1022-05-91
Felix Lazebnik* (lazebnik@math.udel.edu), Department of Mathematical Sciences, University of Delaware, Newark, DE 19716, Oleg Pikhurko (pikhurko@cmu.edu), Department of Mathematical Sciences, Carnegie Mellon University, Pittsburgh, PA 15213, and Andrew J.
Woldar (andrew.woldar@villanova.edu), Department of Mathematical Sciences, Villanova University, Villanova, PA 10085. Maximum Number of Colorings of (2k, k²)-Graphs.

Let \mathcal{F}_{2k,k^2} consist of all simple graphs on 2k vertices and k^2 edges. For a simple graph G and a positive integer λ , let $P_G(\lambda)$ denote the number of proper vertex colorings of G in at most λ colors, and let $f(2k, k^2, \lambda) = \max\{P_G(\lambda) : G \in \mathcal{F}_{2k,k^2}\}$. Let $K_{k,k}$ denote the complete bipartite graph with each partition having k vertices. We prove that $f(2k, k^2, 3) = P_{K_{k,k}}(3)$ and $K_{k,k}$ is the only extremal graph. We also prove that $f(2k, k^2, 4) = (6 + o(1))4^k$ as $k \to \infty$. (Received September 10, 2006)