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Frelinghuysen Rd, Piscataway, NJ 08854. *On the singularity probability of discretely random
complex matrices.*

Let n be a large integer and M_n be a random matrix whose entries are independent random variables taking values in the complex numbers. For constants $0 < q \leq p < 1$ and a constant integer r , we will define a property (p, q) -bounded of exponent r and prove that if the entries of M_n satisfy this property, then the probability that M_n is singular is at most $(p^{1/r} + o(1))^n$. Our work generalizes that of Kahn, Komlós, and Szemerédi and that of Tao and Vu. In the special case where the entries of M_n take the values $+1, -1$ each with probability $1/2$, our result proves that the probability that M_n is singular is at most $\left(\frac{1}{\sqrt{2}} + o(1)\right)^n$, improving on the previous best known upper bound of $\left(\frac{3}{4} + o(1)\right)^n$ proved by Tao and Vu. We present a few other corollaries of our main result, including proving that the $\left(\frac{1}{\sqrt{2}} + o(1)\right)^n$ bound holds for any random complex matrix M_n so long as the distribution of each entry is symmetric and a simple uniformity condition is satisfied. We also show that our main result can be extended to the case where M_n has $f \leq O(\ln n)$ rows containing fixed, non-random entries. (Received August 06, 2007)