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ON INDUCTIVE LIMITS OF CERTAIN C^*-ALGEBRAS OF THE FORM $C(X) \otimes F$

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ABSTRACT. A certain class of $*$-homomorphisms $C(X) \otimes A \to C(Y) \otimes B$, called compatible with a map defined on Y with values in the set of all closed nonempty subsets of X, is studied. A local description of $*$-homomorphisms $C(X) \otimes A \to C(Y) \otimes B$ is given considering separately the cases $X =$ point and $A = \mathbf{C}$; this is done in terms of continuous "quasifields" of C^*-algebras. Conditions under which an inductive limit $\varinjlim(C(X_k) \otimes A_k, \Phi_k)$, where each Φ_k is of the above type, is $*$-isomorphic with the tensor product of a commutative C^*-algebra with an AF algebra are given. For such inductive limits the isomorphism problem is considered.

The study of inductive limits of C^*-algebras of the form $C(X) \otimes F$ (with F a finite-dimensional C^*-algebra) has been suggested by E. G. Effros in [**5**]. Clearly, for this problem, the structure of the $*$-homomorphisms between algebras of the above form is important. This question has been considered in [**1, 2, 8, 9, 10, 11** and **12**].

The main result of the present paper gives a sufficient condition for the triviality of the inductive limits, i.e., so that they are tensor products of commutative algebras and AF-algebras.

After some preliminaries in §1, we consider in §2 $*$-homomorphisms $\Phi\colon C(X) \otimes A \to C(Y) \otimes B$ compatible (2.3) with a map $\theta\colon Y \to K(X)$ ($K(X)$ the closed subsets of X) which generalize the homomorphisms compatible with a covering considered in [**8**]. Our results are more precise in the following two situations:

1°. $\theta(y) = \varphi^{-1}(y)$, $\varphi\colon X \to Y$ a continuous surjection;

2°. $\theta(y) = \{\varphi(y)\}$, $\varphi\colon Y \to X$ continuous (2.7).

Given a homomorphism, we find conditions that insure the existence of a θ as in 1° above with which it is compatible (2.8). We also improve one of our previous results (Proposition 2.5 in [**8**]) concerning homomorphisms compatible with a p-fold covering (2.9).

In §3 the homomorphisms $C(X) \otimes A \to C(Y) \otimes B$ are unitial, A, B are finite dimensional and the compact spaces X, Y are metrizable (excepting Proposition 3.1). Our results describe the local structure of such homomorphisms in terms of continuous "quasifields" of finite-dimensional C^*-algebras ((3.1) and (3.4)). Using classes of inner equivalent injective homomorphisms between continuous quasifields of finite-dimensional C^*-algebras (see 3.3) we study the set of classes of inner equivalent homomorphisms (injective homomorphisms) from $C(X)$ to $C(Y) \otimes B$

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(3.4). A similar analysis is done for the set of all $*$-homomorphisms (injective $*$-homomorphisms) from $C(X)\otimes A$ to $C(Y)\otimes B$ which are compatible with a given continuous surjective map from X to Y, the fibre of which satisfies a certain continuity property (3.6).

§4 contains the main result of this paper. Consider a system:

$$C(X_1)\otimes A_1 \overset{\Phi_1}{\to} C(X_2)\otimes A_2 \overset{\Phi_2}{\to} \cdots$$

with X_k compact and A_k a finite-dimensional C^*-algebra. We give conditions under which the above inductive limit is "trivial," in the sense that it coincides with the tensor product of a commutative C^*-algebra with an AF-algebra. The assumptions on the spaces X_k involve the vanishing of certain nonabelian cohomologies (this occurs for X_k contractible, for instance). Moreover, it is required that

$$\Phi_k(C(X_k)\otimes 1_{A_k}) \subset C(X_{k+1})\otimes 1_{A_{k+1}}$$

(see (4.3)). For such trivial inductive limits we also consider the isomorphism problem (4.4).

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1. For A and B unital C^*-algebras, $\mathrm{Hom}(A,B)$ (resp. $\mathrm{Hom}_i(A,B)$) will denote the set of all unital $*$-homomorphisms (resp. all unital injective $*$-homomorphisms) from A to B endowed with the topology of pointwise convergence. $Z(A)$ denotes the center and $U(A)$ the group of all unitaries of A. Φ, $\Psi \in \mathrm{Hom}(A,B)$ are called *inner equivalent*, $\Phi\sim\Psi$, if $\Phi = \mathrm{Ad}\,u\circ\Psi$ for some $u\in U(B)$. For $M\subset \mathrm{Hom}(A,B)$, we denote by $M/\sim$ the corresponding set of classes of inner equivalent $*$-homomorphisms.

For a compact topological space X we use the canonical identification $C(X)\otimes A = C(X,A)$. If $f\in C(X)\otimes A$ and $F\subset X$, we denote $\|f\,|_F\,\| := \sup_{x\in F}\|f(x)\|$ if $F\neq\varnothing$ and $\|f\,|_\varnothing\,\| := 0$. For a finite-dimensional C^*-algebra $A=\bigoplus_{i\in I}A_i$ (where each A_i is a finite discrete factor) the inclusions $A_i\subset A$, $i\in I$, induce canonical embeddings $C(X)\otimes A_i\subset C(X)\otimes A$, $i\in I$, and we have $C(X)\otimes A = \bigoplus_{i\in I} C(X)\otimes A_i$.

If $\varphi\colon X\to Y$ is a continuous map between compact spaces, we denote by $\varphi^*\colon C(Y)\to C(X)$ the map $\varphi^*(f)=f\circ\varphi$, $f\in C(Y)$.

Let G be a topological group, G_c the sheaf of germs of continuous G-valued functions on X and $H^1(X,G_c)$ the corresponding cohomology set; for a contractible compact space X, $H^1(X,G_c)$ reduces to the trivial element [**7**].

2. Throughout this section X, Y will denote compact spaces and A a finite-dimensional C^*-algebra.

2.1. Consider $A=\bigoplus_{i\in I}A_i$, where I is a finite set and each A_i is a finite discrete factor.

Denote $K(X) := \{F | F \text{ is a nonempty closed (i.e., compact) subset of } X\}$. Consider $\Phi\in\mathrm{Hom}(C(X)\otimes A, C(Y)\otimes B)$, where B is a unital C^*-algebra. For any $y\in Y$, let $X_{y,\Phi}\in K(X)$ be such that $\{g\in C(X) | g|X_{y,\Phi}=0\}$ is the kernel of the unital $*$-homomorphism:

$$C(X)\ni g\to\Phi(g\otimes 1_A)(y)\in B.$$

Then, for each $y\in Y$, $X_{y,\Phi}\in K(X)$ is determined by the condition

$$\|\Phi(g\otimes 1_A)(y)\| = \|g|X_{y,\Phi}\|,\qquad g\in C(X).$$

In a similar way one sees that for any $y \in Y$ and $i \in I$ there is a unique closed subset $X^i_{y,\Phi}$ of X such that

$$\|\Phi(f_i)(y)\| = \|f_i|X^i_{y,\Phi}\|, \qquad f_i \in C(X) \otimes A_i.$$

Note that $X^i_{y,\Phi}$ can be the empty set. Clearly $X_{y,\Phi} = \bigcup_{i\in I} X^i_{y,\Phi}$.

2.2. For any $f = \bigoplus_{i\in I} f_i \in \bigoplus_{i\in I} C(X) \otimes A_i$ and $y \in Y$ we have

(1) $\|\Phi(f)(y)\| = \max_{i\in I} \|f_i|X^i_{y,\Phi}\|$,

(2) $\|\Phi(f)(y)\| \leq \|f|X_{y,\Phi}\|$, since

$$\begin{aligned}\|\Phi(f)(y)\| &= \left\|\sum_{i\in I} \Phi(f_i)(y)\right\| = \max_{i\in I} \|\Phi(f_i)(y)\| \\ &= \max_{i\in I} \|f_i|X^i_{y,\Phi}\| \leq \max_{i\in I} \|f_i|X_{y,\Phi}\| = \|f|X_{y,\Phi}\|.\end{aligned}$$

Moreover

(3) Φ is injective $\Leftrightarrow \bigcup_{y\in Y} X^i_{y,\Phi} = X$ for any $i \in I$. Indeed, by (1) we have

$$\|\Phi(f)\| = \max_{i\in I} \left\| f_i| \bigcup_{y\in Y} X^i_{y,\Phi} \right\|$$

and each $\bigcup_{y\in Y} X^i_{y,\Phi}$ is closed.

2.3. Consider a map $\theta\colon Y \to K(X)$. We say that a $*$-homomorphism $\Phi \in \mathrm{Hom}(C(X) \otimes A, C(Y) \otimes B)$, where B is any unital C^*-algebra, is *θ-compatible* if

(1) $X_{y,\Phi} \subset \theta(y)$, $y \in Y$.

This is equivalent to

(2) $\|\Phi(f)(y)\| < \|f|_{\theta(y)}\|$, $f \in C(X) \otimes A$, $y \in Y$. Indeed, (1) $\Rightarrow$ (2) by 2.2(2). Conversely, for any $g \in C(X)$ and $y \in Y$ we have $\|g|X_{y,\Phi}\| = \|\Phi(g \otimes 1_A)(y)\| \leq \|g|\theta(y)\|$ and since $X_{y,\Phi}$ is closed in X it follows that $X_{y,\Phi} \subset \theta(y)$.

The above argument also shows that $X_{y,\Phi}$ is the smallest nonempty closed subset F of X such that $\|\Phi(f)(y)\| \leq \|f|_F\|$ for any $f \in C(X) \otimes A$.

2.4. Consider $\Phi \in \mathrm{Hom}(C(X) \otimes A,\ C(Y) \otimes B)$, where $A = \bigoplus_{i\in I} A_i$, I is a finite set and each A_i is a finite discrete factor, and a map $\theta\colon Y \to K(X)$. Then, the following are equivalent:

(1) $\|\Phi(f)(y)\| = \|f|\theta(y)\|$, $f \in C(X) \otimes A$, $y \in Y$.

(2) $X^i_{y,\Phi} = \theta(y)$, $y \in Y$, $i \in I$.

Indeed (2) $\Rightarrow$ (1) by 2.2(1). Conversely, for every $i \in 1$ and $y \in Y$, we have $\|f_i|X^i_{y,\Phi}\| = \|\Phi(f_i)(y)\| = \|f_i|\theta(y)\|$, $f_i \in C(X) \otimes A_i$, and since each $X^i_{y,\Phi}$ is closed in X, we deduce $X^i_{y,\Phi} = \theta(y)$.

2.5. Suppose moreover that $(\theta(y))_{y\in Y}$ is a partition of X and that Φ is compatible with θ. Then the following are equivalent:

(1) Φ is injective,

(2) $\|\Phi(f)(y)\| = \|f|\theta(y)\|$, $f \in C(X) \otimes A$, $y \in Y$.

Indeed, (2) $\Rightarrow$ (1) by 2.2(3) and 2.4. Conversely, suppose there are $i_0 \in I$, $y_0 \in Y$ such that

$$X^{i_0}_{y_0,\Phi} \subsetneqq \theta(y_0).$$

Since Φ is compatible with θ, we have $X^{i_0}_{y,\Phi} \subset \theta(y)$, $y \in Y$. Then, using 2.2(3) and the fact that $(\theta(y))_{y\in Y}$ is a partition of X, one has

$$X = \bigcup_{y\in Y} X^{i_0}_{y,\Phi} \subsetneqq \bigcup_{y\in Y} \theta(y) = X,$$

a contradiction. Hence $X^{i}_{y,\Phi} = \theta(y)$, $y \in Y$, $i \in I$, and the conclusion is obtained using again 2.4.

2.6. PROPOSITION. *Consider* $\Phi \in \mathrm{Hom}(C(X) \otimes A,\ C(Y) \otimes B)$ *and a map* $\theta\colon Y \to K(X)$ *and suppose there is a unital embedding* $B \subset M_n$, *for some* $n \in \mathbf{N}$. *Then* Φ *is* θ*-compatible if and only if*

$$\mathrm{tr}(\Phi(g \otimes 1_A)(y)) \in n \cdot \mathrm{co}\, g(\theta(y)), \qquad g \in C(X),\ y \in Y, \tag{1}$$

where tr *denotes the usual trace on* M_n.

PROOF. For any $y \in Y$, consider the unital finite-dimensional $*$-representation $C(X) \otimes A \ni f \to \Phi(f)(y) \in M_n$. Since this is a direct sum of irreducible $*$-representations, it follows that for any $x \in X_{y,\Phi}$ there is a unital $*$-representation $\Pi_{x,y}$ of A such that

$$\Phi(f)(y) = \bigoplus_{x\in X_{y,\Phi}} \Pi_{x,y}(f(x)) \in M_n \tag{2}$$

for all $f \in C(X) \otimes A$. In particular, in this case, each $X_{y,\Phi}$ is a finite set.

Suppose that Φ is θ-compatible. Using the above discussion, for $g \in C(X)$ and $y \in Y$ we get

$$\begin{aligned}\mathrm{tr}(\Phi(g \otimes 1_A)(y)) &= \sum_{x\in X_{y,\Phi}} g(x) \cdot \dim \Pi_{x,y} \\ &= n \cdot \left(\sum_{x\in X_{y,\Phi}} g(x) \cdot n^{-1} \cdot \dim \Pi_{x,y} \right) \in n \cdot \mathrm{co}\, g(\theta(y))\end{aligned}$$

since $X_{y,\Phi} \subset \theta(y)$ and Φ being unital, $\sum_{x\in X_{y,\Phi}} n^{-1} \cdot \dim \Pi_{x,y} = 1$.

Conversely, assume (1) and suppose there is $y_0 \in Y$ such that $X_{y_0,\Phi} \not\subset \theta(y_0)$. Then there is $x_0 \in X_{y_0,\Phi} \setminus \theta(y_0)$ and $g_0 \in C(X)$ such that $g_0(x_0) = 1$ and $g_0|\theta(y_0) \cup (X_{y_0,\Phi} \setminus \{x_0\}) = 0$.

Using (1) and (2) we have

$$\begin{aligned}\mathrm{tr}(\Phi(g_0 \otimes 1_A)(y_0)) &= \sum_{x\in X_{y_0,\Phi}} g_0(x) \cdot \dim \Pi_{x,y_0} \\ &= \dim \Pi_{x_0,y_0} \notin \{0\} = n \cdot \mathrm{co}\, g_0(\theta(y_0)),\end{aligned}$$

a contradiction.

2.7. Consider in particular the map $\theta\colon Y \to K(X)$ given by $\theta(y) := \{\varphi(y)\}$, $y \in Y$, where $\varphi\colon Y \to X$ is a continuous map. Then Φ is θ-compatible if and only if

(1) $\Phi(g \otimes 1_A) = g \circ \varphi \otimes 1_B$, $g \in C(X)$. Indeed, since $X_{y,\Phi} = \{\varphi(y)\}$, we have $\Phi(g \otimes 1_A)(y) = \Pi_{\varphi(y),y}(g(\varphi(y)) \cdot 1_A) = g(\varphi(y)) \cdot 1_B$, for any $g \in C(X)$ and $y \in Y$. Conversely, if (1) holds then for any $g \in C(X)$ and $y \in Y$ we have $\|g|X_{y,\Phi}\| = \|\Phi(g \otimes 1_A)(y)\| = \|g(\varphi(y))\|$ and since each $X_{y,\Phi}$ is closed, $X_{y,\Phi} = \{\varphi(y)\}$.

On the other hand let B be a finite-dimensional C^*-algebra and $\varphi: X \to Y$ a continuous surjective map. A $*$-homomorphism $\Phi: C(X) \otimes A \to C(Y) \otimes B$ is said to be *φ-compatible* if

$$\Phi(g \circ \varphi \otimes 1_A) = g \otimes 1_B, \qquad g \in C(Y).$$

If Φ is injective, then φ is uniquely determined by Φ since we can use 2.5; we have that $(X_{y,\Phi})_{y \in Y}$ is a partition of X and $\varphi^{-1}(y) = X_{y,\Phi}$, $y \in Y$.

Let B, Φ be as in Proposition 2.6 and consider the map $\theta: Y \to K(X)$ given by $\theta(y) := \varphi^{-1}(y)$, $y \in Y$, where $\varphi: X \to Y$ is a continuous surjection. In this situation the following assertions are equivalent:

(2) Φ is θ-compatible.

(3) Φ is φ-compatible.

(4) $\operatorname{tr}(\Phi(g \circ \varphi \otimes 1_A)(y)) = n \cdot g(y)$, $g \in C(Y)$, $y \in Y$. (tr denotes the usual trace on M_n.)

(2) $\Rightarrow$ (3). For any $g \in C(Y)$ and $y \in Y$ we have

$$\Phi(g \circ \varphi \otimes 1_A)(y) = \bigoplus_{x \in X_{y,\Phi}} \Pi_{x,y}(g(\varphi(x)) \cdot 1_A) = g(y) \cdot 1_B$$

since $X_{y,\Phi} \subset \varphi^{-1}(y)$ (we use the notation and remarks made in the proof of Proposition 2.6).

(3) $\Rightarrow$ (4) is obvious.

(4) $\Rightarrow$ (2) By assumption, for any $g \in C(Y)$ and $y \in Y$ we have

$$n \cdot g(y) = \sum_{x \in X_{y,\Phi}} g(\varphi(x)) \cdot \dim \Pi_{x,y} = \sum_{t \in \varphi(X_{y,\Phi})} c_y(t) g(t),$$

where each $c_y(t) > 0$. Now fix $y_0 \in Y$, suppose there is $t_0 \in \varphi(X_{y_0,\Phi}) \setminus \{y_0\}$ and let $g_0 \in C(Y)$ be such that $g_0(t_0) = 1$,

$$g_0 | \{y_0\} \cup (\varphi(X_{y_0,\Phi}) \setminus \{t_0\}) = 0;$$

then $g = g_0$ and $y = y_0$ will contradict the above form of assumption (4). Hence $\varphi(X_{y,\Phi}) = \{y\}$, $y \in Y$.

2.8. The following proposition gives sufficient conditions for a homomorphism Φ to be compatible with some good φ.

PROPOSITION. *Let B be a finite-dimensional C^*-algebra and consider $\Phi \in \operatorname{Hom}(C(X) \otimes A, C(Y) \otimes B)$. Assume that the cardinality of $X_{y,\Phi}$ is locally constant on Y and $(X_{y,\Phi})_{y \in Y}$ is a partition of X. Then the map $\varphi: X \to Y$, $\varphi(X_{y,\Phi}) = \{y\}$, $y \in Y$, is a covering map and Φ is φ-compatible.*

PROOF. Fix $y' \in Y$. The assumptions imply that there are $n \in \mathbf{N}$ and $U \in \mathscr{V}(y')$ such that $X_y := X_{y,\Phi}$ has exactly n elements for all $y \in U$. Say $X_{y'} = \{z_1(y'), \ldots, z_n(y')\}$ and let $V_p' = \overline{V_p'} \in \mathscr{V}(z_p(y'))$, $p = 1, 2, \ldots, n$, with $V_p' \cap V_q' = \varnothing$ for $p \neq q$.

Now, for fixed $p \in \{1, 2, \ldots, n\}$ we claim there is $W \in \mathscr{V}(y')$, $W \subset U$, such that $X_y \cap V_p' \neq \varnothing$ for any $y \in W$. Indeed, in the contrary case there is a net $(y_i)_{i \in I}$ in U which converges to y' such that $X_{y_i} \cap V_p' = \varnothing$. But for $g \in C(X)$, $g(z_p(y')) = 1$, $\operatorname{supp} g \subset V_p'$ we have

$$\begin{aligned} 1 = |g(z_p(y'))| &\leq \|g|X_{y'}\| = \|\Phi(g \otimes 1_A)(y')\| \\ &= \lim_i \|\Phi(g \otimes 1_A)(y_i)\| = \lim_i \|g|X_{y_i}\| = 0, \end{aligned}$$

a contradiction which proves the claim. Therefore we can choose $V \in \mathscr{V}(y')$, $V \subset U$, such that $X_y \cap V_p' \neq \varnothing$, $y \in V$, $p = 1, 2, \ldots, n$.

We prove that φ is continuous. Indeed, if a net $(x_j)_{j \in J}$ in X converges to $x \in X$ but $\varphi(x_j) \not\to \varphi(x)$, then, X being compact, we may suppose that $\varphi(x_j) \to y_0 \neq \varphi(x)$.

For $g \in C(X)$, $g(x) = 1$, $g|X_{y_0} = 0$ we have

$$\begin{aligned} 0 = \|g|X_{y_0}\| = \|\Phi(g \otimes 1_A)(y_0)\| &= \lim_j \|\Phi(g \otimes 1_A)(\varphi(x_j))\| \\ = \lim_j \|g|X_{\varphi(x_j)}\| &\geq \lim_j |g(x_j)| = |g(x)| = 1, \end{aligned}$$

a contradiction.

For each $y \in V$, let $z_p(y)$ be the unique element of $X_y \cap V_p'$, $p = 1, 2, \ldots, n$. Each map $z_p \colon V \to V_p := z_p(V)$ is a bijection since $\varphi \circ z_p = \mathrm{id}_V$; note that $V_p = \varphi^{-1}(V) \cap V_p' \in \mathscr{V}(z_p(y'))$. Moreover, each z_p is continuous. Indeed, if a net $(y_k)_{k \in K}$ in V converges to $\tilde{y} \in V$ and $z_p(y_k) \not\to z_p(\tilde{y})$, we may consider $z_p(y_k) \to \tilde{x}$ for some $\tilde{x} \in \overline{V}_p \subset \overline{V}_p' = V_p$, $\tilde{x} \neq z_p(\tilde{y})$ and we have $\tilde{y} = \lim_k y_k = \lim_k \varphi(z_p(y_k)) = \varphi(\tilde{x})$, that is, $\tilde{x} \in \varphi^{-1}(\tilde{y}) \cap V_p' = X_{\tilde{y}} \cap V_p'$; hence $\tilde{x} = z_p(\tilde{y})$, a contradiction.

Thus each $\varphi_p = \varphi|_{V_p} \colon V_p \to V$ is a homeomorphism with inverse z_p. Hence φ is a covering map.

Since $X_{y,\Phi} = \varphi^{-1}(y)$, $y \in Y$, it follows from 2.7 that Φ is φ-compatible.

2.9. The next proposition gives the structure of homomorphisms compatible with a finite covering, which improves the result in [**8**, Proposition 2.5] by replacing the absolute retract assumption with contractibility and by using a shorter argument.

PROPOSITION. *Let $\varphi \colon X \to Y$ be a p-fold covering map ($p \in \mathbf{N}$), where X, Y are compact metric spaces and assume Y is contractible. Then there is a partition $(U_i)_{i=1}^p$ of X into clopen sets and there exist homeomorphisms $z_i \colon Y \to U_i$ satisfying $\varphi \circ z_i = \mathrm{id}_Y$ $(1 \leq i \leq p)$ such that if $\Phi \colon C(X) \otimes A \to C(Y) \otimes B$ is a φ-compatible $*$-homomorphism, then there are $u \in C(Y, U(B))$ and $*$-homomorphisms $\Psi_1, \Psi_2, \ldots, \Psi_p \colon A \to B$ such that*

$$\Phi(f)(y) = \mathrm{Ad}\, u(y) \left(\bigoplus_{k=1}^{p} \Psi_k(f(z_k(y))) \right)$$

for all $f \in C(X) \otimes A$ and $y \in Y$.

PROOF. Since Y is simply connected, there is a homeomorphism $H \colon X \to Y \times \{1, 2, \ldots, p\}$ such that the diagram

$$\begin{array}{ccc} X & \xrightarrow{H} & Y \times \{1, 2, \ldots, p\} \\ \varphi \downarrow & \swarrow \psi & \\ Y & & \end{array}$$

commutes, where ψ is the canonical projection. For each $1 \leq i \leq p$ we define $U_i = H^{-1}(Y \times \{i\})$, the homeomorphism $h_i \colon Y \to Y \times \{i\}$ given by $h_i(y) = (y, i)$, $y \in Y$ and $z_i \colon Y \to U_i$, $z_i := H^{-1} \circ h_i$.

Using Proposition 2.4 from [**8**] and the fact that Y is connected, we find $*$-homomorphisms $\Psi_1, \ldots, \Psi_p \colon A \to B$, a *proper* open covering $(V_i)_{i \in I}$ of Y (see

[**7**, p. 17]) and $u_i \in C(V_i, U(B))$ such that

$$\Phi(f)(y) = \operatorname{Ad} u_i(y) \left(\bigoplus_{k=1}^{p} \Psi_k(f(z_k(y))) \right)$$

for $f \in C(X) \otimes A$, $y \in V_i$, $i \in I$. (The set of Ψ's in [**8**, 2.4] depends on the local neighborhood but they can be chosen canonical [**4**], that is, in a finite set, so that this locally constant choice of the Ψ's is actually constant.) The continuous maps $g_{ij} \colon V_i \cap V_j \to G :=$ the topological group of all unitaries of the relative commutant of $\bigoplus_{k=1}^{p}(\Psi_k(A))$ in B, defined by $g_{ij}(y) := u_i(y)^* u_j(y)$, $y \in V_i \cap V_j$, $i, j \in I$, satisfy $g_{ij} \cdot g_{jk} = g_{ik}$ on $V_i \cap V_j \cap V_k$ and hence $\{V_i, g_{ij}\}_{i \in I}$ defines an element in $H^1(Y, G_c)$. Since Y is contractible, $H^1(Y, G_c)$ reduces to the distinguished element. Therefore, we may assume that, for any $i \in I$ there exists a continuous map $v_i \colon V_i \to G$ such that $g_{ij}(y) = v_i(y) v_j(y)^*$, $y \in V_i \cap V_j$, $i, j \in I$. We define $u \colon Y \to U(B)$ by $u(y) := u_i(y) v_i(y)$, $y \in V_i$, $i \in 1$. Since $u_i(y)v_i(y) = u_j(y)v_j(y)$ for $y \in V_i \cap V_j$, $i, j \in I$, the map u is well defined and continuous.

It is easy to verify that

$$\Phi(f)(y) = \operatorname{Ad} u(y) \left(\bigoplus_{k=1}^{p} \Psi_k(f(z_k(y))) \right), \qquad f \in C(X) \otimes A, \ y \in Y.$$

3. Throughout this section X, Y will denote compact metric spaces (excepting Proposition 3.1) and A, B finite-dimensional C^*-algebras.

In this section we give a local description of homomorphisms from $C(X) \otimes A$ to $C(Y) \otimes B$ by considering separately the cases $X=$ point and $A = \mathbf{C}$. We also consider certain classes of inner equivalent homomorphisms.

3.1. PROPOSITION. *Consider $\Phi \in \operatorname{Hom}(A, C(Y) \otimes B)$, where Y is a compact space. For every $y' \in Y$ there exist $V \in \mathscr{V}(y')$, $\Psi \in \operatorname{Hom}(A, B)$ and $u \in C(V, U(B))$ such that*

$$\Phi(a)(y) = \operatorname{Ad} u(y)(\Psi(a)), \qquad a \in A, \ y \in V.$$

PROOF. It is enough to consider the case when $A = \bigoplus_{i=1}^{n} M_{k_i}$, $B = M_l$.

For every $y \in Y$, consider the unital finite-dimensional $*$-representation $A \ni a \to \Phi(a)(y) \in M_l$. Since this is a direct sum of irreducible $*$-representations, it follows that $(\exists)$ $p_i(y) \in \{0, 1, 2, \dots\}$ and $u'(y) \in U(l)$ such that

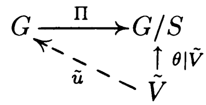
$$\Phi(a)(y) = \operatorname{Ad} u'(y) \left(\bigoplus_{i=1}^{n} a_i \otimes 1_{p_i(y)} \right)$$

for any $a = \bigoplus_{i=1}^{n} a_i \in \bigoplus_{i=1}^{n} M_{k_i}$ and $y \in Y$.

Since for any i, the map $Y \ni y \to \operatorname{tr}(\Phi(1_{k_i})(y)) = k_i \cdot p_i(y) \in \{0, 1, 2, \dots\}$ is continuous (here tr denotes the usual trace on M_l), $(\exists) V' \in \mathscr{V}(y')$ and $(\exists) \tilde{\Psi} \in \operatorname{Hom}(A, B)$ such that

$$\Phi(a)(y) = \operatorname{Ad} u'(y)(\tilde{\Psi}(a)), \qquad a \in A, \ y \in V'.$$

We denote $G := U(B)$, $S := U(\tilde{\Psi}(A)^c)$ (here $\tilde{\Psi}(A)^c$ is the relative commutant of $\tilde{\Psi}(A)$ in B), $G/S := \{gS \mid g \in G\}$ and $\Pi \colon G \to G/S$ the canonical map. G/S will be embedded into the topological space $\operatorname{Hom}(\tilde{\Psi}(A), B)$ by the formula $\Pi(g)(\tilde{\Psi}(a)) = \operatorname{Ad} g(\tilde{\Psi}(a))$, $g \in G$, $a \in A$. It follows that we can define a continuous map $\theta\colon V' \to G/S$ by $\theta(y)(\tilde{\Psi}(a)) = \Phi(a)(y)$, $y \in V'$, $a \in A$. Since S is a closed subgroup of the Lie group G, Π has smooth local sections. Thus, there is $\tilde{V} \in \mathscr{V}(y')$, $\tilde{V} \subset V'$ and $\tilde{u} \in C(\tilde{V}, G)$ such that the diagram:



commutes, which ends the proof.

3.2. We consider on $K(X)$ the topology given by the Pompeiu-Hausdorff metric $\tilde{d}$, defined by

$$\tilde{d}(F, G) := \max\left(\sup_{x\in F} d(x, G), \sup_{y\in G} d(F, y)\right),$$

$F, G \in K(X)$. Here d is a metric which gives the topology of X. Denote by $F(X)$ the set of all finite nonempty subsets of X. Then $F(X) \subset K(X)$ is endowed with the induced topology.

The proof of the following lemma is elementary and will be omitted.

LEMMA. *Let W be a metric space and a map $\theta\colon W \to F(X)$. The following assertions are equivalent*:

(1) $\theta \in C(W, F(X))$,

(2) *the map* $W \ni w \to \|f|_{\theta(w)}\| \in \mathbf{R}$ *is continuous for every* $f \in C(X)$.

3.3. Let T be a compact space and for each $t \in T$ let $E(t)$ be a C^*-algebra. We say that $((E(t))_{t\in T}, \Gamma)$ is a *continuous quasifield of C^*-algebras* if Γ is a continuity structure for T and the $\{E(t)\}$ in the sense of J. M. G. Fell [**6**], i.e., every $a \in \Gamma$ is a map defined on T such that $a(t) \in E(t)$ for any $t \in T$ and

(1) Γ is a $*$-algebra under the pointwise operations,

(2) $\overline{\{a(t) | a \in \Gamma\}} = E(t)$, $t \in T$,

(3) for any $a \in \Gamma$, the map $T \ni t \to \|a(t)\| \in \mathbf{R}$ is continuous.

Any continuous field of C^*-algebras [**3**] is a continuous quasifield.

Let $\mathscr{E}_i = ((E_i(t))_{t\in T}, \Gamma_i)$, $i = 1, 2$, be two continuous quasifields of C^*-algebras. We say that $\Psi = (\Psi_t)_{t\in T}$ is a *homomorphism from $\mathscr{E}_1$ to $\mathscr{E}_2$* if (1°) every Ψ_t is a $*$-homomorphism of C^*-algebras from $E_1(t)$ to $E_2(t)$; (2°) Ψ takes Γ_1 into Γ_2 (if we consider quasifields of unital C^*-algebras, each Ψ_t is assumed unital). We say that Ψ is *injective* if each Ψ_t is injective.

We denote by $\operatorname{Hom}(\mathscr{E}_1, \mathscr{E}_2)$ (resp. $\operatorname{Hom}_i(\mathscr{E}_1, \mathscr{E}_2)$) the set of all homomorphisms (resp., injective homomorphisms) from $\mathscr{E}_1$ to $\mathscr{E}_2$.

In the unital case we say that $\Psi^{(i)} = (\Psi_t^{(i)})_{t\in T} \in \operatorname{Hom}(\mathscr{E}_1, \mathscr{E}_2)$, $i = 1, 2$, are *inner equivalent*, written $\Psi^{(1)} \sim \Psi^{(2)}$, if there is $u \in \Gamma_2$ such that $u(t) \in U(E_2(t))$ and $\Psi_t^{(1)} = \operatorname{Ad} u(t) \circ \Psi_t^{(2)}$ for any $t \in T$.

3.4. Let B be a C^*-algebra, $B \simeq M_{n_1} \oplus M_{n_2} \oplus \cdots \oplus M_{n_k}$, $n := n_1 + n_2 + \cdots + n_k$, $\mathscr{F}_n(X) := \{F \in F(X) | F \text{ has at most } n \text{ elements}\}$.

For any $\theta \in C(Y, \mathscr{F}_n(X))$ consider $E_\theta(y) := C(\theta(y))$, $y \in Y$ (each $\theta(y)$ is a discrete topological space) and $\Gamma_\theta := \{Y \ni y \to f|_{\theta(y)} \in E_\theta(y) | f \in C(X)\}$. Using

Lemma 3.2, we see that $\mathscr{E}(\theta) := ((E_\theta(y))_{y\in Y}, \Gamma_\theta)$ is a continuous quasifield of C^*-algebras.

Let $C := \mathrm{Hom}(C(X), C(Y)\otimes B)$, $C_i := \mathrm{Hom}_i(C(X), C(Y)\otimes B)$ and let $\mathscr{F}$ be the constant continuous field on Y, of fibre B. We define a map

$$F\colon C \to \bigcup_{\theta\in C(Y,\mathscr{F}_n(X))} \mathrm{Hom}_i(\mathscr{E}(\theta), \mathscr{F})$$

by

$$F(\Phi) := (\Psi_{y,\Phi})_{y,Y} \in \mathrm{Hom}_i(\mathscr{E}(X_\Phi), \mathscr{F})$$

where $\Psi_{y,\Phi}(f|_{X_{y,\Phi}}) := \Phi(f)(y)$ for $f \in C(X)$, $y\in Y$ and $X_\Phi\colon Y \ni y \to X_{y,\Phi} \in \mathscr{F}_n(X)$ is continuous by virtue of Lemma 3.2.

PROPOSITION. *The map F is a bijection which induces in a canonical way a bijection of $C/\sim$ onto $\bigcup_{\theta\in C(Y,\mathscr{F}_n(X))}(\mathrm{Hom}_i(\mathscr{E}(\theta),\mathscr{F})/\sim)$.*

Moreover, F restricts to a bijection of C_i onto $\bigcup_{\theta\in\tilde{C}(Y,\mathscr{F}_n(X))} \mathrm{Hom}_i(\mathscr{E}(\theta),\mathscr{F})$ which induces a bijection of $C_i/\sim$ onto $\bigcup_{\theta\in\tilde{C}(Y,\mathscr{F}_n(X))}(\mathrm{Hom}_i(\mathscr{E}(\theta),\mathscr{F})/\sim)$, where $\tilde{C}(Y,\mathscr{F}_n(X)) := \{f\in C(Y,\mathscr{F}_n(X)) \mid \bigcup_{y\in Y} f(y) = X\}$.

PROOF. Consider $F(\Phi_i) = (\Psi^{(i)}_{y,\Phi_i})_{y\in Y}$, $i = 1, 2$, with $F(\Phi_1) = F(\Phi_2)$, that is, $\Psi^{(1)}_{y,\Phi_1} = \Psi^{(2)}_{y,\Phi_2}$, $X_{y,\Phi_1} = X_{y,\Phi_2}$ for any $y\in Y$. Then

$$\Phi_1(f)(y) = \Psi^{(1)}_{y,\Phi_1}(f|X_{y,\Phi_1}) = \Psi^{(2)}_{y,\Phi_2}(f|X_{y,\Phi_2}) = \Phi_2(f)(y)$$

for $f\in C(X)$, $y \in Y$; hence F is injective.

For the surjectivity of F consider $\Psi = (\Psi_y)_{y\in Y} \in \mathrm{Hom}_i(\mathscr{E}(\theta),\mathscr{F})$, where $\theta\in C(Y,\mathscr{F}_n(X))$ and define $\Phi\in C$ by $\Phi(f)(y) := \Psi_y(f|\theta(y))$, $f\in C(X)$, $y\in Y$. Using the definition of $X_{y,\Phi}$ $(y\in Y)$ and the fact that each Ψ_y is injective, we have $\|f|_{X_{y,\Phi}}\| = \|\Phi(f)(y)\| = \|f|\theta(y)\|$, $y\in Y$, which implies $X_{y,\Phi} = \theta(y)$ for any $y \in Y$. It follows that $F(\Phi) = \Psi$.

Finally, using 2.2(3) it follows that $F(C_i) = \bigcup_{\theta\in\tilde{C}(Y,\mathscr{F}_n(X))} \mathrm{Hom}_i(\mathscr{E}(\theta),\mathscr{F})$.

3.5. REMARK. Consider the continuous map $\varphi\colon \mathbf{T}\to\mathbf{T}$ given by $\varphi(y) := y^2$, $y\in\mathbf{T}$ $(:= \{y\in\mathbf{C} \mid |y| = 1\})$. Define $\theta\in C(\mathbf{T}, F_1(\mathbf{T}))$ by $\theta(y) := \{\varphi(y)\} = \{y^2\}$, $y\in\mathbf{T}$ and two continuous maps $f, g\colon \mathbf{T}\to\mathbf{C}$ by $f(y) = 1$, $g(y) = y$, $y\in\mathbf{T}$.

Then $f\in\Gamma_\theta$ and $g\cdot f\notin\Gamma_\theta$; thus $((E_\theta(y))_{y\in\mathbf{T}}, \Gamma_\theta)$ is not a continuous field of C^*-algebras (see [**3**, 10.1.9]).

3.6. Let $\varphi\colon X\to Y$ be a continuous surjective map such that $\varphi^{-1}(y)$ is a finite subset of X for any $y\in Y$ and the map $Y\ni y\to\varphi^{-1}(y)\in F(X)$ is continuous. This condition is satisfied if, for instance, φ is a covering map with a finite fibre.

Denote by $C(\varphi)$ the set of all φ-compatible $*$-homomorphisms from $C(X)\otimes A$ to $C(Y)\otimes B$ and by $C_i(\varphi)$ the set $C(\varphi)\cap\mathrm{Hom}_i(C(X)\otimes A, C(Y)\otimes B)$.

Let $\mathscr{E} := ((E(y))_{y\in Y}, \Gamma)$ be the continuous field of C^*-algebras given by $E(y) := C(\varphi^{-1}(y))\otimes A$, $y\in Y$, $\Gamma := \{Y\ni y\to f|\varphi^{-1}(y)\in E(y) \mid f\in C(X)\otimes A\}$. (To see that $\mathscr{E}$ is indeed a continuous field use Lemma 3.2 and standard partition of unity arguments.) Let $\mathscr{F}$ be the constant continuous field on Y, of fibre B. Define a map $G\colon C(\varphi)\to\mathrm{Hom}(\mathscr{E},\mathscr{F})$ by $G(\Phi) := (\Psi_y)_{y\in Y}$ where $\Psi_y(f|\varphi^{-1}(y)) := \Phi(f)(y)$, $f\in C(X)\otimes A$, $y\in Y$.

Using 2.5 we easily obtain the following:

PROPOSITION. *The map G is a bijection which induces a bijection from $C(\varphi)/\sim$ onto* $\mathrm{Hom}(\mathscr{E},\mathscr{F})/\sim$.

Moreover G maps $C_i(\varphi)$ onto $\mathrm{Hom}_i(\mathscr{E},\mathscr{F})$ *and induces a bijection from $C_i(\varphi)/\sim$ onto* $\mathrm{Hom}_i(\mathscr{E},\mathscr{F})/\sim$.

4. In this section we prove our main result concerning the stability under inductive limits of C^*-algebras of the form $C(X)\otimes A$ and isomorphisms of such C^*-algebras.

4.1. We first clarify the local structure of θ-compatible homomorphisms with $\theta(y)=\{\varphi(y)\}$ where $\varphi\colon Y\to X$ is continuous.

PROPOSITION. *Let X, Y be compact spaces, A, B finite-dimensional C^*-algebras, $\varphi\colon Y\to X$ a continuous map and consider* $\Phi\in\mathrm{Hom}(C(X)\otimes A, C(Y)\otimes B)$ *such that*

$$\Phi(g\otimes 1_A)=g\circ\varphi\otimes 1_B,\qquad g\in C(X).$$

Then, for each $y'\in Y$ there exist a neighborhood V of y', a continuous map $u\colon V\to U(B)$ and a $*$-homomorphism $\Psi\in\mathrm{Hom}(A,B)$ *such that*

$$\Phi(f)(y)=\mathrm{Ad}\,u(y)(\Psi(f(\varphi(y))))$$

for $f\in C(X)\otimes A$, $y\in V$.

PROOF. Fix $V\in\mathscr{V}(y')$, $\Psi\in\mathrm{Hom}(A,B)$ and $u\in C(V,U(B))$ given by Proposition 3.1 for the homomorphism $A\ni a\to\Phi(1_{C(X)}\otimes a)\in C(Y)\otimes B$. Then, for any $g\in C(X)$, $a\in A$ and $y\in V$ we have

$$\begin{aligned}\Phi(g\otimes a)(y)&=(\Phi(g\otimes 1_A)(y))\cdot(\Phi(1_{C(X)}\otimes a)(y))\\&=((g\circ\varphi)(y)\cdot 1_B)\cdot(\mathrm{Ad}\,u(y)(\Psi(a)))\\&=\mathrm{Ad}\,u(y)(\Psi(g\otimes a(\varphi(y)))),\end{aligned}$$

which completes the proof.

4.2. In the situation of the above proposition suppose that Y is connected. Then there are $\Psi\in\mathrm{Hom}(A,B)$, a proper open covering $(U_i)_{i\in I}$ of Y and $u_i\in C(U_i,U(B))$ such that

$$\Phi(f)(y)=\mathrm{Ad}\,u_i(y)(\Psi(f(\varphi(y))))$$

for $f\in C(X)\otimes A$, $y\in U_i$, $i\in I$. For $y\in Y$, denote by $(\Phi(C(X)\otimes A)(y))^c$ the relative commutant of $\Phi(C(X)\otimes A)(y)$ in B. Since for any y_1, $y_2\in Y$ there is a (inner) $*$-automorphism of B (depending on y_1 and y_2) which maps $\Phi(C(X)\otimes A)(y_1)$ onto $\Phi(C(X)\otimes A)(y_2)$, $(\Phi(C(X)\otimes A)(y_1))^c$ and $(\Phi(C(X)\otimes A)(y_2))^c$ are $*$-isomorphic and hence

$$U((\Phi(C(X)\otimes A)(y_1))^c)\simeq U((\Phi(C(X)\otimes A)(y_2))^c),\qquad y_1,y_2\in Y$$

(as topological groups). Assume also that $H^1(Y,U((\Phi(C(X)\otimes A)(y))^c)_c)$ is reduced to the distinguished element for some $y\in Y$ (and hence for all $y\in Y$).

PROPOSITION. $\Phi \sim \varphi^* \otimes \Psi$.

PROOF. Define continuous maps $g_{ij}\colon U_i \cap U_j \to G$, where G is the unitary group of the relative commutant of $\Psi(A)$ in B, by $g_{ij}(y) = u_i(y)^* u_j(y)$, $y \in U_i \cap U_j$, $i, y \in I$.

Since $g_{ij} \cdot g_{jk} = g_{ik}$ on $U_i \cap U_j \cap U_k$, $\{U_i, g_{ij}\}_{i \in I}$ defines an element in $H^1(Y, G_c)$. As $H^1(Y, G_c)$ is trivial, we may assume that for any $i \in I$ there is a continuous map $v_i\colon U_i \to G$ such that $g_{ij}(y) = v_i(y) v_j(y)^*$, $y \in U_i \cap U_j$, $i, j \in I$. Define $u\colon Y \to U(B)$ by $u(y) := u_i(y) v_i(y)$, $y \in U_i$, $i \in I$. Since $u_i(y)v_i(y) = u_j(y)v_j(y)$ for $y \in U_i \cap U_j$, $i, j \in I$, the map u is well defined and continuous, and we have $\Phi = \operatorname{Ad} u \circ (\varphi^* \otimes \Psi)$.

4.3. Now consider a system

$$C(X_1) \otimes A_1 \xrightarrow{\Phi_1} C(X_2) \otimes A_2 \xrightarrow{\Phi_2} \cdots,$$

where for each k, X_k is a compact space, A_k is a finite-dimensional C^*-algebra, Φ_k is an isometric $*$-homomorphism such that

$$\Phi_k(g \otimes 1_{A_k}) = g \circ \varphi_k \otimes 1_{A_{k+1}}, \qquad g \in C(X_k),$$

with $\varphi_k\colon X_{k+1} \to X_k$ a surjective continuous map. Let $X := \varprojlim(X_k, \varphi_k)$.

Assume that for any $k \geq 2$, X_k is connected and

$$H^1(X_k, U((\Phi_{k-1}(C(X_{k-1}) \otimes A_{k-1})(x))^c)_c)$$

is reduced to the distinguished element for some $x \in X_k$ (and hence for all $x \in X_k$). Here $(\Phi_{k-1}(C(X_{k-1}) \otimes A_{k-1})(x))^c$ is the relative commutant of

$$\Phi_{k-1}(C(X_{k-1}) \otimes A_{k-1})(x) \quad \text{in } A_k.$$

Then, by Proposition 4.2, for any $k \geq 1$ there exists $\Psi_k \in \operatorname{Hom}_i(A_k, A_{k+1})$ (unique, up to inner equivalence) such that $\Phi_k \sim \varphi_k^* \otimes \Psi_k$. Let $A := \varinjlim(A_k, \Psi_k)$.

We thus obtain the following:

THEOREM. *The C^*-algebra $\varinjlim(C(X_k) \otimes A_k, \Phi_k)$ is $*$-isomorphic to the (spatial) C^*-tensor product $C(X) \otimes A$.*

4.4. The isomorphism problem for the above considered inductive limits can be settled in certain cases by using the following result. We give a proof for the sake of the completeness.

PROPOSITION. *Let X, Y be compact spaces and A, B unital C^*-algebras with trivial centers. Then $C(X) \otimes A \simeq C(Y) \otimes B$ if and only if X and Y are homeomorphic and $A \simeq B$.*

PROOF. Suppose that $\Phi\colon C(X) \otimes A \to C(Y) \otimes B$ is a $*$-isomorphism. Since Φ maps $Z(C(X) \otimes A)$ onto $Z(C(Y) \otimes B)$, $C(X) \simeq C(Y)$, i.e., X and Y are homeomorphic.

Let m be a maximal ideal in $C(X)$ and let χ be the corresponding character of $C(X)$. We consider the surjective $*$-homomorphism $\chi \otimes \mathrm{id}_A\colon C(X) \otimes A \to \mathbf{C} \otimes A$. Since $\ker(\chi \otimes \mathrm{id}_A) = m \otimes A$, we have $A \simeq \mathbf{C} \otimes A \simeq C(X) \otimes A / m \otimes A$. But $\Phi(m \otimes 1_A) = m' \otimes 1_B$ with m' a maximal ideal in $C(Y)$, since Φ maps $C(X) \otimes 1_A$ $(= Z(C(X) \otimes A))$ onto $C(Y) \otimes 1_B$ $(= Z(C(Y) \otimes B))$. We have $A \simeq C(X) \otimes A / m \otimes A \simeq \Phi(C(X) \otimes A) / \Phi(m \otimes A) = C(Y) \otimes B / m' \otimes B \simeq B$, which completes the proof.

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