Meeting: 1007, Santa Barbara, California, PUTINAR, Invited Address

1007-12-27 **Mihai Putinar*** (`mputinar@math.ucsb.edu`), Mathematics Department, University of California, Santa Barbara, CA 93106.
Positive polynomials, a hilbertian perspective. Preliminary report.

This will be a survey, following the historical path, of the structure theory of positive polynomials and its many recent and spectacular applications. We will start with number theoretic questions (a la Waring problem) and the early contributions of Minkowski and Hilbert. Then we will follow Hilbert's programme, culminating with Artin-Schreier's theory of real fields. The early developments of functional analysis (the spectral theorem and moment problems) will bring into focus the contributions of F.Riesz, M.Riesz and M.G. Krein. Tarski's novel ideas (his transfer principle, and the resulting elimination of quatifiers) will connect us to mathematical logic. The geometrization idea of real spectrum of an ordered ring (a la Grothendieck) will open the gate to the modern real algebraic geometry and its many facets (still fascinating and far from being completed). Returning to the Hilbert space interpretation, we will show how the spectral theorem for tuples of commuting operators can produce refined Positivestellensatze. Finally, we will show how these ideas have recently revolutionized optimization theory and control theory. The presentation will be non-technical, easily accessible to an upper division undergraduate student in mathematics or engineering. (Received December 11, 2004)