc. 163 (1972), 189–205. MR 46 #7800.

9. ———, *Measure algebras and functions of bounded variation on idempotent semigroups*, Bull. Amer. Math. Soc. 75 (1969), 1396–1400. MR 40 #4778.

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