

**CORRECTION TO “VANISHING THEOREMS AND  
KÄHLERITY FOR STRONGLY PSEUDOCONVEX MANIFOLDS”**

BY  
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It has been brought to my attention by N. Coltoiu that there was a gap in the proof of Theorem II in [3]. By using the same notations as in [3], the proof of our Theorem II can be corrected as follows.

**THEOREM II.** *Let  $(X, S)$  be a strongly pseudoconvex manifold. If  $\dim S = 1$ , then  $X$  is Kählerian. In particular, any strongly pseudoconvex surface is Kählerian.*

First of all the following result is needed.

**LEMMA [1].** *Let  $S$  be a compact  $\mathbf{C}$  analytic space and let  $L$  be a holomorphic line bundle on  $S$ . Let us assume that, for every positive dimensional subspace  $T$  of  $S$ , there exist an integer  $n$  and a nonzero holomorphic section of  $L^n \otimes O_T$  which vanishes at some point of  $T$ . Then  $L$  is positive.*

*Notations.* From now on, a *positive* line bundle (resp. *semipositive* line bundle) will be denoted by  $L > 0$  (resp.  $L \geq 0$ ).

**PROOF OF THEOREM II.** Without loss of generality, one can assume that  $S$  is irreducible. So let  $x$  be a point on  $S$  and let  $\pi: \hat{X} \rightarrow X$  be the blowing up of  $X$  at  $x$  inducing a biholomorphism  $\hat{X} \setminus D \simeq X \setminus \{x\}$ .

Let  $T$  be the strict transform of  $S$  under  $\pi$ ; it is clear the  $\hat{X}$  is a strongly pseudoconvex manifold with its exceptional set  $\hat{S} = D \cup T$ .

**CLAIM.** There exists a positive line bundle  $L$  on  $\hat{X} \setminus D$ .

In fact, let  $L_1 := [D]$  be the line bundle determined by  $D$ . Since  $\dim T = 1$ , the Lemma above tells us that  $L_1|_T > 0$ . In view of the compactness of  $T$ , one can find a relative compact neighborhood  $V$  of  $T$  in  $\hat{X}$  such that

(\*)  $L_1 > 0$  on  $V$  and by construction  $L_1 \geq 0$  on  $\hat{X} \setminus D$ .

Since  $\hat{X}$  is strongly pseudoconvex, one can find a line bundle  $L_2$  on  $\hat{X}$  such that

(\*\*)  $L_2 > 0$  on  $\hat{X} \setminus \hat{S}$  and  $L_2 \geq 0$  on  $\hat{X}$ .

In view of (\*) and (\*\*), it follows that, for some  $N \gg 0$ , the line bundle  $L := L_1 \otimes L_2^N > 0$  on  $(\hat{X} \setminus \hat{S}) \cup V \supset \hat{X} \setminus D$ . Hence our claim is proved.

Consequently,  $\hat{X} \setminus D \simeq X \setminus \{x\}$  is Kählerian. The result in [2] tells us that  $X$  itself is Kählerian. Q.E.D.

## REFERENCES

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