

**Meeting:** 1000, Albuquerque, New Mexico, SS 14A, Special Session on Braids and Knots

1000-20-201            **Lucas A Sabalka\*** (sabalka@math.uiuc.edu), 273 Altgeld Hall, Urbana, IL 61801, and **Daniel Farley** (farley@math.uiuc.edu), 273 Altgeld Hall, Urbana, IL 61801. *Braid Groups on Graphs.*

Configuration spaces of graphs arise naturally in problems about robotics and motion planning. Let  $G$  be any finite graph, and fix a natural number  $n$ . The *labelled configuration space*  $LC^nG$  is the  $n$ -fold Cartesian product of  $G$ , with the set  $\Delta = \{(x_1, \dots, x_n) \mid x_i = x_j \text{ for some } i \neq j\}$  removed. The *unlabelled configuration space*  $C^nG$  is the quotient of  $LC^nG$  by the natural action of the symmetric group. The fundamental group of  $LC^nG$  (respectively,  $C^nG$ ) is called the *pure braid group (respectively, the braid group) of  $G$  on  $n$  strands*. We apply a version of Morse theory to the spaces  $C^nG$  for any  $G$  and any  $n$ . As a result, we can compute presentations for the braid groups of an arbitrary tree for any number of strands. For any  $n$  and  $G$ , we show that  $C^nG$  strong deformation retracts on a subcomplex of dimension at most  $k$ , where  $k$  is the number of vertices of  $G$  having degree at least 3. (This last theorem was first proved by Ghrist, but from a different point of view.) Our methods provide a very good description of the critical cells of the space  $C^nG$ , which are vital to understanding its topology. (Received August 24, 2004)