

Let $F \supset \mathbb{Q}$ be an algebraic number field containing an element t with $t^6 - 3t^3 + 1 = 0$. The map

$$x \mapsto X = \begin{pmatrix} A & B & C \\ 0 & 0 & 1 \\ D & E & -A \end{pmatrix},$$

$$y \mapsto Y = \begin{pmatrix} D & E & -A \\ 3(Bt - D) & -D & -Ct \\ 1 & 0 & 0 \end{pmatrix}.$$

where $A = -3t^4 + 8t$, $B = -4t^4 + 11t$, $C = 2t^3 - 6$, $D = -5t^5 + 14t^2$ and $E = -7t^5 + 19t^2$ extends to a representation of G'_1 over F .

$(XY)^2$ has trace 3 which implies that XY has infinite order. Indeed, if XY would be of finite order then $(XY)^2$, as a matrix of finite order, would be diagonalizable and, hence, having trace 3, it would be the identity matrix I which is not the case. Hence, in particular, G_1 is infinite and ab has infinite order in G_1 .

(b) For the computations for the second group G_2 we used the computer system *GAP* developed at Lehrstuhl D für Mathematik (J. Neubüser) at the RWTH Aachen. G_2 was found to contain a subgroup H of index 24 and generated by a , bab^{-1} , $b^{-1}abab^{-1}ab$, $b^{-1}ab^{-1}ab^{-1}abab^{-1}ababab$, $b^{-1}ab^{-1}abababab^{-1}ab^{-1}ab^{-1}abab$ and

$$b^{-1}ab^{-1}abab^{-1}ababab^{-1}ab^{-1}abab^{-1}abab.$$

The core N of H has index 13824. To get a presentation for N one constructs the coset table of N using the regular representation of G_2 on H . One then feeds the resulting coset table in the reduced Reidemeister-Schreier algorithm to get a presentation for N in 1208 generators and 5745 relators. This presentation can be reduced via Tietze transformations to a presentation with 7 generators and 148 relators of total length 1584. The latter presentation is manageable with the Todd-Coxeter algorithm to get the order of G_2 which is $2^{20} \cdot 3^4 \cdot 5$. In fact, G_2 is a normal product of $SL(2, 5)$ and a solvable group with amalgamated central subgroup of order 2. \square

Remarks. (1) The group G_2 is a counterexample to the conjecture at the end of Chapter I in [6]. Besides this group the conjecture seems to hold.

(2) There is a certain difference between G_1 and G_2 that is quite suggestive. In the case of G_1 there are certain subgroups for which the p -quotient algorithm revealed larger and larger p -factors. On the other hand if one takes G_2 and looks at various subgroups one does not get nilpotent factors of large class, all lower central series stop very soon. This fact was one of the reasons for suspecting that G_2 might be finite.

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