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The algebras  $\text{NCSym}_n$  and  $\text{Sym}_n$  ( $n \in \mathbb{N}_+$ ) are defined to be the  $\mathfrak{S}_n$ -invariants inside  $\mathbb{Q}\langle A_n \rangle$  (resp.  $\mathbb{Q}[X_n]$ ), the polynomial functions on a noncommutative alphabet  $A_n$  (resp. commutative,  $X_n$ ) of cardinality  $n$ . The abelianization ( $a_i \mapsto x_i$ ) realizes  $\text{Sym}_n$  as a quotient of  $\text{NCSym}_n$ . Here, we view it as a subspace. We realize  $\text{Sym}_n$  as the  $\mathfrak{S}_n$ -invariants inside  $\text{NCSym}_n$  for a second, natural action of the symmetric group on  $\text{NCSym}_n$  and describe the coinvariants explicitly. Some surprising identities on the ordinary generating function for the Bell numbers appear as an immediate corollary. In case  $n = \infty$ , we obtain new information on the (Hopf) algebraic structure of  $\text{NCSym}_n$ .

Time permitting, we outline similar results for Hivert's  $r$ - $\text{QSym}_n$  algebras ( $r, n \in \mathbb{N}_+ \cup \{\infty\}$ ) and their noncommutative analogues. The algebra  $\text{Sym}_n$  and Gessel's quasisymmetric functions appear at the extremal values of  $r$ . (Received July 06, 2006)