

1007-05-170

**Nathan Linial** and **Isabella Novik\*** ([novik@math.washington.edu](mailto:novik@math.washington.edu)), Department of Mathematics, Box 354350, University of Washington, Seattle, WA 98195. *How neighborly can a centrally symmetric polytope be?*

A centrally symmetric (cs, for short)  $d$ -dimensional polytope  $P$  is  $k$ -neighborly if every set of its  $k$  vertices that does not contain two antipodal ones is the vertex set of a face of  $P$ . In contrast with the polytopes without symmetry, the neighborliness of cs polytopes appears to be quite restricted: a cs  $d$ -polytope with at least  $2(d+2)$  vertices cannot be more than  $\lfloor (d+1)/3 \rfloor$ -neighborly. This result is due to McMullen and Shephard (1968) who also conjectured that a cs  $d$ -polytope with  $2(d+n)$  vertices cannot be more than  $\lfloor (d+n-1)/(n+1) \rfloor$ -neighborly for all  $n \geq 3$ . Their conjecture was shown to be false by Halsey (1972) and then by Schneider (1975), but only for the case of  $d \gg n$ . We extend their result to all  $d$  and  $n$  by verifying that for every  $d$  and  $n$  there exists a cs  $d$ -polytope with  $2(d+n)$  vertices that is at least  $\lfloor Cd / \ln(1 + (n+d)/d) \rfloor$ -neighborly. (Here  $C > 0$  is an absolute constant independent of  $n$  and  $d$ .) (Received February 20, 2005)