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**Adam J Hammett\*** (hammett@math.ohio-state.edu), 231 W. 18th Avenue, Columbus, OH 43210, and **Boris G Pittel**, 231 W. 18th Avenue, Columbus, OH 43210. *On the Likelihood of Comparability in Bruhat Order.*

Two permutations of  $[n] := \{1, 2, \dots, n\}$  are comparable in the *Bruhat order* if one can be obtained from the other by a sequence of transpositions decreasing the number of inversions. We show that the total number of pairs of permutations  $(\pi, \sigma)$  with  $\pi \leq \sigma$  is of order  $(n!)^2/n^2$  at most. Equivalently, if  $\pi, \sigma$  are chosen uniformly at random and independently of each other, then  $Pr\{\pi \leq \sigma\}$  is of order  $n^{-2}$  at most. By a direct probabilistic argument we prove  $Pr\{\pi \leq \sigma\}$  is of order  $(0.708)^n$  at least, so that there is currently a wide qualitative gap between the upper and lower bounds.

For the *weak* Bruhat order “ $\preceq$ ” – when only adjacent transpositions are admissible – we use a non-inversion set criterion to prove that  $P_n^* := Pr\{\pi \preceq \sigma\}$  is submultiplicative, thus showing existence of  $\rho = \lim \sqrt[n]{P_n^*}$ . We demonstrate that  $\rho$  is 0.362 at most. Moreover, we prove the lower bound  $\prod_{i=1}^n (H(i)/i)$  for  $P_n^*$ , where  $H(i) := \sum_{j=1}^i 1/j$ . In light of numerical experiments, we conjecture that for each order the upper bound is qualitatively close to the actual behavior. (Received August 22, 2006)